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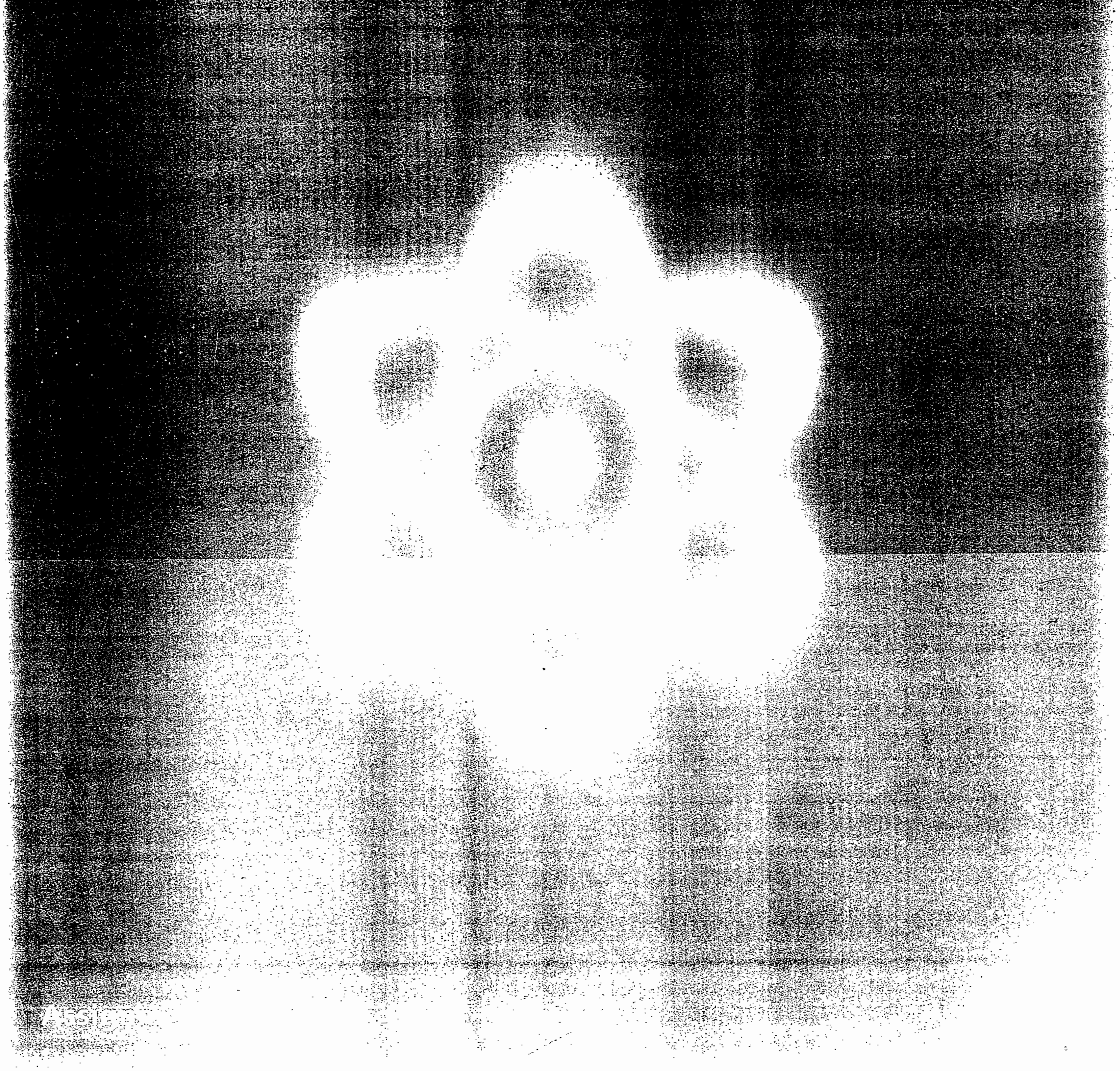

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ATOMIC & MOLECULAR PHYSICS

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$1\,A^{\circ} = 10^{-10}$ m

$m_e = 9.1 \times 10^{-3}$[illegible] = [illegible]

K = Boltzmann constant → $K_B = 1.38 \times 10^{-23}$

Speed of e in [illegible] orbit [illegible] 137

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Assignment-1 : Atomic Spectra (I)

1. An electron is initially in the n = 3 state and the average lifetime of this state is 10^{-8} sec. The number of revolution that the electron takes before making a transition to the n = 2 state, is
(a) 2.4×10^{4} (b) 2.4×10^{5} (c) 2.4×10^{6} (d) 2.4×10^{7}

2. The wavelength of seventh line of the Balmer series of hydrogen atom is
[✓ (a) 384 nm] (b) 424 nm (c) 456 nm (d) 512 nm

3. The energy required to remove an electron from a singly-ionized Helium atom which in the n = 8 state, is
(a) 0.213 eV (b) 0.51 eV [✓ (c) 0.85 eV] (d) 1.11 eV

4. The momentum of the photon emitted by a hydrogen atom making a direct transition from an excited state n = 10 to the ground state, is
(a) $2.8\times10^{-27}\,kg\,m\,s^{-1}$ [✓ (b) $7.2\times10^{-27}\,kg\,m\,s^{-1}$] (c) $7.2\times10^{-26}\,kg\,m\,s^{-1}$ (d) $2.8\times10^{-26}\,kg\,m\,s^{-1}$

5. An accelarated beam of electrons bombards a sample of hydrogen and the first line of Paschen series is emiited. The electrons must have been accelarated through a minimum potential difference of
[✓ (a) 12.8 V] (b) 14.1 V (c) 16.7 V (d) 19.1 V

6. Suppose E_H be the binding energy of hydrogen atom and E_p be the binding energy of the positronium atom which consists of a positron and an electron. Then, which of the following statements is true?
(a) $E_p = E_H$ (b) $E_p = \frac{1}{3}E_H$ [✓ (c) $E_p = \frac{1}{2}E_H$] (d) $E_p = 2E_H$

7. Suppose λ_H be the spectral wavelength of hydrogen atom and λ_P be the spectral wavelength of the positronium atom which consists of a positron and an electron. Then, which of the following statements is true?
(a) $\lambda_P = \lambda_H$ [✓ (b) $\lambda_P = 2\lambda_H$] (c) $\lambda_P = \frac{1}{2}\lambda_H$ (d) $\lambda_P = \frac{1}{3}\lambda_H$

A μ^- meson (Charge -e, Mass $207m_e$) can be captured by a proton to form a hydrogen-like atom muonic atom.

8. The radius of the first Bohr orbit is
[✓ (a) 0.285 pm] (b) 0.345 pm (c) 0.415 pm (d) 455 pm

9. The binding energy of the atom is
(a) +1750 eV (b) +2140 eV [✓ (c) -2530 eV] (d) +3120 eV

10. The first line of the Balmer series is
[✓ (a) 6.5 A^0] (b) 18.3 A^0 (c) 25.3 A^0 (d) 35.3 A^0

11. A mixture of ordinary hydrogen and tritium is excited and its spectrum is studied. The shift in wavelength for the H_α lines of the two kinds of hydrogen, is
[✓ (a) 2.4 A^0] (b) 3.3 A^0 (c) 5.4 A^0 (d) 6.7 A^0

12. In a transition to a state of excitation energy 10.19 eV, an hydrogen like atom emits a 4890 A^0 photon. The binding energy of the initial state is
(a) 7.66 eV (b) 8.92 eV (c) 10.19 eV (d0 13.6 eV

13. The temparature will the average molecular kinetic energy in the gaseous hydrgen be equal to the binding energy of the hydrogen atom, is
(a) $10^4 K$ (b) $10^5 K$ (c) $10^6 K$ (d) $10^7 K$

14. In the hydrogen spectrum, the ratio of the wavelengths for Lyman-α radiation to Balmer-α radiation is
(a) 5/48 (b) 1/3 (c) 5/27 (d) 3

15. The wavelength of one of the visible lines in the spectrum of the hydrogen atom is approximately 6560 A^0. It is known that this corresponds to a transition between the states with principal quantum number n = 2 and n = 3. The frequency corresponding to the transition from the ground state to the first excited state will be
(a) $4.56\times10^{14} Hz$ (b) $1.65\times10^{15} Hz$ (c) $2.47\times10^{15} Hz$ (d) $3.29\times10^{15} Hz$

16. Consider the spectral line resulting from the transition n = 2 and n = 1 in the atoms and ions given below. The shoertest wavelength will be produced by
(a) Hydrogen atom (b) Deutron atom
(c) Singly ionized helium (d) Doubly ionized lithium

17 A gas of monochromatic hydrogen is bombarded with a stream of electrons that have been accelerated from rest through a potenial difference 12.75 Volts. In the emission spectra one cannot observe
(a) Lymam series (b) Balmer Series (c) Paschen Series (d) Pfund Series

18. If the principal quantum number and the azimuthal quantum number in the relativistic model of the atom are 3 and 1 respectively, then the magnitude of the semi-minor axis 'b' in terms of the semi major axis 'a' is given by
(a) b = a/3 (b) b = 2a/3 (c) b = a/2 (d) b = a

19. Given that the ground state energy of the hydrogen atom is -13.6 eV, then the ground state energy of positronium (which is a bound state of an electron and a positron) is **[NET Dec. 2011]**
(a) 6.8 eV (b) -6.8 eV (c) -13.6 eV (d) 13.6 eV

20. A muon $\left(\mu^-\right)$ from cosmic rays is trapped by a proton to form a hydrogen-like atom. Given that a muon is approximately 200 times heavier than an electron, the longest wavelength of the spectral line (in the analogoue of the Lyman series) of such an atom will be **[NET June 2013]**
(a) 5.62 A^0 (b) 6.67 A^0 (c) 3.75 A^0 (d) 13.3 A^0

21. If R_1 is the value of the Rydeberg constant assuming the mass of the nucleus to be infinitely large compared to that of an electron and R_2 is the value of the Rydeberg constant taking the nuclear mass to be 7500 times the mass of the electron, then the ratio $\frac{R_2}{R_1}$ is **[GATE 2002]**
(a) a little less than unity (b) a little more than unity
(c) infinitely small (d) infinitely large

22. If the wavelength of the first line of the Balmer series in the hydrogen spectrum is λ, then the wavelength of the first line of the Lyman series is **[GATE 2002]**
(a) $\frac{27}{5}\lambda$ (b) $\frac{5}{27}\lambda$ (c) $\frac{32}{27}\lambda$ (d) $\frac{27}{32}\lambda$

23. Consider the Bohr model of the hydrogen atom. If α is the fine structure constant, then the velocity of the electron in its lowest orbit is **[JEST 2012]**
(a) $\frac{c}{1+\alpha}$ (b) $\frac{c}{1+\alpha^2}$ or $(1-\alpha)c$ (c) $\alpha^2 c$ (d) αc

24. The binding energy of the hydrogen atom (electron bound to proton) is -13.6 eV. The binding energy of positronium (electron bound to positron) is **[JEST 2012]**
(a) 13.6/2 eV (b) 13.6/1810 eV (c) 13.6× 1810 eV (d) 13.6 × 2 eV

25. If a proton were ten times lighter, then the ground state energy of the electron in a hydrogen atom would be B.E → less **[JEST 2013]**

(a) less [✓ (b) more] (c) same

(d) less, more or equal depending on the electron mass

26. A hydrogen atom in its ground state is collided with an electron of kinetic energy 13.377 eV. The maximum factor by which the radius of the atom would increase is **[JEST 2014]**

(a) 7 (b) 8 [✓ (c) 49] (d) 64

27. If a proton were ten times lighter, then the ground state energy of the electron in a hydrogen atom would have been **[JEST 2014]**

(a) less [✓ (b) more]

(c) same (d) depends on the electron mass

28. Given that the ionization energies of Hydrogen $\left({}^{1}H\right)$ and Lithium $\left({}^{3}Li\right)$ are 13.6 eV and 5.39 eV respectively. The effective nuclear charge experienced by the valence electron of the ${}^{3}Li$ atom may be estimated in terms of proton charge e as **[TIFR 2011]**

(a) 3.00 e (b) 1.59 e [✓ (c) 1.26 e] (d) 0.63 e

29. Consider the high excited states of a hydrogen atom corresponding to large values of the principal quantum number (n>>1). The wavelength λ of a photon emitted due to an electron undergoing a transition between two such states with consecutive values of n $\left(i.e.\ \psi_{n+1} \rightarrow \psi_{n}\right)$ is related to the wavelength λ_α of the K_α line of hydrogen by **[TIFR 2012]**

(a) $\lambda = \frac{n^3}{8}\lambda_\alpha$ [✓ (b) $\lambda = \frac{3n^3}{8}\lambda_\alpha$] (c) $\lambda = n^2\lambda_\alpha$ (d) $\lambda = \frac{4}{n^2}\lambda_\alpha$

30. The velocity of an electron in the ground state of a hydrogen atom is v_H. If v_p be the velocity of an electron in the ground state of positronium, then **[TIFR 2013]**

[✓ (a) $v_p = v_H$] (b) $v_p = 2v_H$ (c) $v_p = \frac{v_H}{2}$ (d) $v_p = \sqrt{2}v_H$

31. A sample of ordinary hydrogen $\left({}_{1}^{1}H\right)$ gas in a discharge tube was seen to emit the usual Balmer spectrum. On careful examination, however, it was found that the H_α line in the spectrum was split into two fine lines, one an intense line at 656.28 nm and the other a faint line at 656.04 nm. From this, one can conclude that the gas sample had a small impurity of **[TIFR 2015]**

[✓ (a) ${}_{1}^{3}H$] (b) ${}_{1}^{2}H$ (c) ${}_{2}^{4}He$ (d) H_2O

32. The Bohr model gives the value for the ionisation potential of Li^{2+} ion as

(a) 13.6 eV (b) 27.2 eV (c) 40.8 eV [✓ (d) 122.4 eV]

33. If the wavelength of the first line of the Balmer series in the hydrogen spectrum is λ, then the wavelength of the first line of the Lyman series is :

(a) $(27/5)\lambda$ [✓ (b) $(5/27)\lambda$] (c) $(32/27)\lambda$ (d) $(27/32)\lambda$

34. The radius of a hydrogen atom in its ground state is: 0.53 Å

(a) 10^{-4} cm (b) 10^{-6} cm [✓ (c) 10^{-8} cm] (d) 10^{-10} cm

35. The electron in a hydrogen atom with a radius equal to first Bohr radius has a velocity equal to

(a) $\frac{C}{5}$ (b) $\frac{C}{10}$ [✓ (c) $\frac{C}{137}$] (d) $\frac{C}{8}$

36. The total energy of the electron in the nth orbit of hydrogen atom is:

(a) $\frac{e^2}{4\pi\varepsilon_0 r_n}$ (b) $\frac{e^2}{4\pi\varepsilon_0 r^2_n}$ (c) $\frac{e^2}{8\pi\varepsilon_0 r_n}$ [✓ (d) $-\frac{e^2}{8\pi\varepsilon_0 r_n}$]

37. The kinetic energy of an electron in atom is:
[✓ (a) Half of its potential energy]
(b) Twice its potential energy
(c) Equal to its potential energy
(d) Thrice its potential energy

38. The numerical value of the radius of the first orbit of hydrogen as
(a) 0.529 nm
(b) 0.0529 Å
(c) 5.29 Å
[✓ (d) 0.0529 nm]

39. The total energy of the electron in hydrogen atom is:
(a) $-(13.6/n)$eV
(b) $(13.6/n)$eV
[✓ (c) $-(13.6/n^2)$eV]
(d) $(13.6/n^2)$eV

40. When an electron jumps from the fourth orbit to the second orbit, one gets
(a) The first line of P fund series
(b) Second line of Lyman series
(c) Second line of Paschen series
[✓ (d) Second line of Balmer series]

41. The Bohr formula when utilised to calculate ionization energy. He is found to yield a result higher than experimental value because of
[✓ (a) Electron- electron interaction]
(b) Spin orbit interaction
(c) Spin-spin interaction
(d) None of above

42. The first line of the Lyman series of a ten times ionized Na atom has a wavelength of about
(a) 0.1 Å
(b) 1Å
(c) 10Å
(d) 100Å

43. The range of frequencies of the lines of visible region in hydrogen spectrum is:
(a) $6000 \times 10^{14} S^{-1}$
(b) $600 \times 10^{14} S^{-1}$
(c) $60 \times 10^{14} S^{-1}$
[✓ (d) $6 \times 10^{14} S^{-1}$]

44. The shortest wavelength of Balmer series in hydrogen spectrum is given by the equation
(a) $\frac{1}{\lambda} = R_H\left(\frac{1}{4} - \frac{1}{9}\right)$
(b) $\frac{1}{\lambda} = R_H\left(\frac{1}{2} - \frac{1}{\infty}\right)$
(c) $\frac{1}{\lambda} = R_H\left(1 - \frac{1}{\infty}\right)$
[✓ (d) $\frac{1}{\lambda} = R_H\left(\frac{1}{4} - \frac{1}{\infty}\right)$]

45. If v is the velocity of the electron in a stationary orbit of radius r, the orbital frequency of the electron is:
[✓ (a) $\frac{\upsilon}{2\pi r}$]
(b) $\frac{\upsilon}{r}$
(c) $\frac{\upsilon}{\pi r}$
(d) $\frac{2\pi r}{\upsilon}$

46. Consider a spectal line resulting from the transition $n = 2 \rightarrow n = 1$ in the atoms and ions given below. The shortest wavelength produced by
(a) singly ionised helium
(b) hydrogen atom
[✓ (c) doubly ionised lithium]
(d) deuterium atom

47. The length of the semi-major axis of an electron in an elliptical orbit is determined
[✓ (a) only by the principal quantum number]
(b) only by the azimuthal quantum number
(c) both by principal quantum number and azimuthal quantum numbers
(d) none of these

48. When the azimuthal quantum number n_ϕ is zero
[✓ (a) The ellipse reduces to a straight line]
(b) The ellipse reduces into a circle
(c) The ellipse reduces to a parabola
(d) none of these

49. The doublets observed in alkali spectra are due to
(a) Screening of the K-electrons
[✓ (b) Spin-orbit interaction of the electron]
(c) Pressure of isotopes
(d) None of these

50. In outer atomic shells of atoms 4s level is filled first by electrons than 3d level because
[✓ (a) 4s electrons are less screened from the nucleus than 3d electrons]
(b) Spin-spin interaction dominates for 3d level
(c) Spin orbit intersection between the electrons in the different levels reduces energy of 4s electrons below that of 3d electrons.
(d) None of these

Higher l value orbital is less penetrating
4s > 3d
3d is less penetrating.

51. Two states of an atom have definite parities. An electric dipole transition between these states is:
(a) Allowed if both the states have even parity.
(b) Allowed if both the states have odd parity.
[✓ (c)] Allowed if the two states have opposite parities.
(d) Not allowed unless a static electric field is applied.

52. Let E_1, E_2, E_3 be the first three energy levels of a hydrogen atom. Consider the ratio $(E_3 - E_2)/(E_2 - E_1)$. Neglecting the fine structure condition this ratio is approximately equal to

(a) $\frac{27}{5}$ (b) $\frac{1}{27}$ (c) $\frac{27}{4}$ (d) $\frac{5}{27}$

53. The shortest wavelength in the Balmer series of the Hydrogen atom occurs due to
(a) transition from the 3s to 2s
[✓ (b)] ionization from the 2s level
(c) ionization form th 1s level
(d) transition from 2s to 1s level

54. A hydrogen atom is in the 2p state. To what state or states can it go by radiating a photon in an allowed transition? $\Delta l \neq 0$

(a) 1s, 2s, 3p, 3d (b) 1s, 2s, 3s, 3p (c) 2s, 3p, 3d (d) 1s, 2s

55. An atom in an excited sate $\psi_{n\ell m}$ can make a radiative transition to a state $\psi_{n'\ell'm'}$ if

(a) $\int_{-\infty}^{\infty} \psi^*_{n'\ell'm'}\, \bar{r}\, \psi_{n\ell m}\, dV \neq 0$
(b) $\int_{-\infty}^{\infty} \psi^*_{n'\ell'm'}\, \bar{r}^2\, \psi_{n\ell m}\, dV \neq 0$

(c) $\int_{-\infty}^{\infty} \psi^*_{n'\ell'm'}\, \psi_{n\ell m}\, dV = \delta_{nn'}\delta_{\ell\ell'}\delta_{mm'}$
(d) $\int_{-\infty}^{\infty} \psi^*_{n'\ell'm'}\, \bar{r}\, \psi_{n\ell m}\, dV = 0$

56. The longest wavelength in the spectral series of the Hydrogen atom occurs due to
(a) transition from $n = \infty$ to $n = 1$
(b) ionization from 1s level
(c) transition from 2s to 1s level
[✓ (d)] ionization from $n = 100$

57. If E_H is the binding energy of the electron in the Bohr model of the Hydrogen atom, and both the charge and mass of the electron are doubled, the binding energy becomes $E_H \propto m e^4 \propto 2 \times (2)^4 = 32$

[✓ (a)] $32\,E_H$ (b) $2\,E_H$ (c) $4\,E_H$ (d) $16\,E_H$

Statement for Linked Answer type Q.58 and Q.59.
Positronium is an atom formed by an electron and a positron. The mass of a positron is the same as that of an electron and its charge is equal in magnitude but opposite in sign to that of an electron. The positronium atom is thus similar to the hydrogen atom with the positron replacing the proton.

58. The binding energy of a positronium atom is:
(a) 13.6 eV [✓ (b)] 6.8 eV (c) 27.2 eV (d) 3.4 eV

59. If a positronium atom makes a transition from the state with n = 3 to a state with n = 2, the energy of the photon that is emitted in this transition is closest to
(a) 1.88 eV [✓ (b)] 0.94 eV (c) 1.13 eV (d) 2.27 eV

60. Which of the following transitions is NOT allowed in the case of an atom, according to the electric dipole radiation selection rule? **[GATE 2016]**

[✓ (a)] $2s - 1s$ (b) $2p - 1s$ (c) $2p - 2s$ (d) $3d - 2p$

61. For an electron in hydrogen atom, the states are characterized by the usual quantum numbers n, l, m_l. The electric dipole transition between any two states requires that
(a) $\Delta\ell = 0, \Delta m_\ell = 0, \pm 1$
(b) $\Delta\ell = \pm 1, \Delta m_\ell = \pm 1, \pm 2$
[✓ (c)] $\Delta\ell = \pm 1, \Delta m_\ell = 0, \pm 1$
(d) $\Delta\ell = \pm 1, \Delta m_\ell = 0, \pm 2$

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Assignment-2 : Atomic Spectra (II)

1. The two possible orientations of $\vec{S}$ with respect to an external magnetic field $\vec{B}$ along z-axis, are

(a) $\cos^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$ (b) $\cos^{-1}\left(\pm\frac{1}{\sqrt{3}}\right)$ (c) $\cos^{-1}\left(\pm\frac{1}{3}\right)$ (d) $\cos^{-1}\left(\pm\frac{1}{2}\right)$

2. The magnitude of magnetic dipole moment corresponding to spin of an electron is

(a) μ_B (b) $2\mu_B$ (c) $\sqrt{2}\mu_B$ (d) $\sqrt{3}\mu_B$

3. A beam of electrons (each of rest mass m, charge e) enters in a uniform magnetic field B along z-axis. The energy separation between the electrons having spin parallel and anti-parallel with magnetic field, is

(a) $\frac{eB\hbar}{m}$ (b) $\frac{eB\hbar}{2m}$ (c) $\frac{eB\hbar}{4m}$ (d) 0

4. The number of possible spectroscopic terms of a one electron atom corresponding to n = 4 is

(a) 3 (b) 5 (c) 7 (d) 9

5. The angle between the orbital angular momentum and spin angular momentum for the term $^2D_{3/2}$ is

(a) 45^0 (b) 62^0 (c) 90^0 (d) 135^0

6. Which of the following spectroscopic term is not allowed?

(a) $^2D_{3/2}$ (b) $^2F_{5/2}$ (c) $^2P_{3/2}$ (d) $^2D_{1/2}$

7. Consider the state in which $l = 4, s = 1/2$. The orientation of total angular momentum w.r.t z-axis for the state with largest possible j, m_j, is

(a) 25.2^0 (b) 39.8^0 (c) 51.2^0 (d) 74.8^0

8. Which of the following statements is NOT CORRECT?
The difference of spin-orbit correction to energy between the spin-down and spin-up electron in a hydrogen like atom will
(a) increase with increase in atomic number.
(b) decrease with the principal quantum number
(c) increase with principal quantum number
(d) decrease with orbital quantum number

9. The number of allowed transitions in the fine structure of H_α line is

(a) 3 (b) 4 (c) 5 (d) 6

10. Find the gound state term of the following atoms:

(i) $^{14}_{7}N$ (ii) $^{12}_{6}C$ (iii) $^{16}_{8}O$ (iv) $^{24}_{11}Na$ (v) C^{+} (vi) Na^{+}

The electronic configuration of $_{25}Mn$ atom is $1s^2 2s^2 2p^6 3s^2 3p^6 3d^5 4s^2$.

11. The ground state term of Mn atom will be

(a) $^6S_{5/2}$ (b) $^6P_{5/2}$ (c) $^1S_0$ (d) $^3P_0$

12. The ground state term of Mn^{3+} ion will be

(a) $^5D_0$ (b) $^5D_4$ (c) $^6S_{5/2}$ (d) $^6P_{5/2}$

13. If the doublet splitting of the first excited state $2^2D_{5/2} - 2^2D_{3/2}$ of He^+ is 3.67 cm^{-1}, then the corresponding separation of H atom is
(a) 0.23 cm^{-1} (b) 0.36 cm^{-1} (c) 0.52 cm^{-1} (d) 0.76 cm^{-1}

14. Sodium atom has 11 electrons. If the sequence in which the energy levels are filled in 1s, 2s, 2p, 3s, 3p, 4s, 3d...., the ground state of sodium is:
(a) ${}^3p_{1/2}$ (b) ${}^2p_{1/2}$ (c) ${}^1p_{1/2}$ (d) ${}^2s_{1/2}$

15. There are four electrons in the 3*d* shell of an isolated atom. The total magnetic moment of the atom in units of Bohr magneton is ___0___. **[GATE 2016]**

16. The number of spectroscopic terms resulting from the L.S. coupling of a 3*p* electron and a 3*d* electron is ___12___ **[GATE 2016]**

17. The orbital quantum numbers of two atomic electrons are $\ell_1 = 1$ and $\ell_2 = 2$. The possible values of the total angular momentum quamtum number J under LS coupling are
(a) –1, 1 only (b) 1, 2, 3 only (c) 0, 1, 2, 3, 4 (d) –2, –1, 0, 1, 2

18. Which of the following states exist?
(a) $2\,{}^2P_1$ (b) $2\,{}^2P_{3/2}$ (c) $2\,{}^2P_{5/2}$ (d) $2\,{}^2P_{7/2}$

19. Under the LS coupling scheme, the possible spectral terms ${}^{2s+1}L_j$ for the electronic configuration 2s3s are
(a) ${}^2S_{1/2}, {}^2P_{3/2}, {}^2P_{1/2}$ (b) ${}^1S_0, {}^3P_1$ (c) ${}^1S_0, {}^3S_1$ (d) ${}^3S_0, {}^3S_1$

20. Which of the following is the spectroscopic ground state for Mn^{3+} ions of electronic configuration $1s^2 2s^2 2p^6 3s^2 3p^6 3d^4$ predicted by Hund's rule?
(a) ${}^5D_0$ (b) ${}^5D_4$ (c) ${}^5D_3$ (d) ${}^5D_2$

21. Possible values of the total angular momentum quantum number of a single f-electron are
(a) $j = \frac{7}{2}, \frac{5}{2}$ (b) $j = \pm 3, 2, 1, 0$ (c) $j = \pm\frac{7}{2}, \pm\frac{5}{2}$ (d) None of these

22. The electronic configuration $(np)^2$ can have only which one of the following term
(a) ${}^2D, {}^3P$ or ${}^1S$ (b) ${}^1D, {}^3P$ or ${}^1S$ (c) ${}^3D, {}^3P$ or ${}^2S$ (d) None of these

23. Which of the following has the order of increasing energy?
(a) ${}^1D_2, {}^3D_2, {}^3F_2$ (b) ${}^3F_2, {}^3D_2, {}^1D_2$ (c) ${}^3D_2, {}^3F_2, {}^1D_2$ (d) ${}^1D_2, {}^3F_2, {}^3D_2$

24. The electronic configuration of the ground state of the Na atom is ${}^2S_{1/2}$. This implies that
(a) $S = 2,\ L = 0,\ J = 2$ (b) $S = 0, L = 1/2,\ J = 1/2$
(c) $S = 1/2,\ L = 0,\ J = 1/2$ (d) $S = 0, L = 2, J = 2$

25. A proton and a neutron are both subject to a uniform magnetic field. Which of the following is true?
(a) Both particles undergo Larmor precession because they have non-zero intrinsic magnetic moments.
(b) The neutron does not precess because its intrinsic magnetic moment is zero.
(c) The proton does not precess because its intrinsic magnetic moment is zero.
(d) Both the particles precess and the direction of the precession is the same for the two particles.
magnetic moment of neutron is –ve

26. The term symbol for a particular atomic state is ${}^4P_{5/2}$. The values of S (spin angular momentum), L (orbital angular momentum), J (total angular momentum) of the state are
(a) $S = \frac{3}{2}, L = 0, J = \frac{3}{2}$ (b) $S = \frac{1}{2}, L = 1, J = \frac{3}{2}$
(c) $S = \frac{1}{2}, L = 2, J = \frac{5}{2}$ (d) $S = \frac{3}{2}, L = 1, J = \frac{5}{2}$

a → spin orbit coupling constant

27. The spin-orbit interaction in an atom is given by $H = a\vec{L}.\vec{S}$, where $\vec{L}$ and $\vec{S}$ denote the orbital and spin angular momenta respectively, of the electron. The splitting between the levels $^2P_{3/2}$ and $^2P_{1/2}$ is **[NET June 2012]**

higher J value

ΔE

(a) $\frac{3}{2}a\hbar^2$ (b) $\frac{1}{2}a\hbar^2$ (c) $3a\hbar^2$ (d) $\frac{5}{2}a\hbar^2$

28. Consider a system of three spins S_1, S_2 and S_3 each of which can take values +1 and −1. The energy of the system is given by $E = -J[S_1S_2 + S_2S_3 + S_3S_1]$, where J is a positive constant. The minimum energy and the corresponding number of spin configurations are, respectively, **[NET Dec 2012]**

(a) J and 1 (b) −3J and 1 (c) −3J and 2 (d) −6J and 2

29. How does the total angular quantum number J change in the transition $Cr(3d^6)$ atom ionizes to $Cr^{2+}(3d^4)$? **[NET June 2014]**

(a) Increases by 2 (b) Decreases by 2 (c) Decreases by 4 (d) Does not change

30. The effective spin-spin interaction between the electron spin $\vec{s}_e$ and the proton spin $\vec{s}_p$ in the ground state of the Hydrogen atom is given by $H' = a\vec{s}_e \cdot \vec{s}_p$. As a result of this interaction, the energy levels split by an amount **[NET Dec 2014]**

(a) $\frac{1}{2}a\hbar^2$ (b) $2a\hbar^2$ (c) $a\hbar^2$ (d) $\frac{3}{2}a\hbar^2$

31. The Lande g-factor for the $^3P_1$ level of an atom is: **[GATE 2001]**

(a) 1/2 (b) 3/2 (c) 5/2 (d) 7/2

32. The spin-orbit coupling constant for the upper state of sodium atom which emits D lines of wave numbers 16956.2 and 16973.4 cm^{-1} is : **[GATE 2003]**

(a) 15 cm^{-1} (b) 11.4 cm^{-1} (c) 12.5 cm^{-1} (d) 15.1 cm^{-1}

33. The spectral term for the atom with 70% subshell and only s = 3/2 is **[GATE 2003]**

filled

(a) $^3P_0$ (b) $^4F_{9/2}$ (c) $^3F_{1/2}$ (d) $^4P_{1/2}$

34. The spectral terms for a certain electronic configuration are given by ^3D, ^1D, ^3P, ^1P, ^5S, ^3S. The term with the lowest energy is: **[GATE 2004]**

(a) ^5S (b) ^3P (c) ^3D (d) ^3S

35. The degeneracy of the spectral term ^3F is: **[GATE 2004]**

(a) 7 (b) 9 (c) 15 (d) 21

36. The Lande g factor for the level $^3D_3$ is: **[GATE 2004]**

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

37. The L, S and J quantum numbers corresponding to the ground state electronic configuration of Boron(Z = 5) are **[GATE 2004]**

(a) $L = 1, S = \frac{1}{2}, J = \frac{3}{2}$ (b) $L = 1, S = \frac{1}{2}, J = \frac{1}{2}$

(c) $L = 1, S = \frac{3}{2}, J = \frac{1}{2}$ (d) $L = 0, S = \frac{3}{2}, J = \frac{3}{2}$

38. The LS configurations of the ground state of ^{12}Mg, ^{13}Al, ^{17}Cl and ^{18}Ar are, respectively.

(1) $^3S_1$, $^2P_{1/2}$, $^2P_{1/2}$ and $^1S_0$ (2) $^3S_1$, $^2P_{3/2}$, $^2P_{3/2}$ and $^3S_1$ **[NET Dec. 2015]**

(3) $^1S_0$, $^2P_{1/2}$, $^2P_{3/2}$ and $^1S_0$ (4) $^1S_0$, $^2P_{3/2}$, $^2P_{1/2}$ and $^3S_1$

39. The degeneracies of the J-states arising from the ^{3}P term with spin-orbit inteaction are **[GATE 2004]**

(a) 1, 3, 5 (b) 1, 2, 3 (c) 3, 5, 7 (d) 2, 6, 10

40. Deuteron in its ground state has a total angular momentum J = 1 and a positive parity. The corresponding orbital angular momentum L and spin S combinations are **[GATE 2004]**

(a) L = 0, S = 1 and L = 2, S = 0 (b) L = 0, S = 1 and L = 1, S = 1
(c) L = 0, S = 1 and L = 2, S = 1 (d) L = 1, S = 1 and L = 2, S = 1

41. For an electron in hydrogen atom, the states are characterized by the usual quantum numbers n, l, m_l. The electric dipole transition between any two states requires that **[GATE 2005]**

(a) $\Delta\ell = 0, \Delta m_\ell = 0, \pm 1$ (b) $\Delta\ell = \pm 1, \Delta m_\ell = \pm 1, \pm 2$
(c) $\Delta\ell = \pm 1, \Delta m_\ell = 0, \pm 1$ (d) $\Delta\ell = \pm 1, \Delta m_\ell = 0, \pm 2$

42. In a two electron atomic system having orbital and spin angular momenta ℓ_1, ℓ_2 and $s_1\ s_2$ respectively, the coupling strengths are defined as $\Gamma_{\ell_1\ell_2}, \Gamma_{s_1s_2}, \Gamma_{\ell_1s_1}, \Gamma_{\ell_2s_2}, \Gamma_{\ell_1s_2}$ and $\Gamma_{\ell_2s_1}$. For the J-J coupling scheme to be applicable, the coupling strengths MUST satisty the condition. **[GATE 2005]**

(a) $\Gamma_{\ell_1\ell_2}, \Gamma_{s_1s_2} > \Gamma_{\ell_1s_1}, \Gamma_{\ell_2s_2}$ (b) $\Gamma_{\ell_1s_1}, \Gamma_{\ell_2s_2} > \Gamma_{\ell_1\ell_2}, \Gamma_{s_1s_2}$
(c) $\Gamma_{\ell_1s_2}, \Gamma_{\ell_2s_1} > \Gamma_{\ell_1\ell_2}, \Gamma_{s_1s_2}$ (d) $\Gamma_{\ell_1s_2}, \Gamma_{\ell_2s_1} > \Gamma_{\ell_1s_1}, \Gamma_{s_2s_2}$

43. The allowed states for He($2p^2$) configuration are **[GATE 2007]**

(a) $^1S_0$, $^3S_1$, $^1P_1$, $^3P_{0,1,2}$ $^1D_2$ and $^3D_{1,2,3}$ (b) $^1S_0$, $^3P_{0,1,2}$ and $^1D_2$
(c) $^1P_1$ and $^3P_{0,1,2}$ (d) $^1S_0$ and $^1P_1$

44. In accordance with the selection rules of electric dipole transitions, the 4^3P_1 state of helium can decay by photon emission to the states **[GATE 2007]**

(a) $2^1S_0, 2^1P_1$ and 3^1D_2 (b) $3^1P_1, 3^1D_2$ and 3^1S_0
(c) $3^3P_2, 3^3D_3$ and 3^3P_0 (d) $2^3S_1, 3^3D_2$ and 3^3D_1

45. If an atom is in the $^3D_3$ state, the angle between its orbital and spin angular momentum vectors ($\vec{L}$ and $\vec{S}$) is: **[GATE 2007]**

(a) $\cos^{-1}\frac{1}{\sqrt{3}}$ (b) $\cos^{-1}\frac{2}{\sqrt{3}}$ (c) $\cos^{-1}\frac{1}{2}$ (d) $\cos^{-1}\frac{\sqrt{3}}{2}$

46. The last two terms of the electronic configuration of manganese(Mn) atom is $3d^5 4s^2$. The term factor of Mn^{4+} ion is : **[GATE 2008]**

(a) $^4D_{1/2}$ (b) $^4F_{3/2}$ (c) $^3F_{9/2}$ (d) $^3D_{7/2}$

47. For a multi-electron atom, ℓ, L and S specify the one-electron orbital angular momentum, total orbital angular momentum and total spin angular momentum, respectively. The selection rules for electric dipole transition between the two electronic energy levels, specified by ℓ, L and S are **[GATE 2011]**

(a) $\Delta L = 0, \pm 1; \Delta S = 0; \Delta\ell = 0, \pm 1$ (b) $\Delta L = 0, \pm 1; \Delta S = 0; \Delta\ell = \pm 1$
(c) $\Delta L = 0, \pm 1; \Delta S = \pm 1; \Delta\ell = 0, \pm 1$ (d) $\Delta L = 0, \pm 1; \Delta S = \pm 1; \Delta\ell = \pm 1$

48. The degeneracy of an first excited state of nitrogen atom having electronic configuration $1s^2 2s^2 2p^2 3d^1$ is **[GATE 2011]**

(a) 6 (b) 10 (c) 15 (d) 150

49. The terms $\{j_1, j_2\}$, arising from $2s^1\ 3d^1$ electronic configuration in $j-j$ coupling scheme are **[GATE 2012]**

(a) $\left\{\frac{1}{2},\frac{3}{2}\right\}_{2,1}$ and $\left\{\frac{1}{2},\frac{5}{2}\right\}_{3,2}$ (b) $\left\{\frac{1}{2},\frac{1}{2}\right\}_{1,0}$ and $\left\{\frac{1}{2},\frac{3}{2}\right\}_{2,1}$

(c) $\left\{\frac{1}{2},\frac{1}{2}\right\}_{1,0}$ and $\left\{\frac{1}{2},\frac{5}{2}\right\}_{3,2}$ (d) $\left\{\frac{3}{2},\frac{1}{2}\right\}_{2,1}$ and $\left\{\frac{1}{2},\frac{5}{2}\right\}_{3,2}$

M.F is not given.

50. The number of spectral lines allowed in the spectrum for the $3^2D \rightarrow 3^2P$ transitions in sodium are ___3___ **[GATE 2013]**

51. The energy difference between the $3p$ and $3s$ levels in Na is 2.1 eV. Spin-orbit coupling splits the $3p$ level, resulting in two emission lines differing by 6 Å. The splitting of the $3p$ level is approximately, **[JEST 2015]**

(a) 2 eV (b) 0.2 eV (c) 0.02 eV (d) 2 meV

52. Which of the following statements is true for the energies of the terms of the carbon atom in the ground state electronic configuration $1s^2 2s^2 2p^2$? **[JEST 2015]**

(a) ${}^3P < {}^1D < {}^1S$ (b) ${}^3P < {}^1S < {}^1D$ (c) ${}^3P < {}^1F < {}^1S$ (d) ${}^3P < {}^1F < {}^1D$

53. In the ground state electronic configuration of nitrogen $\left({}^{14}_{7}N\right)$ the L, S and J quantum numbers are **[TIFR 2015]**

(a) $L=1,\ S=\frac{1}{2},\ J=\frac{1}{2}$ (b) $L=1,\ S=\frac{1}{2},\ J=\frac{3}{2}$

(c) $L=0,\ S=\frac{1}{2},\ J=\frac{1}{2}$ (d) $L=0,\ S=\frac{3}{2},\ J=\frac{3}{2}$ ✓

54. If an atom is in the ${}^3D_3$ state, the angle between its orbital and spin angular momentum vectors $\left(\vec{L} \text{ and } \vec{S}\right)$ is:

(a) $\cos^{-1}\frac{1}{\sqrt{3}}$ (b) $\cos^{-1}\frac{2}{\sqrt{3}}$ (c) $\cos^{-1}\frac{1}{2}$ (d) $\cos^{-1}\frac{\sqrt{3}}{2}$

55. The Hydrogen α - line H_α of wavelength $\lambda = 656.3$ nm corresponds to a transition between the principal quantum states $n=3$ and $n=2$. The wavelength of the line H_β corresponding to the transition between $n=4$ and $n=2$ is

(a) 486.1nm (b) 886.0 nm (c) 437.5 nm (d) 1093.7 nm

56. If the ϕ – dependent part of the eigen function of an electron in a hydrogen atom is $e^{2i\phi}$ the minimum pricipal and orbital quantum numbers n and i respectively for the eigen function will be

$e^{im\phi}$

(a) n2, 1, 1 (b) n3, 1, 2 (c) n4, 1, 3 (d) n3, 1, 1

φ dependent part of H-atom.

57. The Helium atom is excited into the 1s 2s configuration. The following spectral terns ^{2s+1}L are the only ones allowed

(a) $^1S, ^1P$ (b) $^1S, ^3P$ (c) $^1S, ^3S$ (d) $^2S, ^2P$

58. The possible values of the total angular momentum J resulting from the addition of two angular momenta $J_1 = 1$ and $J_2 = 2$ are

(a) 1, 2 (b) 1, 3 (c) 0, 1, 2 (d) 1, 2, 3

59. The spectral term for the atom with 70% filled subshell and only S = 3/2 is :

(a) $^3P_0$ (b) $^4F_{9/2}$ (c) $^3F_{1/2}$ (d) $^4P_{1/2}$

60. In a hydrogen atom, the accidental or Coulomb degeneracy for the n = 4 state is:

(a) 4 (b) 16 (c) 18 (d) 32

61. The electronic configuration of the ground state of a nitrogen is $1s^2 2s^2 2p^3$. The degeneracy of this configuration is:

(a) 3 (b) 20 (c) 6 (d) 0

62. An atomic state of hydrogen is represented by the following wave function :

$$\psi(r,\theta,\varphi) = \frac{1}{\sqrt{2}}\left(\frac{1}{a_0}\right)^{3/2}\left(1-\frac{r}{2a_0}\right)e^{-r/2a_0}\cos\theta$$

where a_0 is a constant. The quantum number of the state are:

(a) $\ell = 0, m = 0, n = 1$ (b) $\ell = 1, m = 1, n = 2$

(c) $\ell = 1, m = 0, n = 2$ (d) $\ell = 2, m = 0, n = 3$

63. The radial wave function of the electrons in the state of n = 1 and $\ell = 0$ in a hydrogen atom is: $R_{10} = \frac{2}{a_0^{3/2}}\exp\left(\frac{-r}{a_0}\right)$, a_0 is the Bohr radius. The most probable value of r for electron is:

(a) a_0 (b) $2a_0$ (c) $4a_0$ (d) $8a_0$

64. One Bohr magneton is approximately

(a) 10^{23} Am² (b) 10^{-23} Am² (c) 10^{10} Am² (d) 10^{-10} Am²

65. The magnetic moment associated with the first orbit in hydrogen atom is given by

(a) $\frac{h}{4\pi me}$ (b) $\frac{4\pi me}{he}$ (c) $\frac{eh}{4\pi m}\times\sqrt{3}$ (d) $\frac{ehm}{4\pi}$

66. What is the maximum number of electrons with spin +1/2 in a p orbital ?

(a) 6 (b) 3 (c) 8 (d) 1

67. In the j-j coupling scheme of an atom with two electrons in the 'p' orbital, possible mj values are

(a) ±1/2 only (b) ±1, 0 (c) $\pm 3/2, \pm\frac{1}{2}$ (d) ± 3, ± 2, ± 1

68. The ground state of sodium atom $\left({}^{11}Na\right)$ is a $^2S_{1/2}$ state. The difference in energy levels arising in the presence of a weak external magnetic field B, given in terms of Bohr magneton, μ_B, is

(a) $\mu_B B$ (b) $2\mu_B B$ (c) $4\mu_B B$ (d) $6\mu_B B$

69. The terms $\{j_1, j_2\}$, arising from $2s^1\,3d^1$ electronic configuration in $j-j$ coupling scheme are

(a) $\left\{\frac{1}{2},\frac{3}{2}\right\}_{2,1}$ and $\left\{\frac{1}{2},\frac{5}{2}\right\}_{3,2}$ (b) $\left\{\frac{1}{2},\frac{1}{2}\right\}_{1,0}$ and $\left\{\frac{1}{2},\frac{3}{2}\right\}_{2,1}$

(c) $\left\{\frac{1}{2},\frac{1}{2}\right\}_{1,0}$ and $\left\{\frac{1}{2},\frac{5}{2}\right\}_{3,2}$ (d) $\left\{\frac{3}{2},\frac{1}{2}\right\}_{2,1}$ and $\left\{\frac{1}{2},\frac{5}{2}\right\}_{3,2}$

70. The spatial part of a two-electron state is symmetric under exchange. If $|\uparrow\rangle$ and $|\downarrow\rangle$ represent the spin-up and spin-down states respectively of each particle, the spin-part of the two-particle state is

(a) $|\uparrow\rangle|\uparrow\rangle$ (b) $|\uparrow\rangle|\downarrow\rangle$ (c) $|\downarrow\rangle|\uparrow\rangle-|\uparrow\rangle|\downarrow\rangle/\sqrt{2}$ (d) $|\downarrow\rangle|\uparrow\rangle+|\uparrow\rangle|\downarrow\rangle/\sqrt{2}$

71. The ground state electronic configuration for a carbon atom is

$$(1s)^2(2s)^2(2p)^2$$

The first excited state of this atom would be achieved by

(a) re-alignment of the electron spins within the 2p orbital

(b) transition of an electron from the 2s orbital to the 2p orbital.

(c) transition of an electron from the 2p orbital to the 3s orbital.

(d) transition of an electron from 2s orbital to the 3s orbital.

72. In the ground state electronic configuration of nitrogen $\left({}^{14}_{7}\mathrm{N}\right)$ the L, S and J quantum numbers are **[TIFR 2015]**

(a) $L=1,\ S=\frac{1}{2},\ J=\frac{1}{2}$ (b) $L=1,\ S=\frac{1}{2},\ J=\frac{3}{2}$

(c) $L=0,\ S=\frac{1}{2},\ J=\frac{1}{2}$ (d) $L=0,\ S=\frac{3}{2},\ J=\frac{3}{2}$

73. Which of the following statements is true for the energies of the terms of the carbon atom in the ground state electronic configuration $1s^2 2s^2 2p^2$? **[JEST 2015]**

(a) ${}^3P<{}^1D<{}^1S$ (b) ${}^3P<{}^1S<{}^1D$ (c) ${}^3P<{}^1F<{}^1S$ (d) ${}^3P<{}^1F<{}^1D$

74. Of the following term symbols of the np^2 atomic configurations, ${}^1S_0$, ${}^3P_0$, ${}^3P_1$, ${}^3P_2$ and ${}^1D_2$ which is the grounded state? **[NET June 2015]**

(a) ${}^3P_0$ (b) ${}^1S_0$ (c) ${}^3P_2$ (d) ${}^3P_1$

75. The ground state electronic configuration of ${}^{22}\mathrm{Ti}$ is $[\mathrm{Ar}]3d^2 4s^2$. Which state, in the standard spectroscopic notations, is not possible in this configuration? **[NET June 2016]**

(a) ${}^1F_3$ (b) ${}^1S_0$ (c) ${}^1D_2$ (d) ${}^3P_0$

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Assignment-3 : Atomic Spectra (III)

1. A sample of certain element is placed in a magnetic field $\vec{B}$. The wavelength separation between the Zeeman components of a spectral line of wavelength λ will be

(a) $\frac{\mu_B B}{hc}\lambda$ (b) $\frac{\mu_B B}{hc\lambda}$ (c) $\frac{hc\lambda}{\mu_B B}$ (d) $\frac{\mu_B B}{hc}\lambda^2$

2. In the presence of external weak magnetic field, the number of substates into which the state $^2D_{5/2}$ will split, is

(a) 3 (b) 4 (c) 5 (d) 6

3. In the presence of external weak magnetic field, the number of substates into which the state $^4D_{1/2}$ will split, is

(a) 2 (b) 3 (c) 4 (d) none of these

4. In the presence of weak magnetic field B, the separation between the splitted sublevels of the $^3D_2$ level will be

(a) $\frac{7eB}{24\pi mc}$ (b) $\frac{7eB}{12\pi mc}$ (c) $\frac{7eB}{6\pi mc}$ (d) none of these

5. The number of Zeeman components for $^2D_{3/2}-^2P_{3/2}$ transition in one-electron atom will be

(a) 6 (b) 8 (c) 10 (d) 12

6. The number of Zeeman components for $^3D_3-^3P_2$ transition in one-electron atom will be

(a) 6 (b) 10 (c) 12 (d) 15

7. The Zeeman pattern of a line consists of six equidistant components. The upper state term is known to be $^2P_{3/2}$. The lower state term will be

(a) $^1P_1$ (b) $^2S_{1/2}$ (c) $^1S_0$ (d) none of these

8. The Zeeman pattern of a line consists of nine equidistant components. The upper state term is known to be $^3P_2$. The lower state term will be

(a) $^3P_0$ (b) $^2S_{1/2}$ (c) $^3S_1$ (d) $^3P_{3/2}$

9. The sodium doublet lines are due to transitions from $^2P_{3/2}$ and $^2P_{1/2}$ levels to $^2S_{1/2}$ level. On application of a weak magnetic field, the total number of the allowed transitons are

(a) 4 (b) 6 (c) 8 (d) 10

10. The spectroscopic symbol for the ground state of $_{13}$Al is $^2P_{1/2}$. Under the action of a strong magnetic field (when L-S coupling can be neglected) the ground state energy level will split into **[NET Dec. 2013]**

(a) 3 levels (b) 4 levels (c) 5 levels (d) 6 levels

11. A spectral line due to a transition from an electronic state p to an s state splits into three Zeeman lines in the presence of a strong magnetic field. At intermediate field strengths the number of spectral lines is **[NET June 2014]**

weak Zeeman

(a) 10 (b) 3 (c) 6 (d) 9

12. An atomic transition $^1P \rightarrow {}^1S$ in a magnetic field 1 Tesla shows Zeeman splitting. Given that the Bohr magneton $\mu_B = 9.27 \times 10^{-24}$ J/T, and the wavelength corresponding to the transition is 250 nm, the separation in the Zeeman spectral lines is approximately **[NET Dec. 2014]**

[✓ (a)] 0.01 nm (b) 0.1 nm (c) 1.0 nm (d) 10 nm

13. The total number of Zeeman components observed in an electronic transition $^2D_{5/2} \rightarrow {}^2P_{3/2}$ of an atom in a weak field is: **[GATE 2001]**

(a) 4 (b) 6 (c) 12 (d) 10

Common data for Q.14 and Q.15

In the presence of an inhomogeneous weak magnetic field, spectral lines due to transitions between two sets of states were observed.

$(1)\ {}^5I_5 \rightarrow {}^5H_4$ and $(2)\ {}^2D_{5/2} \rightarrow {}^2P_{3/2}$ **[GATE 2003]**

14. The types of Zeeman effect observed in (1) and (2) respectively are

(a) normal, normal (b) anomalous, anomalous

(c) anomalous, normal (d) normal, anomalous

15. The number of levels into which each of the above four terms split into respectively is: 2×4+1 = 9 lines

(a) 6, 4, 10, 8 (b) 4, 6, 10, 12 [✓ (c)] 11, 9, 6, 4 (d) 9, 5, 12, 10

16. The D_1 and D_2 lines of Na atom will split on the application of a weak magnetic field into

(a) 4 and 6 lines (b) 3 lines each (c) 6 and 4 lines (d) 6 lines each

Common Data Q. 17 and Q.18

Consider the Zeeman splitting of a single electron system for the 3d → 3p electric dipole transition.

17. The Zeeman spectrum is : **[GATE 2009]**

(a) Randomly polarized (b) Only π polarized

(c) Only σ polarized [✓ (d)] Both π and σ polarized

18. The fine structure line having the longest wavelength will split into :

(a) 17 components [✓ (b)] 10 components

(c) 8 components (d) 4 components

Common Data for Q.19 and Q. 20

In the presence of a weak magnetic field, atomic hydrogen undergoes the transition:

$$^2p_{1/2} \rightarrow {}^2s_{1/2}$$

by emission of radiation. **[GATE 2010]**

19. The number of distinct spectral lines that are observed in the resultant Zeeman spectrum is

(a) 2 (b) 3 [✓ (c)] 4 (d) 6

20. The spectral line corresponding to the transition

$$^2P_{1/2}\left(m_j = +\frac{1}{2}\right) \rightarrow {}^1S_{1/2}\left(m_j = -\frac{1}{2}\right)$$

is observed along the direction of the applied magnetic field. The emitted electromagnetic field is:

[✓ (a)] Circularly polarized

(b) Linearly polarized

(c) Unpolarized

(d) Not emitted along the magnetic field direction

21. An atom with one outer electron having orbital angular momentum l is placed in a weak magnetic field. The number of energy levels into which the higher total angular momentum state splits, is **[GATE 2011]**

(a) $2\ell+2$ (b) $2\ell+1$ (c) 2ℓ (d) $2\ell-1$

22. In a normal Zeeman effect experiment, spectral splitting of the line at the wavelength 643.8 nm corresponding to the transition $5\,^1D_2 \to 5\,^1P_1$ of cadmium atom is to be observed. The spectrometer has a resolution of 0.01 nm. The minimum magnetic field needed to observe this is ($m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6\times10^{-19}$ C, $c = 3 \times 10^8$ m/s) **[GATE 2013]**

(a) 0.26 T (b) 0.52 T (c) 2.6 T (d) 5.2 T

23. The number of normal Zeeman splitting components of $^1P \to {}^1D$ transition is **[GATE 2014]**

(a) 3 (b) 4 (c) 8 (d) 9

24. The number of permitted transitions from $^2P_{3/2} \to {}^2S_{1/2}$ in the presence of a weak magnetic field is ___6___ **[GATE 2015]**

25. When a pure element is vaporised and placed in a uniform magnetic field B_0, it is seen that a particular spectral line of wavelength l, corresponding to a J = 1 ® J = 0 transition, gets split into three components $\lambda, \lambda \pm \Delta\lambda$. It follows that the Lande g-factor for the transition J = 1 ® = 0 is given by **[TIFR 2013]**

(a) $g = \frac{hc}{\mu_B B_0}\frac{\Delta\lambda^2}{\lambda}$ (b) $g = \frac{hc}{\mu_B B_0}\frac{\lambda^2}{\Delta\lambda}$ (c) $g = \frac{hc}{\mu_B B_0}\frac{\lambda}{\Delta\lambda^2}$ (d) $g = \frac{hc}{\mu_B B_0}\frac{\Delta\lambda}{\lambda^2}$

26. The D1 and D2 lines of Na$\left(3^2P_{1/2} \to 3^2S_{1/2}, 3^2P_{3/2} \to 3^2S_{1/2}\right)$ will split on the application of a weak magnetic field into

(a) 4 and 6 lines respectively (b) 3 lines each
(c) 6 and 4 lines respectively (d) 6 lines each

27. Pauli exclusion principle state that

(a) Two electrons can have all the quantum numbers same
(b) No two electrons can have all the quantum numbers same
(c) Particles with integer and half integer spin cannot exist in the same state
(d) None of the above

28. The spectroscopic symbol for the ground state of $_{13}$Al is $^2P_{1/2}$. Under the action of a strong magnetic field (when L-S coupling can be neglected) the ground state energy level will split into

(a) 3 levels (b) 4 levels (c) 5 levels (d) 6 levels

29. In Zeeman effect, a spectral-line, upon the application of magnetic field, splits into more than three componets because of

(a) Energy levels split into 2J + 1
(b) In magnetic field $\Delta m_J = 0, \pm 1$ no longer holds
(c) Variation of Lande g-factor from one level to another
(d) None of the above

30. In the Zeeman effect the light emitted along and perpendicular to the applied magnetic field are respectively

(a) linearly and circularly polarized (b) circularly and linearly polarized
(c) both linearly polarized (d) both circularly polarized

31. Which of the following transitions give rise to the $K_{\alpha 1}$ line of X-ray spectra?

(a) $2\,^2S_{1/2} \to 1\,^2S_{1/2}$ (b) $2\,^2P_{1/2} \to 1\,^2S_{1/2}$ (c) $2\,^2P_{5/2} \to 1\,^2S_{1/2}$ (d) $2\,^2P_{3/2} \to 1\,^2S_{1/2}$

32. The number of spectral lines allowed in the spectrum for the $3\,^2D \to 3\,^2P$ transition in sodium is __3.__

33. In a normal Zeeman effect experiment using a magnetic field of strength 0.3 T, the splitting between the components of a 660 nm spectral line is **[NET June 2016]**

(a) 12 pm (b) 10 pm (c) 8 pm (d) 6 pm

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Assignment-4 : Atomic Spectra (IV)

1. The number of components in the hyperfine structure corresponding to the spectral line due to the transition $^2D_{3/2} \to ^2P_{1/2}$ within a certain atom (having nuclear spin I = 9/2), is
(a) 4 [✓ (b) 6] (c) 8 (d) 10

Common Data for Q.2 and Q.3

A certain atom has a excited state $^2D_{5/2}$, which splits into six hyperfine levels. The distance between the two consecutive hyperfine levels are 0.236 cm^{-1}, 0.312 cm^{-1}, 0.391 cm^{-1}, 0.471 cm^{-1}, 0.551 cm^{-1} respectively.

2. The nuclear spin quantum number of the atom is
(a) 1/2 (b) 3/2 (c) 7/2 [✓ (d) 9/2]

3. The hyperfine structure constant of the molecule is
(a) 0.039 cm^{-1} [✓ (b) 0.078 cm^{-1}] (c) 0.098 cm^{-1} (d) 0.112 cm^{-1}

4. The number of hyperfine components observed in the electronic transition $^2p_{1/2} \to ^2S_{1/2}$ of an atom with nuclear spin 1/2 is :
[✓ (a) 3] (b) 4 (c) 6 (d) 5

5. The hyperfine splitting of the spectral lines of an atom is due to :
(a) The coupling between the spins of two or more electron
(b) The coupling between the spins and the orbital angular momenta of the electron
[✓ (c) The coupling between the electron spins and the nuclear spin]
(d) The effect of external electromagnetic field

6. The hyperfine structure of $\mathrm{Na}\left(3^2\mathrm{P}_{3/2}\right)$ with nuclear spin I = 3/2 has
(a) 1 state (b) 2 states (c) 3 states [✓ (d) 4 states]

7. In hyperfine interaction, there is coupling between the electron angular momentum $\vec{J}$ and nuclear angular momentum $\vec{I}$, forming resultant angular momentum $\vec{F}$. The selection rules for the corresponding quantum number F in hyperfine transitions are
(a) $\Delta F = \pm 2$ only (b) $\Delta F = \pm 1$ only [✓ (c) $\Delta F = 0, \pm 1$] (d) $\Delta F = \pm 1, \pm 2$

8. Cesium has a nuclear spin of 7/2. The hyperfine spectrum of the D lines of the Cesium atom will consist of
[✓ (a) 10 lines] (b) 4 lines (c) 6 lines (d) 14 lines

9. The wavelength of a photon emitted from a moving source is appeared to be 500.1 nm, whereas the actual wavelength is 500 nm. The speed and direction of the movement of the source w.r.t the observer is
(a) 30 km/sec, away from the observer [✓ (b) 60 km/sec, away from the observer]
(c) 30 km/sec, towards the observer (d) 60 km/sec, towards the observer

10. The dependence of Doppler broadened line width of a laser transition on temperature T is given by
(a) T (b) $T^{-1/2}$ [✓ (c) $T^{1/2}$] (d) T^2

11. The minimum voltage that must be applied to a X-ray tube to produce X-ray photons of wavelength 0.01 nm, is
(a) 50 KV (b) 75 KV (c) 100 KV [✓ (d) 125 KV]

12. If the wavelength limit of the K-series for a certain element is about 0.1 nm, then the atomic number of the element will be

(a) 19 (b) 25 [✓ (c) 31] [✓ (d) 39]

13. The Copper (Z = 29) target in an X-ray tube has some impurity in it. In the X-ray spectrum emitted from the tube, there is an additional line of wavelength 0.54 A^0 alongwith the K_α line of wavelength 1.54 A^0. The impurity present in the target of the X-ray tube may be

(a) Na (b) K (c) Cl [✓ (d) Cd]

14. The continuous X-ray spectrum is the result of the
(a) Photoelectric effect [✓ (b) Inverse photoelectric effect]
(c) Compton effect (d) Auger effect

15. The energy E of K_α X-rays emitted from targets of different atomic number Z varies as
[✓ (a) Z^2] (b) $Z^{2/3}$ (c) Z (d) $Z^{1/2}$

16. Which one of the following would not cause the broadening in spectral lines?
(a) Doppler effect (b) Heisenberg's uncertainty
[✓ (c) Absorption of e.m. radiation] (d) Collisions

17. The linear Stark effect is possible in a hydrogen atom but not in a sodium atom because

(a) The principal quantum number for the ground state of the sodium atom is different from that of the hydrogen atm in the ground state
(b) Spin-orbit interaction is stronger in sodium than in hydrogen
(c) The electronic energy levels of sodium exhibit orbital-degeneracy
[✓ (d) The electronic energy levels of hydrogen exhibit orbital-degeneracy]

18. The possible values of the total angular momentum J resulting from the addition of two angular momenta J_1 = 1 and J_2 = 2 are

(a) 1, 2 (b) 1, 3 (c) 0, 1, 2 [✓ (d) 1, 2, 3]

19. The normal Zeeman effect is:
[✓ (a) Observed only in atoms with an even number of electrons]
(b) Observed only in atoms with an odd number of electrons
[✓ (c) A confirmation of space quantization] → different l split into different m values known as
(d) Not a confirmation of space quantization

20. In a Stern-Gerlach experiment the atomic beam whose angular momentum state is to be determined, must travel through
(a) Homogeneous radio frequency magnetic field
(b) Homogeneous static magnetic field
[✓ (c) Inhomogeneous static magnetic field]
(d) Inhomogeneous radio frequency magnetic field

21. The L_β line of X-rays emitted from an atom with principal quantum numbers n = 1, 2, 3, ... arises from the transition.
[✓ (a) n = 4 → n = 2] (b) n = 3 → n = 2 (c) n = 5 → n = 2 (d) n = 3 → n = 2

22. 39. The principal series of spectral lines of lithium is obtained by transitions between

[✓ (a) nS and 2P, n > 2] (b) nD and 2P, n > 2
[✓ (c) nP and 2S, n > 1] (d) nF and 3D, n > 3

27 X 24 x 28 x 29 x

23. In the linear Stark effect, the application of an electric field

(a) completely lifts the degeneracy of n = 2 level on hydrogen atom amd splits n = 2 level into four levels

(b) Partially lifts the degeneracy of n = 2 level on hydrogen atom amd splits n = 2 level into three levels

[✓ (c)] Partially lifts the degeneracy of n = 2 level on hydrogen atom and splits n = 2 level into two levels

(d) Does not affect the n = 2 levels

24. Group I lists some physical phenomena while Group II gives some physical parameters. Match the phenomena with the corresponding parameter

Goup I	**Group II**
P. Doppler Broadening	1. Moment of inertia
Q. Natural Broadening	2. Refractive index
R. Rotational spectrum	3. Lifetime of the energy level
S. Total internal reflection	4. Pressure

[✓ (a)] P-4, Q-3, R-1, S-2 (b) P-3, Q-2, R-1, S-4

(c) P-2, Q-3, R-4, S-1 (d) P-1, Q-4, r-2, S-3

25. Cesium has a nuclear spin of 7/2. The hyperfine spectrum of the D lines of the Cesium atom will consist of

[✓ (a)] 10 lines (b) 4 lines (c) 6 lines (d) 14 lines

26. The ratio of intensities of the D_1 and D_2 lines of sodium at high temperature

(a) 1:1 (b) 2:3 (c) 1:3 [✓ (d)] 1:2

27. As a consequence of the interaction between the electron's orbital angular momentum and the nuclear spin, the $^2S_{1/2}$ electronic level of the hydrogen atom

(a) shifts up by a constant amount [✓ (b)] splits into two hyperfine levels

[✓ (c)] shifts down by a constant amount (d) splits into three hyperfine levels

28. The electronic energy levels in a hydrogen atom are given by $E_n = -13.6/n^2$ eV. If a selective excitation to the n = 100 level is to be made using a laser, the maximum allowed frequency line-width of the laser is

(1) 6.5 MHz [✓ (2)] 6.5 GHz (3) 6.5 Hz (4) 6.5 kHz

29. A continuous monochromatic ($\lambda = 600$ nm) laser beam is chopped into 0.1 ns pulses using some sort of shutter. Find the resultant linewidth $\Delta\lambda$ of the beam in units of 10^{-3} nm. [JEST 2016]

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Assignment-5 : Atomic Spectra (V)

1. A stable H_2 molecule is possible because
   (a) The electronic ground state is spin triplet
   [✓ (b)] The magnitude of the electronic charge density is maximal in the internuclear region
   (c) The electronic charge density is depleted in the internuclear region
   (d) Of strong spin-orbit interaction in the molecule.

2. The energy separation between two consecutive stokes lines in Raman Scattering depends on
   (a) Energy separation between vibrational levels in the excited state
   (b) Wavelength of the incident light
   [✓ (c)] Energy separation between vibrational levels in the ground state
   (d) Intensity of the incident light

3. Typical energy of the rotational modes in a polyatomic molecule like NH_3 is:
   (a) 10^{-6} eV (b) 10^{-3} eV [✓ (c)] 10^{-4} eV (d) 1 eV

4. There is no infrared absorption for nitrogen molecule because:
   (a) Its polarizability is zero [✓ (b)] It has no vibrational levels
   (c) It has no rotational levels (d) Its dipole moment is zero

5. The vibrational constants ω_0 and $\omega_0 x_0$ of HCl are 2937.5 cm^{-1} and 51.6 cm^{-1}. The first Raman stokes line will be observed at (in cm^{-1})
   (a) 2989.1 (b) 2885.9 [✓ (c)] 2834.3 (d) 3040.7

6. The intensity of the electronic 0-0 band of a diatomic molecule is very intense when the minimum of the potential curve for the upper electronic state lies (here r is the internuclear distance).
   [✓ (a)] at the same value r as that of the lower potential curve
   (b) at a smaller value of r than that of the lower potential curve
   (c) at a larger value of r than that of the lower potential curve
   (d) above the dissociation level of the lower potential curve

7. Which one of the following molecules does not exhibit a rotational spectrum?
   [✓ (a)] H_2 (b) CO (c) HCl (d) HBr

8. Resonance (NMR, ESR etc.) studies in solids provide information about
   [✓ (a)] The electronic structure of single defects
   [✓ (b)] The motion of the spin or of the surrounding
   [✓ (c)] Collective spin excitations
   [✓ (d)] Internal magnetic fields sampled by the spin

9. Infrared absorption can be observed in which of the following molecules?
   (a) N_2 (b) O_2 [✓ (c)] HCl (d) C_2

10. The pure rotational levels of a molecule in the far-infrared region follows the formula F(J) = BJ (J+ 1), where F(J) is the energy of the rotational level with quantum number J and B is the rotational constant. The lowest rotational energy gap in rotational Raman spectrum is :

(a) 2B (b) 4B [✓ (c) 6B] (d) 8B

11. The rotational partition function for a diatomic molecule of moment of inertia I at a temperature T is given by

(a) $\frac{Ik_BT}{\hbar^2}$ [✓ (b) $\frac{2Ik_BT}{\hbar^2}$] (c) $\frac{3Ik_BT}{\hbar^2}$ (d) $\frac{Ik_BT}{2\hbar^2}$ [GATE 2001]

12. The vibrational spectrum of a molecule exhibits a strong line with P and R branches at a frequency u_1 and a weaker line at a frequency u_2. The frequency u_3 isnot shown up. Its vibrational Raman spectrum shows a strongly polarized line at frequency u_3 and no feature at u_1 and u_2

(a) The molecule could be linear
(b) The molecule lacks a center of inversion
(c) $\mathbf{u}_1$ arises from a symmetric stretching mode
(d) $\mathbf{u}_3$ arises from a bending mode

13. Three values of rotational energies of molecules are given below in different units

P: 10 cm^{-1} **Q:** 10^{-23} J **R:** 10^4 MHz

Choose the correct arrangement in the increasing order of energy

(a) P, Q, R [✓ (b) R, Q, P] (c) R, P, Q (d) Q, R, P

14. Consider the following statements about molecular spectra

P: CH_4 does not give pure rotational Raman lines
Q: SF_6 could be studied by rotational Raman spectroscopy
R: N_2 shows infrared absorption spectrum
S: CH_3CH_3 shows vibrational Raman and infrared absorptin lines
T: H_2O_2 shows pure rotational spectrum

Choose the right combination of correct statements

(a) P and Q (b) P, R and T [✓ (c) P, S and T] (d) Q and R

15. All vibrations producing a change in the electric dipole moment of a molecule yield

(a) Raman spectra [✓ (b) Infrared spectra]
(c) Ultra-violet spectra (d) X-ray spectra

16. Consider the pure rotational spectrum of a diatomic rigid rotor, The separation between two consecutive lines ($\Delta\bar{V}$) in the spectrum

$\Delta\bar{\nu} = 2B = \frac{2h}{8\pi^2 I c} \propto \frac{1}{I}$

(a) Is directly proportional to the moment of inertial of the rotor
[✓ (b) In inversely proportional to the moment of inertia of the rotor]
(c) Depends on the angular momentum
(d) Is directly proportional to the square of the interatomic separation

17. Light of wavelength 1.5 μm incident on a material with a characteristic Raman frquency of 20×10^{12} Hz results in a stokes-shifted line of wavelength $\left[\text{Given}: c = 3\times10^8 m.s^{-1}\right]$

(a) 1.47 μm (b) 1.57 μm [✓ (c) 1.67 μm] (d) 1.77 μm

18. In a Raman scattering experiment, light of frequency ν from a laser is scattered by diatomic molecules having the moment of inertia I. the typical Raman shifted frequency depends on

(a) u and I (b) only u [✓ (c) only I] (d) neither u nor I

19. For a diatomic molecule with the vibrational quantum number n and rotational quantum number J, the vibrational level spacing $\Delta E_n = E_n - E_{n-1}$ and the rotational level spacing $\Delta E_J = E_J - E_{J-1}$ are approximately

(a) $\Delta E_n = \text{constant}, \Delta E_J = \text{constant}$ [✓ (b) $\Delta E_n = \text{constant}, \Delta E_J \propto J$]

(c) $\Delta E_n \propto n, \Delta E_J \propto J$ (d) $\Delta E_n \propto n, \Delta E_J \propto J^2$

20. The typical wavelengths emitted by diatomic molecules in purely vibrational and purely rotational transitions are respectively in the region of

(a) Infrared and visible (b) Visible and infrared

[✓ (c) Infrared and microwave] (d) Microwave and infrared

21. In the microwave spectrum of identical rigid diatomic molecules, the separation between the spectral lines is recorded to be 0.7143 cm^{-1}. The moment of inertia of the molecule, in kg m^2, is :

(a) 2.3×10^{-36} (b) 2.3×10^{-40} (c) 7.8×10^{-42} (d) 7.8×10^{-46}

22. A vibrational-electronic spectrum of homonuclear binary molecules, involving electronic ground state ε'' and excited ε', exhibits a continuum at $\bar{\upsilon}$ cm^{-1}. If the total energy of the dissociated atoms in the excited state exceeds the total energy of the dissociated atoms in the ground state by $E_{ex} cm^{-1}$, then dissociation energy of the molecule in the ground state is :

(a) $(\bar{\upsilon} + E_{ex})/2$ (b) $(\bar{\upsilon} - E_{ex})/2$ [✓ (c) $(\bar{\upsilon} - E_{ex})$] (d) $\sqrt{(\bar{\upsilon}^2 - E_{ex}{}^2)}$

23. Match the following:

P: Franck-Hertz experiment 1. Electronic excitation of molecules

Q: Hartree-Fock method 2. 2. Wave function of atom

R: Stern-Gerlach experiment 3. Spin angular momentum of atoms

S: Franck-Condon principle 4. Energy levels in atoms

[✓ (a)]	(b)	(c)	(d)
P-4	P-1	P-3	P-4
Q-2	Q-4	Q-2	Q-1
R-3	R-3	R-4	R-3
S-1	S-2	S-1	S-2

24. The allowed rotational energy levels of a rigid hetero-nuclear diatomic molecule are expressed as $\varepsilon_J = BJ(J+1)$, where B is the rotational constant and J is a rotational quantum number.

In a system of such diatomic molecules of reduced mass μ, some of the atoms of one element are replaced by a heavier isotope, such that the reduced mass is changed to 1.05μ. In the rotational spectrum of the system, the shift in the spectral line, corresponding to a transition $J = 4 \rightarrow J = 5$, is :

[✓ (a) 0.475B] (b) 0.50 B (c) 0.95 B (d) 1.0 B

25. The number of fundamental vibrational modes of CO_2 molecule is :

(a) Four: 2 are Raman active and 2 are infrared active

[✓ (b) Four: 1 is Raman active and 3 are infrared active]

(c) Three: 1 is Raman active and 2 are infrared active

(d) Three: 2 are Raman active and 1 is infrared active

26. A pure rotational Raman spectrum of a linear diatomic molecule is recorded using electromagnetic radiation of frequency υ_e. The frequency of two consecutive stokes lines are : difference is 4B

[✓ (a) $\upsilon_e - 10B, \upsilon_e - 14B$] (b) $\upsilon_e - 2B, \upsilon_e - 4B$

(c) $\upsilon_e + 10B, \upsilon_e + 14B$ (d) $\upsilon_e + 2B, \upsilon_e + 4B$

27. Which one of the following statement is INCORRECT in vibrational spectroscopy with anharmonicity?
(a) The selection rule for vibrational spectroscopy is $\Delta v = \pm 1, \pm 2, ...$
(b) Anharmonicity leads to multiple absorption lines
[✓ (c)] The intensities of hot band lines are stronger than the fundamental absorption.
[✓ (d)] The frequencies of hot band lines are smaller than the fundamental absorption.

28. The molecular spectra of two linear molecules O-C-O and O-C-S are recorded in the microwave region. Which one of the following statement is correct?
(a) Both the molecules would show absorption lines
(b) Both the molecules woulb not show absorption lines
(c) O-C-O would show absorption lines, but not O-C-S.
[✓ (d)] O-C-S would show absorption lines, but not O-C-O.

29. The separation between the first stokes and corresponding anti-stokes lines of the rotational Raman spectrum in terms of the rotational constant, B is :
(a) 2B (b) 4B (c) 6B (d) 12B

30. The spectrum of radiation emitted by a black body at a temperature 1000 K peaks in the
(a) Visible range of frequencies [✓ (b)] Infrared range of frequencis.
(c) Ultraviolet range of frequencies (d) Microwave range of frequencies.

31. The three principal moments of inertia of a methanol (CH_3OH) molecule have the property $I_x = I_y = I$ and $I_z \neq I$. The rotational energy eigenvalues are
[✓ (a)] $\frac{\hbar^2}{2I}\ell(\ell+1)+\frac{\hbar^2 m_l^2}{2}\left(\frac{1}{I_z}-\frac{1}{I}\right)$ (b) $\frac{\hbar^2}{2I}\ell(\ell+1)$
(c) $\frac{\hbar^2 m_l^2}{2}\left(\frac{1}{I_z}-\frac{1}{I}\right)$ (d) $\frac{\hbar^2}{2I}\ell(\ell+1)+\frac{\hbar^2 m_l^2}{2}\left(\frac{1}{I_z}+\frac{1}{I}\right)$

32. CO_2 molecule has the first few energy levels uniformly separated by approximately 2.5 meV At a temperature of 300 K, the ratio of the number of molecules in the 4th excited state to the number in the 2nd excited state is about.
(a) 0.5 (b) 0.6 [✓ (c)] 0.8 (d) 0.9

33. You are shown a spectrum consisting of a series of equally spaced lines. This could be
[✓ (a)] The rotational spectrum CO. (b) The vibrational spectrum of N_2.
(c) The NMR spectrum of CH_4 (d) The Mossbauer spectrum of Fe_3O_4.

34. The rotational energy levels of a rigid diatomic molecules are given by $E_J = B_e J(J+1)$ where J is the rotational quantum number and B_e is the constant. The rotational absorption spectrum of the molecules therefore consists of
(a) One resonance line
[✓ (b)] Lines that are equally spaced.
(c) Lines where the spacing increases with frequency
(d) Lines where the spacing decreases with frequency.

35. Consider the CO molecule as a diatomic rigid rotor with a bond length of 1.12 Å. The reduced mass of the system is obtained from the atomic masses of C and O. The rotational energies are defined in terms of B(the rotational constant) and J (the rotational quantum number). If ν_1 and ν_2 denotes the frequency of the first rotational resonance lines for the moleules ${}^{12}C\,{}^{16}O$ and ${}^{13}C\,{}^{18}O$ respectively, their ratio ν_1/ν_2 is approximately.
(a) 1.5 [✓ (b)] 1.1 (c) 0.9 (d) 1.01

36. The strongest three lines in the emission spectrum of an interstellar gas cloud are found to have wavelengths $\lambda_0, 2\lambda_0$ and $6\lambda_0$ respectively, where λ_0 is a known wavelength. From this we can deduce that the radiating particles in the cloud behave like

(a) Free particles　　(b) Particles in a box
(c) Harmonic oscillators　　[✓ (d) Rigid rotators]

shift in the

37. A laser operating at 500 nm is used to excite a molecule. If the Stokes line is observed at 770 cm^{-1}, the approximate positions of the Stokes and the anti-Stokes lines are

[✓ (a) 481.5 nm and 520 nm]　　(b) 481.5 nm and 500 nm
(c) 500 nm and 520 nm　　(d) 500 nm and 600 nm

38. Which of the diatomic molecules H_2, HD, D_2 and HT has the highest vibrational frequency?

(a) HT　　(b) D_2　　(c) HD　　(d) H_2

all are not shown [illegible]

39. If a molecule has a centre of symmetry then

(a) Raman active vibrations are infrared active
[✓ (b) Raman active vibrations are infrared inactive and vice versa]
(c) some, but not all, of the Raman active vibrations are infrared active
(d) all vibrations are Raman and infrared inactive.

40. In which of the following pairs do both molecules exhibit rotational spectra?

(a) CCl_4 and HF　　(b) CCl_4 and O_2　　[✓ (c) HCN and HF]　　(d) HCN and CCl_4

41. The sharpest sepctrum lines in the microwave rotational spectrum are observed when the sample is a

[✓ (a) solid]　　(b) liquid　　(c) liquid crystal　　(d) gas

42. The unequal spacing in the vibrational energy levels of a diatomic molecule would be due to the

[✓ (a) Presence of anharmonic terms in the potential energy.]
(b) Finite mass of the nuclei.
(c) Frank-Condon principle.
(d) Harmonic nature of the potential energy.

43. The equilibrium vibration frequency for an oscillator observed at $2990 cm^{-1}$. The ratio of the frequencies corresponding to the first overtone and the fundamental spectral lines is 1.96. Considering the oscillator to be anharmonic, the anharmonicity constant is

(a) 0.005　　(b) 0.02　　(c) 0.05　　(d) 0.1

44. The first absorption spectrum of $^{12}C^{16}O$ is at 3.842 cm^{-1} while that of $^{13}C^{16}O$ is at 3.673 cm^{-1}. The ratio of their moments of inertia is

(a) 1.851　　(b) 1.286　　[✓ (c) 1.046]　　(d) 1.038

45. The spacing between vibrational energy levels in CO molecule is found to be 8.441 × 10^{-2} eV. Given that the reduced mass of CO is 1.14 × 10^{-26} kg, Planck's constant is 6.626 × 10^{-34} Js and 1eV = 1.6 × 10^{-19}J. The force constant of the bond is CO molecule is

(a) 1.87 N/m　　(b) 18.7 N/m　　[✓ (c) 187 N/m]　　(d) 1870 N/m

46. Consider the hydrogen deuterium molecule HD. If the mean distance between the two atoms is 0.08 nm and the mass of the hydrogen atom is 938 MeV/c^2, then the energy difference ΔE between the two lowest rotational states is approximately

(1) 10^{-1}eV　　[✓ (2) 10^{-2}eV]　　(3) 2 × 10^{-2}eV　　(4) 10^{-3}eV

47. Raman effect is suppressed in

(a) semiconductors　　(b) dielectrics　　[✓ (c) metals]　　(d) insulators

reflects the visible rad

48. The moment of inertia of a rigid diatomic molecule A is 6 times that of another rigid diatomic molecule B. If the rotational energies of the two molecules are equal, then the corresponding values of the rotational quantum numbers J_A and J_B are

(a) $J_A = 2, J_B = 1$ [✓ (b) $J_A = 3, J_B = 1$] (c) $J_A = 5, J_B = 0$ (d) $J_A = 6, J_B = 1$

49. If the leading anharmonic correction to the energy of the n-th vibrational level of a diatomic molecule is $-x_e\left(n+\frac{1}{2}\right)^2 \hbar\omega$ with $x_e = 0.001$, the total number of energy levels possible is approximately

[NET Dec. 2014]

[✓ (a) 500] (b) 1000 (c) 250 (d) 750

50. In a rigid-rotator of mass M, if the energy of the first excited state is 1 meV, then the fourth excited state energy (in meV) is 10 meV **[GATE 2015]**

51. The excitation wavelength of laser in a Raman effect experiment is 546 nm. If the Stokes line is observed at 552 nm, then the wave number of the anti-Stokes line (in cm^{-1}) is 18.5 [illegible] **[GATE 2015]**

52. A diatomic molecule has vibrational states with energies $E_v = \hbar\omega\left(v+\frac{1}{2}\right)$ and rotational states with energies $E_j = Bj(j+1)$, where v and j are non-negative integers. Consider the transitions in which both the initial and final states are restricted to $v \le 1$ and $j \le 2$ and subject to the selection rules $\Delta v = \pm 1$ and $\Delta j = \pm 1$. Then the largest allowed energy of transition is **[NET June 2015]**

(a) $\hbar\omega - 3B$ (b) $\hbar\omega - B$ [✓ (c) $\hbar\omega + 4B$] (d) $2\hbar\omega + B$

53. The molecule $^{17}O_2$ is **[GATE 2016]**

(a) Raman active but not NMR (nuclear magnetic resonance) active

(b) Infrared active and Raman active but not NMR active x

[✓ (c) Raman active and NMR active]

(d) Only NMR active

54. The H_2 molecule has a reduced mass $M = 8.35 \times 10^{-28}$ kg and an equilibrium internuclear distance $R = 0.742 \times 10^{-10}$ m. The rotational energy in terms of the rotational quantum number J is :

(a) $E_{rot}(J) = 7J(J-1)$ meV (b) $E_{rot}(J) = \frac{5}{2}J(J+1)$ meV **[JEST 2016]**

[✓ (c) $E_{rot}(J) = 7J(J+1)$ meV] (d) $E_{rot}(J) = \frac{5}{2}J(J-1)$ meV

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Assignment-6 : Rotational Spectra

Common Data for Q.1 and Q.2

The $J=0 \rightarrow J=1$ absorption line in CO occurs at a frequency of $1.153\times10^{11}\ s^{-1}$.

(Given: The reduced mass of CO molecule is $1.14\times10^{-26}\ kg$)

1. The moment of inertia of CO molecule is

(a) $0.8\times10^{-46}\ kg-m^2$ [✓ (b) $1.5\times10^{-46}\ kg-m^2$] (c) $0.8\times10^{-42}\ kg-m^2$ (d) $1.5\times10^{-42}\ kg-m^2$

2. The internuclear separation of CO molecule is

(a) $0.48A^0$ (b) $0.84A^0$ [✓ (c) $1.15A^0$] (d) $1.95A^0$

3. The $J=0 \rightarrow J=1$ transition line occurs at a frequency of $1.153\times10^{11}\ s^{-1}$ in $^{12}C^{16}O$ and at a frequency of $1.102\times10^{11}\ s^{-1}$ in $^{?}C^{16}O$. The mass number of the unknown isotope of carbon is

[✓ (a) 13] (b) 14 (c) 15 (d) 16

Common Data for Q.4, Q.5 and Q.6

The separation of lines in the microwave red spectrum of HCl is 20.68 cm^{-1}.

(Given: The reduced mass of HCl molecule is $1.62\times10^{-27}\ kg$)

4. The $J=14 \rightarrow J=15$ transition occurs at

(a) $103.4\ cm^{-1}$ (b) $206.8\ cm^{-1}$ [✓ (c) $310.2\ cm^{-1}$] (d) $413.6\ cm^{-1}$

5. The moment of inertia of the HCl molecule is

(a) $1.34\times10^{-40}\ kg-m^2$ (b) $2.7\times10^{-40}\ kg-m^2$ (c) $1.34\times10^{-47}\ kg-m^2$ [✓ (d) $2.7\times10^{-47}\ kg-m^2$]

6. The internuclear distance of HCl molecule is

(a) $0.48A^0$ (b) $0.84A^0$ [✓ (c) $1.30A^0$] (d) $1.95A^0$

7. The wave numbers (in cm^{-1}) of the lines of a band in the microwave region are given by

$$\nu = 1000(2n-1) \quad \text{for positive } 'n'$$
$$= -1000(2n+1) \quad \text{for negative } 'n'$$

The moment of inertia of the emitter molecule of the spectrum is

[✓ (a) $2.8\times10^{-49}\ kg-m^2$] (b) $5.6\times10^{-49}\ kg-m^2$ (c) $8.4\times10^{-49}\ kg-m^2$ (d) $11.2\times10^{-49}\ kg-m^2$

8. Which one of the following molecule does not exhibit rotational spectra? both

[✓ (a) CCl_4] (b) HF (c) HBr [✓ (d) O_2]

9. The lines in the microwave sprctrum of $^1H^{127}I$ are separated by A cm^{-1}. The lines in the microwave spectrum of $^2H^{127}I$ will be separated by

(a) $\frac{A}{4}$ [✓ (b) $\frac{A}{2}$] (c) $\frac{A}{\sqrt{2}}$ (d) A

10. The spacing between the rotational lines of HF molecule is 40 cm^{-1}. The corresponding spacing between rotational lines in DF molecule is approximately

[✓ (a) 20 cm^{-1}] (b) 30 cm^{-1} (c) 60 cm^{-1} (d) 7.5 cm^{-1}

11. The J = 0 to J = 1 rotational transition for $^{1}H^{79}Br$ occurs at 500.72 GHz. Assuming the moleule to be a rigid rotator, the J = 3 to J = 4transtition occurs at

(a) 50.1 cm^{-1} (b) 66.8 cm^{-1} (c) 16.7 cm^{-1} (d) 83.5 cm^{-1}

12. At a given temparature, for a rigid rotator. the probability that the system is in the rotational state J = 0 is 0.6, in state J = 1is 0.3and in state J = 2 is 0.1. The average enegy of the rotator at the given tempararture will be (Given: Rotational constant of the rotator is B)

(a) 6B (b) 1.2 B (c) 3.6 B (d) 4.8 B

13. Rotational energy of diaatomic molecule is given by $E_{rot} = BhJ(J+1)$ (in Joule). If the rotational constant for H_2 molecule is given as $B = 1.83\times10^{12}\,Hz$, the rotational period of the H_2 molecule in J = 10 level will be

(a) 1.33×10^{-19} sec (b) 5×10^{-15} sec (c) 5.46×10^{-13} sec (d) 7.93×10^{-7} sec

14. Consider the CO molecule as a diatomic rigid rotor with a bond length of 1.12 Å. The reduced mass of the system is obtained from the atomic masses of C and O. The rotational energies are defined in terms of B(the rotational constant) and J (the rotational quantum number). If ν_1 and ν_2 denotes the frequency of the first rotational resonance lines for the moleules $^{12}C\,^{16}O$ and $^{13}C\,^{18}O$ respectively, their ratio ν_1/ν_2 is approximately.

(a) 1.5 (b) 1.1 (c) 0.9 (d) 1.01

15. In which of the following pairs do both molecules exhibit rotational spectra?

(a) CCl_4 and HF (b) CCl_4 and O_2 (c) HCN and HF (d) HCN and CCl_4

16. The ratio of molecules distributed between two states is 9.22×10^{6} at 300 K. The dfference in energy (in kJ mol^{-1}) of the two states is ......40 KJ/mol...

17. The population in the first rotational state of diatomic molecule relative to the lowest state is 3e^{-2}. The relative population in the second rotational state is

(a) 4e^{-3} (b) 5e^{-4} (c) 5e^{-6} (d) 4e^{-5}

18. The relative population in two states with energies E_1 and E_2 satisfying Boltzmann distribution is given by

$$\frac{n_1}{n_2} = \frac{3}{2}\exp\left[-\frac{(E_2-E_1)}{k_BT}\right]$$

The relative degenaracy g_2/g_1 is

(a) 2 (b) 2/3 (c) 3/2 (d) 3

19. The equillibrum population ratio (n_f/n_i) of a doubly degenarate level (E_f) lying at energy 2 units higher than a lower non-degenarate energy level (E_i), will be (assuming $k_BT = 1$ unit)

(a) $\frac{2}{e^2}$ (b) $\frac{1}{e^2}$ (c) e^2 (d) $2e^2$

20. The population of Jth rotational level is given by

$$N_J = N_0(2J+1)\exp\left[-\frac{BJ(J+1)}{k_BT}\right]$$

where N_0 is the population in the ground state. The J value of the rotational level with maximum population is given by,

(a) $\dfrac{\left[\dfrac{2kT}{B}-1\right]}{\sqrt{2}}$ (b) $\dfrac{\left[\sqrt{\dfrac{2kT}{B}}-1\right]}{2}$ (c) $\dfrac{kT}{B}$ (d) $\dfrac{B}{kT}$

21. Three consecutive lines in the microwave spectrum of HBr molecule are found to be at 84.544 cm^{-1}, 101.355 cm^{-1} and 118.112 cm^{-1} respectively. The rotational constant of the HBr molecule is

(a) 3.7 cm^{-1} (b) 5.2 cm^{-1} (c) 6.9 cm^{-1} (d) 8.5 cm^{-1}

22. The first absorption spectrum of $^{12}C^{16}O$ is at $3.842\,cm^{-1}$ while that of $^{13}C^{16}O$ is at $3.673\,cm^{-1}$. The ratio of their moments of inertia is **[NET June 2012]**

(a) 1.851 (b) 1.286 (c) 1.046 (d) 1.038

23. Consider the hydrogen deuterium molecule HD. If the mean distance between the two atoms is 0.08 nm and the mass of the hydrogen atom is 938 MeV/c^2, then the energy difference ΔE between the two lowest rotational states is approximately **[NET June 2013]**

(a) 10^{-1}eV (b) 10^{-2}eV (c) 2×10^{-2}eV (d) 10^{-3}eV

24. Three values of rotational energies of molecules are given below in different units

P: 10 cm^{-1} **Q:** 10^{-23} J **R:** 10^{4} MHz

Choose the correct arrangement in the increasing order of energy **[GATE 2003]**

(a) P, Q, R (b) R, Q, P (c) R, P, Q (d) Q, R, P

25. Consider the pure rotational spectrum of a diatomic rigid rotor. The separation between two consecutive lines $(\Delta\bar{V})$ in the spectrum **[GATE 2004]**

(a) Is directly proportional to the moment of inertial of the rotor
(b) In inversely proportional to the moment of inertia of the rotor
(c) Depends on the angular momentum
(d) Is directly proportional to the square of the interatomic separation

26. In the microwave spectrum of identical rigid diatomic molecules, the separation between the spectral lines is recorded to be 0.7143 cm^{-1}. The moment of inertia of the molecule, in kg m^2, is **[GATE 2006]**

(a) 2.3×10^{-36} (b) 2.3×10^{-40} (c) 7.8×10^{-42} (d) 7.8×10^{-46}

27. The allowed rotational energy levels of a rigid hetero-nuclear diatomic molecule are expressed as $\varepsilon_J = BJ(J+1)$, where B is the rotational constant and J is a rotational quantum number.

In a system of such diatomic molecules of reduced mass μ, some of the atoms of one element are replaced by a heavier isotope, such that the reduced mass is changed to 1.05μ. In the rotational spectrum of the system, the shift in the spectral line, corresponding to a transition $J = 4 \rightarrow J = 5$, is **[GATE 2007]**

(a) 0.475B (b) 0.50 B (c) 0.95 B (d) 1.0 B

28. Three consecutive absorption lines at $64.275\,cm^{-1}$, $77.130\,cm^{-1}$ and $89.985\,cm^{-1}$ have been observed in a microwave spectrum for a linear rigid diatomic molecule. The moments of inertia I_A and I_B are (I_A is with respect to the bond axis passing through the centre of mass and I_B is with respect to an axis passing through the centre of mass and perpendicular to bond axis) **[GATE 2008]**

(a) both equal to $\frac{\hbar^2}{12.855hc}$ gm cm^2 (b) zero and $\frac{\hbar^2}{12.855hc}$ gm cm^2

(c) both equal to $\frac{\hbar^2}{6.427hc}$ gm cm^2 (d) zero and $\frac{\hbar^2}{6.427hc}$ gm cm^2

29. The three principal moments of inertia of a methanol (CH_3OH) molecule have the property $I_x = I_y = I$ and $I_z \neq I$. The rotational energy eigenvalues are **[GATE 2010]**

(a) $\frac{\hbar^2}{2I}\ell(\ell+1)+\frac{\hbar^2 m_l^2}{2}\left(\frac{1}{I_z}-\frac{1}{I}\right)$ (b) $\frac{\hbar^2}{2I}\ell(\ell+1)$

(c) $\frac{\hbar^2 m_l^2}{2}\left(\frac{1}{I_z}-\frac{1}{I}\right)$ (d) $\frac{\hbar^2}{2I}\ell(\ell+1)+\frac{\hbar^2 m_l^2}{2}\left(\frac{1}{I_z}+\frac{1}{I}\right)$

30. CO_2 molecule has the first few energy levels uniformly separated by approximately 2.5 meV. At a temperature of 300 K, the ratio of the number of molecules in the 4th excited state to the number in the 2nd excited state is about. **[GATE 2010]**

(a) 0.5 (b) 0.6 (c) 0.8 (d) 0.9

31. The moment of inertia of a rigid diatomic molecule A is 6 times that of another rigid diatomic molecule B. If the rotational energies of the molecules are equal, then the corresponding values of the roational quantum numbers J_A and J_B are **[GATE 2014]**

(a) $J_A = 2, J_B = 1$ (b) $J_A = 3, J_B = 1$ (c) $J_A = 5, J_B = 0$ (d) $J_A = 6, J_B = 1$

32. In a rigid-rotator of mass *M*, if the energy of the first excited state is 1 *meV*, then the fourth excited state energy (in *meV*) is________ **[GATE 2015]**

33. Typical energy of the rotational modes in a polyatomic molecule like NH_3 is:

(a) 10^{-6} eV (b) 10^{-3} eV (c) 10^{-4} eV (d) 1 eV

34. Which one of the following molecules does not exhibit a rotational spectrum?

(a) H_2 (b) CO (c) HCl (d) HBr

35. The pure rotational levels of a molecule in the far-infrared region follows the formula F(J) = BJ (J+ 1), where F(J) is the energy of the rotational level with quantum number J and B is the rotational constant. The lowest rotational energy gap in rotational Raman spectrum is :

(a) 2B (b) 4B (c) 6B (d) 8B

36. The molecular spectra of two linear molecules O-C-O and O-C-S are recorded in the microwave region. Which one of the following statement is correct?

(a) Both the molecules would show absorption lines
(b) Both the molecules woulb not show absorption lines
(c) O-C-O would show absorption lines, but not O-C-S.
(d) O-C-S would show absorption lines, but not O-C-O.

37. The rotational energy levels of a rigid diatomic molecules are given by $E_J = B_e J(J+1)$ where J is the rotational quantum number and B_e is the constant. The rotational absorption spectrum of the molecules therefore consists of

(a) One resonance line
(b) Lines that are equally spaced.
(c) Lines where the spacing increases with frequency
(d) Lines where the spacing decreases with frequency.

38. The strongest three lines in the emission spectrum of an interstellar gas cloud are found to have wavelengths $\lambda_0, 2\lambda_0$ and $6\lambda_0$ respectively, where λ_0 is a known wavelength. From this we can deduce that the radiating particles in the cloud behave like

(a) Free particles (b) Particles in a box
(c) Harmonic oscillators (d) Rigid rotators

except 40 :

39. The sharpest sepctrum lines in the microwave rotational spectrum are observed when the sample is a

[✓ (a) solid] (b) liquid (c) liquid crystal (d) gas

40. The unequal spacing in the vibrational energy levels of a diatomic molecule would be due to the

[✓ (a) Presence of anharmonic terms in the potential energy.]

(b) Finite mass of the nuclei.

(c) Frank-Condon principle.

(d) Harmonic nature of the potential energy.

41. The H_2 molecule has a reduced mass $M = 8.35 \times 10^{-28}$ kg and an equilibrium internuclear distance $R = 0.742 \times 10^{-10}$ m. The rotational energy in terms of the rotational quantum number J is :

(a) $E_{\text{rot}}(J) = 7J(J-1)$ meV (b) $E_{\text{rot}}(J) = \frac{5}{2}J(J+1)$ meV **[JEST 2016]**

[✓ (c) $E_{\text{rot}}(J) = 7J(J+1)$ meV] (d) $E_{\text{rot}}(J) = \frac{5}{2}J(J-1)$ meV

42. Two homonuclear diatomic molecules produce different rotational spectra, even through the atoms are known to have identical chemical properties. This leads to the conclusion that the atoms must be

[TIFR 2016]

[✓ (a) isotopes, i.e. with the same atomic number] (b) isobars, i.e. with the same atomic weight

(c) isotones, i.e. with the same neutron number (d) isomers, i.e. with the same atomic number and weight

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Assignment-7 : Vibrational Spectra

1. Which of the following molecules has the lowest vibrational frequency?

(a) $^{4}H^{35}Cl$ (b) $^{2}H^{35}Cl$ (c) $^{4}H^{36}Cl$ (d) $^{4}H^{37}Cl$

Linked Answer Type Q. 2 and Q. 3

The IR spectrum of a diatomic molecule exhibits transitions at 2144 cm^{-1}, 4262 cm^{-1}, 6354 cm^{-1} corresponding to the excitations from the ground state to the first, second, third vibration states respectively.

2. The fundamental ~~transition~~ (in cm^{-1}) of the diatomic molecule is at

(a) 2157 ~~(b)~~ 2170 (c) 2183 (d) 2196

3. The anharmonicity ~~constant (in cm~~$^{-1}$) of the diatomic molecule is

(a) 0.018 (b) 0.012 (c) 0.006 (d) 0.003

4. The vibrational energy levels $v'' = 0$ and $v' = 1$ of a diatomic molecule are separated by 2143 cm^{-1}. It's anharmonicity $(\omega_e x_e)$ is 14 cm^{-1}. The values of ω_e (in cm^{-1}) and first vertone (in cm^{-1}) of this molecule are respectively

(a) 2143, 4286 (b) 2157, 4286 (c) 2157, 4314 (d) 2171, 4258

5. The vabrational frequency and anharmonicity constant of an alkali halide are 300 cm^{-1} and 0.0025 respectively. The positions (in cm^{-1}) of its fundamnetal mode and first overtone are respectively

(a) 300, 600 (b) 298.5, 595.5 (c) 301.5, 604.5 (d) 290, 580

6. The Infrared spectrum of HCl gas shows an absorption band centered at 2885 cm^{-1}. The zero point energy of HCl molecule under harmonics oscillator approximation is $E = \frac{1}{2}h\nu$

(a) $2.8865\times10^{-22}J$ (b) $2.8865\times10^{-20}J$ (c) $5.7730\times10^{-20}J$ (d) $5.7730\times10^{-22}J$

7. Overtones are observed in the vibrational spectra of diatomic molecules when

(a) Anharmonicity is large
(b) Anharmonicity is absent
(c) Vibrational and rotational modes are coupled
(d) An alternating electric field is applied

Common Data for Q.8, Q. 9 and Q. 10

The force constant of the bond in CO molecule is 1870 Nm^{-1}

(Given: The reduced mass of CO molecule is $1.14\times10^{-26}\,kg$)

8. The frequncy of vibration of the CO molecule is

(a) $6.45\times10^{13}s^{-1}$ (b) $9.45\times10^{13}s^{-1}$ (c) $9.45\times10^{11}s^{-1}$ (d) $6.45\times10^{11}s^{-1}$

9. The energy of lowest vibartional level of CO molecule is

(a) 0.07 eV (b) 0.13 eV (c) 0.26 eV (d) 0.39 eV

10. The spacing between the vibrational enrgy levels of CO molecule is

(a) 0.07 eV (b) 0.13 eV (c) 0.26 eV (d) 0.39 eV

Common Data for Q. 11 and Q. 12

The fundamental band and the first overtone for CO molecule is centered at 2143.3 cm^{-1} and 4259.7 cm^{-1} respectively. (Given: The reduced mass of CO molecule is $1.14\times10^{-26}\,kg$)

11. The vibrational frequency of CO molecule is

(a) 2170 cm^{-1} (b) 2467 cm^{-1} (c) 1108 cm^{-1} (d) 3456 cm^{-1}

12. The anharmonicity constant of CO molecule will be

(a) 6.8 cm^{-1} (b) 10.2 cm^{-1} (c) 13.5 cm^{-1} (d) 19.5 cm^{-1}

13. The ratio between the fundamental vibrational frequencies of HI and DI is

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

14. Replacement of hydrogen by deuterium atom bound to a heavy atom X in a polyatomic molecule would reduce the vibrational frequency of X-H streching by a factor of

(a) 2 (b) $\sqrt{2}$ (c) 4 (d) 1.3

15. The R branch in the vibrational spectra of AX molecule exhibits a set equally spaced lines with a separation of 10 cm^{-1}. The rotational constant of AX molecule is

(a) 10 cm^{-1} (b) 20 cm^{-1} (c) 5 cm^{-1} (d) 15 cm^{-1}

16. The vibartional-rotational energy of a diatomic molecule may be written as (where ν is the frequency of vibration and I is the moment of inertia of the molecule)

(a) $\left(n+\frac{1}{2}\right)h\nu+\frac{h^2}{2I}J(J+1)$ $[n=1,2,3.....\text{ and } J=0,1,2,3...]$

(b) $\left(n+\frac{1}{2}\right)h\nu+\frac{h^2}{2I}J(J+1)$ $[n=0,1,2,3.....\text{ and } J=0,1,2,3...]$

(c) $\left(n+\frac{1}{2}\right)h\nu+\frac{h^2}{2I}M^2$ $[n=1,2,3.....\text{ and } M=0,\pm1,\pm2,\pm3...]$

(d) $\left(n+\frac{1}{2}\right)h\nu+\frac{h^2}{2I}M^2$ $[n=0,1,2,3.....\text{ and } M=0,1,2,3....]$

17. The selection rules for the appearances of P branch in the rotational vibrational absorption spectra of a di-atomic molecule with rigid rotator- harmonic oscillator model are

(a) $\Delta v=\pm1,\Delta J=\pm1$ (b) $\Delta v=+1,\Delta J=+1$ (c) $\Delta v=+1,\Delta J=-1$ (d) $\Delta v=-1,\Delta J=-1$

18. The roatational constant for CO molecule in the ground state and the first excited states are 1.9 cm^{-1} and 1.6 cm^{-1} respectively. The percentage change in the internuclear distance due to vibrational excitation is

(a) 9 (b) 30 (c) 16 (d) 0

19. In the IR spectrum of $\left[Co(CN)_5H\right]^{3-}$ the $Co-H$ strech is observed at 1840 cm^{-1}. The $Co-D$ strech in $\left[Co(CN)_5D\right]^{3-}$ will appear at nearly

(a) 1300 cm^{-1} (b) 1400 cm^{-1} (c) 1500 cm^{-1} (d) 1600 cm^{-1}

20. The fundamental mode of HCl occurs at 2886 cm^{-1}. The fundamental mode of DCl will occur

(a) 1367 cm^{-1} (b) 2069 cm^{-1} (c) 2778 cm^{-1} (d) 3024 cm^{-1}

21. The moment of inertia of the HCl^{35} molecule in the $v=0$ and $v=1$ levels is $20.8\times10^{-7}\,kg-m^2$.

The wave number difference between the $R(0)$ and $P(1)$ lines of the fundamemtal band for HCl^{35} molecule is

(a) 4164 m^{-1} (b) 1041 m^{-1} (c) 2082 m^{-1} (d) 5205 m^{-1}

22. For a diatomic molecule with the vibrational quantum number n and rotational quantum number J, the vibrational level spacing $\Delta E_n = E_n - E_{n-1}$ and the rotational level spacing $\Delta E_J = E_J - E_{J-1}$ are approximately **[GATE 2005]**

(a) $\Delta E_n = \text{constant}, \Delta E_J = \text{constant}$ [✓ (b)] $\Delta E_n = \text{constant}, \Delta E_J \propto J$

(c) $\Delta E_n \propto n, \Delta E_J \propto J$ (d) $\Delta E_n \propto n, \Delta E_J \propto J^2$

23. Which one of the following statement is INCORRECT in vibrational spectroscopy with anharmonicity?

(a) The selection rule for vibrational spectroscopy is $\Delta v = \pm 1, \pm 2, \ldots$ **[GATE 2008]**
(b) Anharmonicity leads to multiple absorption lines
(c) The intensities of hot band lines are stronger than the fundamental absorption.
[✓ (d)] The frequencies of hot band lines are smaller than the fundamental absorption.

24. The equilibrium vibration frequency for an oscillator observed at 2990cm^{-1}. The ratio of the frequencies corresponding to the first and the fundamental spectral lines is 1.96. Considering the oscillatorto be anharmonic, the anharmonicity constant is **[GATE 2012]**

(a) 0.005 [✓ (b)] 0.02 (c) 0.05 (d) 0.1

25. If the leading anharmonic correction to the energy of the n-th vibrational level of a diatomic molecule is

$-x_e\left(n+\frac{1}{2}\right)^2 \hbar\omega$ with $x_e = 0.001$, the total number of energy levels possible is approximately

(a) 500 (b) 1000 (c) 250 (d) 750

26. There is no infrared absorption for nitrogen molecule because:

(a) Its polarizability is zero (b) It has no vibrational levels
(c) It has no rotational levels [✓ (d)] Its dipole moment is zero

27. Infrared absorption can be observed in which of the following molecules?

(a) N_2 (b) O_2 [✓ (c)] HCl (d) C_2

28. All vibrations producing a change in the electric dipole moment of a molecule yield

(a) Raman spectra [✓ (b)] Infrared spectra
(c) Ultra-violet spectra (d) X-ray spectra

29. The typical wavelengths emitted by diatomic molecules in purely vibrational and purely rotational transitions are respectively in the region of

(a) Infrared and visible (b) Visible and infrared
[✓ (c)] Infrared and microwave (d) Microwave and infrared

30. Which of the diatomic molecules H_2, HD, D_2 and HT has the highest vibrational frequency?

(a) HT (b) D_2 (c) HD [✓ (d)] H_2

31. The spacing between vibrational energy levels in CO molecule is found to be 8.441×10^{-2} eV. Given that the reduced mass of CO is 1.14×10^{-26} kg, Planck's constant is 6.626×10^{-34} Js and $1\text{eV} = 1.6 \times 10^{-19}$J. The force constant of the bond is CO molecule is

(a) 1.87 N/m (b) 18.7 N/m (c) 187 N/m (d) 1870 N/m

32. A diatomic molecule has vibrational states with energies $E_v = \hbar\omega\left(v+\frac{1}{2}\right)$ and rotational states with energies $E_j = Bj(j+1)$, where v and j are non-negative integers. Consider the transitions in which both the initial and final states are restricted to $v \leq 1$ and $j \leq 2$ and subject to the selection rules $\Delta v = \pm 1$ and $\Delta j = \pm 1$. Then the largest allowed energy of transition is **[NET June 2015]**

(a) $\hbar\omega - 3B$ (b) $\hbar\omega - B$ (c) $\hbar\omega + 4B$ (d) $2\hbar\omega + B$

33. The first ionization potential of K is 4.34 eV, the electron affinity of Cl is 3.82 eV and the equilibrium separation of KCl is 0.3nm. The energy required to dissociate a KCl molecule into a K and a Cl atom is

[NET Dec. 2015]

(a) 8.62 eV (b) 8.16 eV (c) 4.28 eV (d) 4.14 eV

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Assignment-8 : Electronic and Raman Spectra

1. In an experiment, the exciting line of of a sample is observed at 5460 A^0 and stokes line is observed at 5520 A^0. The wavelength of the anti-stokes line is
(a) 5420 A^0 (b) 5401 A^0 (c) 5380 A^0 (d) 5580 A^0

2. In th vibrational Raman spectrum of HF molecule, the Raman lines are observed at wavelengths 2670 A^0 and 3430 A^0 respectively. The fundamental vibrational frequency of the molecule is
(a) $0.62\times10^{14}\ Hz$ (b) $1.02\times10^{14}\ Hz$ (c) $1.24\times10^{14}\ Hz$ (d) $1.62\times10^{14}\ Hz$

3. In the Rotational Raman spectrum of HCl molecule, the displacemnts from the exciting line are represented by

$$\Delta\nu = \pm(120+80J)\ cm^{-1}$$

The moment of inertia of the molecule is
(a) $1.4\times10^{-47}\ kg-m^2$ (b) $2.8\times10^{-47}\ kg-m^2$
(c) $4.2\times10^{-47}\ kg-m^2$ (d) $5.6\times10^{-47}\ kg-m^2$

4. A substance shows a Raman line at 4567 A^0 when exciting line 4358 A^0 is used. The positions of the anti-stokes line for the same substance (when exciting line 4047 A^0 is used) will be
(a) 3882 A^0 (b) 3936 A^0 (c) 4162 A^0 (d) 4226 A^0

Common Data for Q.5 and Q.6

The rotational Raman displacement for HCl molecule is 41.6 cm^{-1}.
(Given: The reduced mass of HCl molecule is $1.62\times10^{-27}\ kg$)

5. The moment of inertia of HCl molecule is
(a) $1.34\times10^{-40}\ kg-m^2$ (b) $2.7\times10^{-40}\ kg-m^2$ (c) $1.34\times10^{-47}\ kg-m^2$ (d) $2.7\times10^{-47}\ kg-m^2$

6. The internuclear distance between the atoms of the molecule is
(a) $0.48A^0$ (b) $0.84A^0$ [✓ (c)] $1.30A^0$ (d) $1.95A^0$

7. The selection rule for the allowed rotational Raman lines is
(a) $\Delta J = 0, \pm2$ (b) $\Delta J = \pm1$ (c) $\Delta J = 0$ (d) $\Delta J = 0, \pm1, \pm2$

8. The first line in the rotational Raman spectra of a diatomic molecule appears with a stokes shift of 12 cm^{-1}. The stokes shift for the second line is
(a) 36 cm^{-1} (b) 24 cm^{-1} (c) 18 cm^{-1} [✓ (d)] 20 cm^{-1}

9. When $^{14}N_2$ (with rotational constant 0f 1.99 cm^{-1}) is exposed to 340 nm light, then the stokes and anti-stokes line for the molecule in the second rotational state can be observed at
(a) 29412 cm^{-1}, 29410 cm^{-1} (b) 29366 cm^{-1}, 29402 cm^{-1}
(c) 14779 cm^{-1}, 58529.9 cm^{-1} (d) 29384 cm^{-1}, 29424 cm^{-1}

Common Data for Q. 10 and Q. 11

10. The rotational constant of $^{14}N_2$ is 2 cm^{-1}. The wave number of the incident radiation in a Raman spectrometer is 20487 cm^{-1}. The wave number of the first scattered Stokes line (in cm^{-1}) of $^{14}N_2$ is

(a) 20479 (b) 20475 (c) 20499 (d) 20495

11. The rotational Stokes line is expected at (in cm^{-1})

(a) 20467 (b) 20469 (c) 20471 (d) 20475

12. A laser operating at 500 nm is used to excite a molecule. If the Stokes line is observed at 770 cm^{-1}, the approximate positions of the Stokes and the anti-Stokes lines are **[NET Dec. 2011]**

(a) 481.5 nm and 520 nm (b) 481.5 nm and 500 nm

(c) 500 nm and 520 nm (d) 500 nm and 600 nm

13. Light of wavelength 1.5 μm incident on a material with a characteristic Raman fiquency of 20×10^{12} Hz results in a stokes-shifted line of wavelength $\left[\text{Given} : c = 3\times10^{8} m.s^{-1}\right]$ **[GATE 2002]**

(a) 1.47 μm (b) 1.57 μm (c) 1.67 μm (d) 1.77 μm

14. In the Raman scattering experiment, light of frequency ν from a laser is scatteered by diatomic molecules having moment of inertia I. The typical Raman shifted frequency depends on **[GATE 2005]**

[✓ (a)] ν and I (b) only ν (c) only I (d) neither ν nor I

15. A vibrational-electronic spectrum of homonuclear binary molecules, involving electronic ground state ε'' and excited ε', exhibits a continuum at $\bar{\upsilon}$ cm^{-1}. If the total energy of the dissociated atoms in the excited state exceeds the total energy of the dissociated atoms in the ground state by $E_{ex} cm^{-1}$, then dissociation energy of the molecule in the ground state is : **[GATE 2006]**

(a) $(\bar{\upsilon}+E_{ex})/2$ (b) $(\bar{\upsilon}-E_{ex})/2$ (c) $(\bar{\upsilon}-E_{ex})$ (d) $\sqrt{(\bar{\upsilon}^2-E_{ex}{}^2)}$

16. A pure rotational Raman spectrum of a linear diatomic molecule is recorded using electromagnetic radiation of frequency u_e. The frequency of two consecutive stokes lines are : **[GATE 2008]**

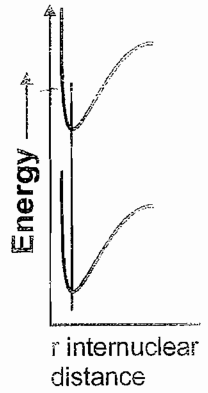
[✓ (a)] $\upsilon_e-10B, \upsilon_e-14B$ (b) $\upsilon_e-2B, \upsilon_e-4B$

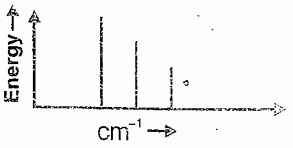
(c) $\upsilon_e+10B, \upsilon_e+14B$ (d) $\upsilon_e+2B, \upsilon_e+4B$

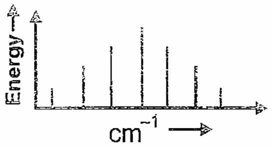
17. The separation between the first stokes and corresponding anti-stokes lines of the rotational Raman spectrum in terms of the rotational constant, B is : **[GATE 2009]**

(a) 2B (b) 4B (c) 6B [✓ (d)] 12B

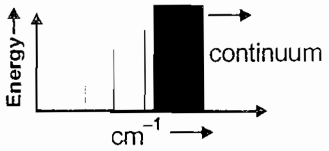
18. In a diatomic molecule, the internuclear separation of the ground and first excited electronic state are the same as shown in the figure. If the molecule is initially in the lowest vibrational state of the ground state, then the absorption spectrum will appear as

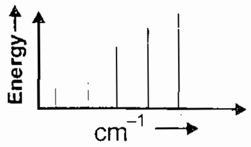


(a) 

(b) 

(18-a)

(c) 

(d) 

[GATE 2009]

19. The far infrared rotational absorption spectrum of diatomic molecule shows equidistant blines with spacing 20 cm^{-1}. The position of the first stokes line in the rotational Raman spectrum of the molecule is **[GATE 2011]**

(a) 20 cm^{-1} (b) 40 cm^{-1} (c) 60 cm^{-1} (d) 120 cm^{-1}

20. The first Stokes line of a rotational Raman spectrum is observed at 12.96 cm^{-1}. Considering rigid rotator approximation, the rotational constant is given by **[GATE 2012]**

(a) 6.48 cm^{-1} (b) 3.24 cm^{-1} (c) 2.16 cm^{-1} (d) 1.62 cm^{-1}

21. The excitation wavelength of laser in a Raman effect experiment is 546 *nm*. If the Stokes line is observed at 552 *nm*, then the wave number of the anti-Stokes line (in cm^{-1}) is____________________

[GATE 2015]

22. The energy separation between two consecutive stokes lines in Raman Scattering depends on
(a) Energy separation between vibrational levels in the excited state
(b) Wavelength of the incident light
(c) Energy separation between vibrational levels in the ground state
(d) Intensity of the incident light

23. The vibrational constants ω_0 and $\omega_0 x_0$ of HCl are 2937.5 cm^{-1} and 51.6 cm^{-1}. The first Raman stokes line will be observed at (in cm^{-1})
(a) 2989.1 (b) 2885.9 (c) 2834.3 (d) 3040.7

24. The intensity of the electronic 0-0 band of a diatomic molecule is very intense when the minimum of the potential curve for the upper electronic state lies (here r is the internuclear distance).
(a) at the same value r as that of the lower potential curve
(b) at a smaller value of r than that of the lower potential curve
(c) at a larger value of r than that of the lower potential curve
(d) above the dissociation level of the lower potential curve

25. Resonance (NMR, ESR etc.) studies in solids provide information about
(a) The electronic structure of single defects
(b) The motion of the spin or of the surrounding
(c) Collective spin excitations
(d) Internal magnetic fields sampled by the spin

26. The vibrational spectrum of a molecule exhibits a strong line with P and R branches at a frequency u_1 and a weaker line at a frequency u_2. The frequency u_3 isnot shown up. Its vibrational Raman spectrum shows a strongly polarized line at frequency u_3 and no feature at u_1 and u_2
(a) The molecule could be linear
(b) The molecule lacks a center of inversion
(c) $\mathbf{u}_1$ arises from a symmetric stretching mode
(d) $\mathbf{u}_3$ arises from a bending mode

27. Consider the following statements about molecular spectra
P: CH_4 does not give pure rotational Raman lines
Q: SF_6 could be studied by rotational Raman spectroscopy
R: N_2 shows infrared absorption spectrum

S: CH_3CH_3 shows vibrational Raman and infrared absorptin lines
T: H_2O_2 shows pure rotational spectrum
Choose the right combination of correct statements
(a) P and Q (b) P, R and T (c) P, S and T (d) Q and R

28. Light of wavelength 1.5 μm incident on a material with a characteristic Raman frquency of 20×10^{12} Hz results in a stokes-shifted line of wavelength [Given : $c = 3\times10^8 m.s^{-1}$]
(a) 1.47 μm (b) 1.57 μm (c) 1.67 μm (d) 1.77 μm

29. Match the following and choose the correct combination **[GATE 2005]**

Group-I	**Group-2**
P. Atomic configuration $1s^2 2s^2 2p^6 3s^2 3p^6$	1. Na
Q. Strongly electropositive	2. Si
R. Strongly electronegative	3. Ar
S. Covalent bonding	4. Cl

(a) P-1, Q-2, R-3, S-4 (b) P-3, Q-2, R-4, S-1
(c) P-3, Q-1, R-4, S-2 (d) P-3, Q-4, R-1, S-2

30. In a Raman scattering experiment, light of frequency u from a laser is scattered by diatomic molecules having the moment of inertia I. the typical Raman shifted frequency depends on
(a) u and I (b) only u (c) only I (d) neither u nor I

31. A vibrational-electronic spectrum of homonuclear binary molecules, involving electronic ground state ε" and excited ε', exhibits a continuum at $\bar{\upsilon}\ cm^{-1}$. If the total energy of the dissociated atoms in the excited state exceeds the total energy of the dissociated atoms in the ground state by $E_{ex} cm^{-1}$, then dissociation energy of the molecule in the ground state is :
(a) $(\bar{\upsilon}+E_{ex})/2$ (b) $(\bar{\upsilon}-E_{ex})/2$ (c) $(\bar{\upsilon}-E_{ex})$ (d) $\sqrt{(\bar{\upsilon}^2-E_{ex}{}^2)}$

32. Match the following: [GATE 2007]
P: Franck-Hertz experiment 1. Electronic excitation of molecules
Q: Hartree-Fock method 2. Wave function of atom
R: Stern-Gerlach experiment 3. Spin angular momentum of atoms
S: Franck-Condon principle 4. Energy levels in atoms

(a)	(b)	(c)	(d)
P-4	P-1	P-3	P-4
Q-2	Q-4	Q-2	Q-1
R-3	R-3	R-4	R-3
S-1	S-2	S-1	S-2

33. The number of fundamental vibrational modes of CO_2 molecule is :
(a) Four: 2 are Raman active and 2 are infrared active
(b) Four: 1 is Raman active and 3 are infrared active
(c) Three: 1 is Raman active and 2 are infrared active
(d) Three: 2 are Raman active and 1 is infrared active

3N-5

34. A pure rotational Raman spectrum of a linear diatomic molecule is recorded using electromagnetic radiation of

frequency u_e. The frequency of two consecutive stokes lines are :

[✓ (a) $\upsilon_e - 10B, \upsilon_e - 14B$]
(b) $\upsilon_e - 2B, \upsilon_e - 4B$
(c) $\upsilon_e + 10B, \upsilon_e + 14B$
(d) $\upsilon_e + 2B, \upsilon_e + 4B$

35. The separation between the first stokes and corresponding anti-stokes lines of the rotational Raman spectrum in terms of the rotational constant, B is :

(a) 2B
(b) 4B
(c) 6B
[✓ (d) 12B]

36. Match the typical spectra of stable molecules with the corresponding wave-number rnage **[GATE 2010]**

1. Electronic spectra	i. 10^6 cm^{-1} and above
2. Rotational spectra	ii. $10^5 - 10^6$ cm^{-1}
3. Molecular dissociation	iii. $10^0 - 10^2$ cm^{-1}

R < V < E < M.
wave no.

(a) 1–ii, 2–i, 3–iii
[✓ (b) 1–ii, 2–iii, 3–i]
[✓ (c) 1–iii, 2–ii, 3–i]
(d) 1–i, 2–ii, 3–iii.

37. You are shown a spectrum consisting of a series of equally spaced lines. This could be

(a) The rotational spectrum CO.
[✓ (b) The vibrational spectrum of N_2.]
(c) The NMR spectrum of CH_4
(d) The Mossbauer spectrum of Fe_3O_4.

38. A laser operating at 500 nm is used to excite a molecule. If the Stokes line is observed at 770 cm^{-1}, the approximate positions of the Stokes and the anti-Stokes lines are

(a) 481.5 nm and 520 nm
(b) 481.5 nm and 500 nm
(c) 500 nm and 520 nm
(d) 500 nm and 600 nm

39. If a molecule has a centre of symmetry then

(a) Raman active vibrations are infrared active
[✓ (b) Raman active vibrations are infrared inactive and vice versa]
(c) some, but not all, of the Raman active vibrations are infrared active
(d) all vibrations are Raman and infrared inactive.

40. Match the typical spectroscopic regions specified in **Group I** With the corresponding type of transitions in **Group II.** [GATE 2012]

Group I	**Group II**
(P) Infra-red region	(i) electronic transitions involving valence electrons
(Q) Ultraviolet- visible region	(ii) nuclear transitions
(R) X-ray region	(iii) vibrational transitions of molecules
(S) γ-ray region	(iv) transitions involving inner shell electrons

(a) $(P,i);(Q,iii);(R,ii);(S,iv)$
(b) $(P,ii);(Q,iv);(R,i);(S,iii)$
[✓ (c) $(P,iii);(Q,i);(R,iv);(S,ii)$]
(d) $(P,iv);(Q,i);(R,ii);(S,iii)$

41. Raman effect is suppressed in

(a) semiconductors
(b) dielectrics
[✓ (c) metals]
(d) insulators

42. Match the phrases in Group I and Group II and identify the correct option. **[GATE 2015]**

Group I	Group II
(P) Electron spin resonance (ESR)	(i) radio frequency
(Q) Nuclear magnetic resonance (NMR)	(ii) visible range frequency
(R) Transition between vibrational states of a molecule	(iii) microwave frequency
(S) Electronic transition	(iv) far-infrared range

(a) (P-i), (Q-ii), (R-iii), (S-iv) (b) (P-ii), (Q-i), (R-iv), (S-iii)
(c) (P-iii), (Q-iv), (R-i), (S-ii) [✓ (d) (P-iii), (Q-i), (R-iv), (S-ii)]

43. The excitation wavelength of laser in a Raman effect experiment is 546 *nm*. If the Stokes line is observed at 552 *nm*, then the wave number of the anti-Stokes line (in cm^{-1}) is 18515 cm^{-1} **[GATE 2015]**

44. The molecule $^{17}O_2$ is **[GATE 2016]**

(a) Raman active but not NMR (nuclear magnetic resonance) active
(b) Infrared active and Raman active but not NMR active
[✓ (c) Raman active and NMR active]
(d) Only NMR active

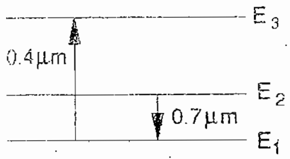
17

$10^{-3} \times 10^{7}$

CSIR-UGC-NET/JRF | GATE PHYSICS

Assignment-9: Laser

1. Consider the energy level diagram (as shown in the figure below) of a typical three level ruby laser system with 1.6×10^{19} Chromium ions per cubic centimeter. All the atoms excited by the $0.4\ \mu m$ radiation decay rapidly to level E_2 which has a lifetime $\tau = 3ms$ **[CSIR June 2011]**



A. Assuming that there is no radiation of wavelength $0.7\ \mu m$ present in the pumping cycle and that the pumping rate is R atoms per cm³, the population density in the level N_2 builds up as:

(a) $N_2(t)=R\tau\left(e^{t/\tau}-1\right)$　　(b) $N_2(t)=R\tau\left(1-e^{-t/\tau}\right)$

(c) $N_2(t)=\frac{Rt^2}{\tau}\left(1-e^{-t/\tau}\right)$　　(d) $N_2(t)=Rt$

B. The minimum pump power required (per cubic centimeter) to bring the system to transparency, i.e. zero gain, is

(a) 1.52 kW　　(b) 2.64 kW　　(c) 0.76 kW　　(d) 1.32 kW

2. Consider a hydrogen atom undergoing a $2P \to 1S$ transition. The lifetime t_{sp} of the 2P state for spontaneous emission is 1.6 ns and the energy difference between the levels is 10.2 eV. Assuming that the refractive index of the medium $n_0 = 1$, the ratio of the Einstein coefficients for stimulated emission $B_{21}(\omega)/A_{21}(\omega)$ is given by

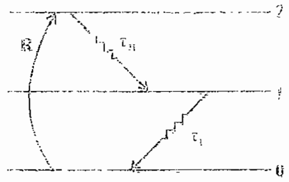
Calculate Einstein Coefficient A and B.

[CSIR Dec. 2012]

(a) $0.683\times10^{12}\ m^3J^{-1}s^{-1}$　　(b) $0.146\times10^{-12}\ J\,s\,m^{-3}$

(c) $6.83\times10^{12}\ m^3\ J^{-1}\ s^{-1}$　　(d) $1.463\times10^{-12}\ J\,s\,m^{-3}$

3. Consider the energy level diagram shown below, which corresponds to the molecular nitrogen laser. **[CSIR Dec. 2012]**



If the pump rate R is 10^{20} atoms $cm^{-3}\,s^{-1}$ and the decay routes are as shown with $\tau_{21} = 20$ ns and $\tau_1 = 1\,\mu s$, the equilibrium populations of states 2 and 1 are, respectively.

(a) $10^{14}\ cm^{-3}$ and $2\times10^{12}\ cm^{-3}$　　(b) $2\times10^{12}\ cm^{-3}$ and $10^{14}\ cm^{-3}$

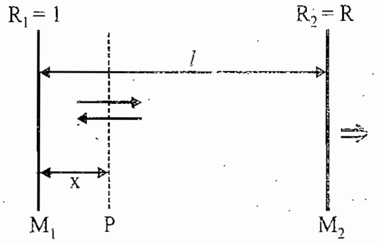
(c) $2\times10^{12}\ cm^{-3}$ and $2\times10^{6}\ cm^{-3}$　　(d) zero and $10^{20}\ cm^{-3}$

4. Consider a He-Ne laser cavity consisting of two mirrors of reflectivities $R_1 = 1$ and $R_2 = 0.98$. The mirrors are separated by a distance d = 20 cm and the medium in between has a refractive index $n_0 = 1$ and absorption coefficient $\alpha = 0$. The values of the separation between the modes $\delta\nu$ and the width $\Delta\nu_p$ of each mode of the laser cavity are: **[NET Dec. 2012]]**

(a) $\delta\nu = 75$ kHz, $\Delta\nu_p = 24$ kHz (b) $\delta\nu = 100$ kHz, $\Delta\nu_p = 100$ kHz

(c) $\delta\nu = 750$ MHz, $\Delta\nu_p = 2.4$ MHz (d) $\delta\nu = 2.4$ MHz, $\Delta\nu_p = 750$ MHz

5. Consider the laser resonator cavity shown in the figure.
If I_1 is the intensity of the radiation at mirror M_1 and α is the gain coefficient of the medium between the mirrors, then the energy density of photons in the plane P at a distance x from M_1 is



$$I = I_0\left(e^{\gamma x} + e^{-\gamma x}\right)$$
$$= \frac{I_1}{c}$$

[NET June 2013]

(a) $(I_1/c)e^{-\alpha x}$ (b) $(I_1/c)e^{\alpha x}$ (c) $(I_1/c)(e^{\alpha x} + e^{-\alpha x})$ (d) $(I_1/c)e^{2\alpha x}$

6. The electronic energy levels in a hydrogen atom are given by $E_n = -13.6/n^2$ eV. If a selective excitation to the n = 100 level is to be made using a laser, the maximum allowed frequency line-width of the laser is **[NET June 2013]**

(a) 6.5 MHz (b) 6.5 GHz (c) 6.5 Hz (d) 6.5 kHz

7. Two monochromatic sources, L_1, and L_2, emit light at 600 and 700 nm respectively. If their frequency bandwidths are 10^{-1} and 10^{-3} GHz, respectively, then the ratio of linewidth of L_1 and L_2 is approximately **[NET Dec. 2013]**

(a) 100 : 1 (b) 1 : 85 (c) 75 : 1 (d) 1 : 75

8. A gas laser cavity has been designed to operate at $\lambda = 0.5\mu m$ with a cavity length of 1m. With this set-up, the frequency is found to be larger than the desired frequency by 100 Hz. The change in the effective length of the cavity required to retune the laser is **[NET Dec. 2013]**

(a) -0.334×10^{-12} m (b) 0.334×10^{-12} m
(c) 0.167×10^{-12} m (d) -0.167×10^{-12} m

9. A three level system of atoms has N_1 atoms in level E_1, N_2 in level E_2, and N_3 in level E_3 ($N_2 > N_1 > N_3$ and $E_1 < E_2 < E_3$). Laser emission is possible betwen the levels **[GATE 2005]**

(a) $E_3 \to E_1$ (b) $E_2 \to E_1$ (c) $E_3 \to E_2$ (d) $E_2 \to E_3$

10. Which one of the following electronic transitions in Neon is NOT responsible for LASER action in a helium-neon laser? **[GATE 2006]**

(a) 6s → 5p (b) 5s → 4p (c) 5s → 3p (d) 4s → 3p

11. In a He-Ne laser, the laser transition takes place in **[GATE 2007]**

(a) He only (b) Ne only
(c) Ne first, then in He. (d) He first, then in Ne

9 → Most populated state is metastable state. 5-c

12. The coherence length of laser light is : **[GATE 2008]**
(a) Directly proportional to the length of the active medium
(b) Directly proportional to the width of the spectral line
[✓ (c)] Inversely proportional to the width of the spectral line
(d) Inversely proportional to the length of the active lasing medium

13. When the refractive index m of the active medium changes by $\Delta\mu$ in a laser resonator of length L, the change in the spectral spacing between the longitudinal modes of the laser is (c is the speed of light in free space) **[GATE 2008]**

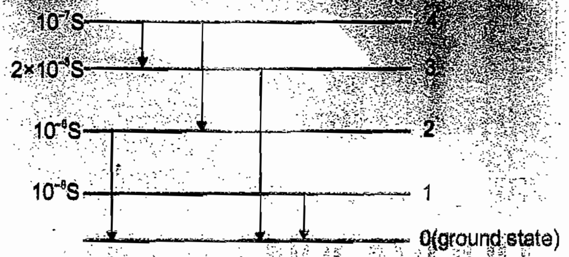
(a) $\frac{c}{2(\mu+\Delta\mu)L}$ (b) $\frac{c}{2\Delta\mu L}$ [✓ (c)] $\frac{c}{2L}\frac{\Delta\mu}{\mu(\mu+\Delta\mu)}$ (d) zero

14. A collection of N atoms is exposed to a strong resonant electromagnetic radiation with N_g atoms in the ground state and N_e atoms in the excited state, such that $N_g + N_e = N$. This collection of two–level atoms will have the following population distribution. **[GATE 2010]**
(a) $N_g << N_e$ (b) $N_g >> N_e$ [✓ (c)] $N_g \approx N_e \approx N/2$ (d) $N_g - N_e \approx N/2$

15. The population inversion in a two level laser material CANNOT be achieved by optical pumping because **[GATE 2011]**
[✓ (a)] The rate of upward transitions is equal to the rate of downward transitions.
(b) The upward transitions are forbidden but downward transitions are allowed.
(c) The upward transitions are allowed but downward transitions are forbidden.
(d) The spontaneous decay rate of the higher level is very low.

16. The intensity of a laser in free space is 150 mW/m². The corresponding amplitude of the electric field of the laser is ______________ V/m **[GATE 2014]**

17. Five energy levels of a system including the ground state are shown below. Their lifetimes and the allowed electric dipole transitions are also marked.



Which one of the following transitions is the most suitable for a continuous wave (CW) laser?
(a) $1 \rightarrow 0$ [✓ (b)] $2 \rightarrow 0$ (c) $4 \rightarrow 2$ (d) $4 \rightarrow 3$

18. For an atom with two energy levels placed in a cavity containing blackbody radiation, the ratio of the probabilities for spontaneous emission and stimulated emission is given by $\frac{A}{B}=\frac{8\pi v^3}{c^3}$, where A and B are the corresponding Einstein coefficients and v is the frequency of the photon emitted. The probability of spontaneous emission.
[✓ (a)] Is independent of the energy difference between the two levels.
(b) Increases with the energy difference between the two levels.
(c) Decreases with the energy difference between the two levels.
(d) Is zero.

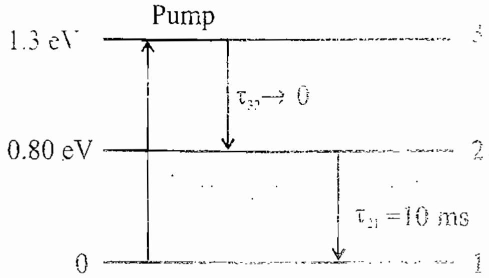
19. The mirrors of a laser cavity are separated by a distance L. If T is the time taken by the light to travel from one mirror to the other and back, the mode separation is:

[✓ (a)] $\frac{1}{T}$ (b) $\frac{2}{T}$ (c) $\frac{1}{2T}$ (d) $\frac{1}{\sqrt{2}\,T}$

20. The central wavelength of a $0.6\,\mu m$ wavelength laser corresponds to the m^{th} cavity mode of a resonator cavity of length 6 cm. The mode number m is

(a) 1000　(b) 2×10^4　[✓ (c) 2×10^5]　(d) 200

21. The typical energy level diagram for Erbium ions in a glass is shown below.



For an Erbium ion concentration of ρ ions/~~c~~m^3, the pump power needed for population inversion is

(a) $80\rho\,eV/m^3s$　(b) $40\rho\,eV/m^3s$　(c) $130\rho\,eV/m^3s$　[✓ (d) $65\rho\,eV/m^3s$]

22. EMT. The approximate force exerted on a perfectly reflecting mirror by an incident laser beam of power 10 mW at normal incidence is

(a) 10^{-13} N　(b) 10^{-11} N　(c) 10^{-9} N　(d) 10^{-15} N

Common Data for Q.23 & Q.24

A laser beam $(\lambda = 694.3\ nm)$ has a power density of 100 mW and an aperture of 5 mm. The beam is focussed with a lens of focal length 0.1 m.

23. The area of the image will be

(a) $220\ \mu m^2$　(b) $290\ \mu m^2$　(c) $340\ \mu m^2$　(d) $380\ \mu m^2$

24. The intensity of the image (in MW/m²) will be

(a) 346　(b) 392　(c) 435　(d) 491

25. A laser beam $(\lambda = 600\ nm)$ has power 10 mW and angular spread 1.5×10^{-4} radian. The coherence length of the laser will be

(a) 2 mm　(b) 4 mm　(c) 6 mm　(d) 8 mm

26. A He-Ne laser on transition from 3s to 2p level gives a laser emission of wavelength 632.8 nm. If the 2p level has energy 9.5 eV. The minimum pumping energy required is (assume there is no loss)

(a) 6.7 eV　(b) 8.9 eV　[✓ (c) 11.5 eV]　(d) 13.9 eV

27. A lamp emits a radiation of wavelength 500 nm at a temparature of 1000K. The ratio of the Stimulated emission rate and spontaneous emission rate will be of the order of

(a) 10^{-7}　(b) 10^{-9}　(c) 10^{-11}　[✓ (d) 10^{-13}]

28. The temparature at which stimulated emission rate and spontaneous emission rates are equal $(\lambda = 500\ nm)$, is

[✓ (a) 41. 6 K]　(b) 83.2 K　(c) 124.8 K　(d) 166.4 K

29. Consider an optical resonator of length 0.3 m. The number of the modes in the wavelength range 2 nm centered about 500 nm, will be

(a) 3600　(b) 4200　[✓ (c) 4800]　(d) 5400

30. The number of photons emitted per second from a 1 Watt Ar-ion laser operating at 488.0 nm is approximately

(a) 10.23×10^{19}　[✓ (b) 2.46×10^{18}]　(c) 10.23×10^{17}　(d) 2.46×10^{15}

31. Spectral line width of the He-Ne laser is 0.01 nm and the cross sectional area of the beam is 0.01 cm². If the output power is 1 milli Watt, the radiation intensity per unit wavelength (in Watt/cm³) is:

(a) 10^{10}　[✓ (b) 10^8]　(c) 10^{-8}　(d) 10^{-10}

32. A laser beam emerging from a laser tube operating at 800 nm has a cross sectional diameter of 2 mm. The diameter of the beam at a distance of 1 km is approximately given by

(a) 10 nm (b) 8 cm (c) 80 cm (d) 10 m

33. In which of the following lasers can the wavelength not be tuned?

(a) semiconductor laser (b) Dye laser
(c) Ti: Sapphire laser (d) Ruby laser

34. Consider a He-Ne laser cavity consisting of two mirrors of reflectivities $R_1 = 1$ and $R_2 = 0.98$. The mirrors are separated by a distance $d = 20$ cm and the medium in between has a refractive index $n_0 = 1$ and absorption coefficient $\alpha = 0$. The values of the separation between the modes $\delta\nu$ and the width $\Delta\nu_p$ of each mode of the laser cavity are:

(a) $\delta\nu = 75$ kHz, $\Delta\nu_p = 24$ kHz (b) $\delta\nu = 100$ kHz, $\Delta\nu_p = 100$ kHz

(c) $\delta\nu = 750$ MHz, $\Delta\nu_p = 2.4$ MHz (d) $\delta\nu = 2.4$ MHz, $\Delta\nu_p = 750$ MHz

35. The emission wavelength for the transition ${}^1D_2 \rightarrow {}^1F_3$ is 3122 Å. The ratio of populations of the final to the initial states at a temperature 5000 K is $(h = 6.626\times10^{-34}\ J.s, c = 3\times10^8\ m/s, k_B = 1.380\times10^{-23}\ J/K)$

(a) 2.03×10^{-5} (b) 4.02×10^{-5} (c) 7.02×10^{-5} (d) 9.83×10^{-5}

36. A He-Ne laser operates by using two energy levels of Ne separated by 2.26 eV. Under steady state conditions of optical pumping, the equivalent temperature of the system at which the ratio of the number of atoms in the upper state to that in the lower state will be 1/20, is approximately (the Boltzmann constant $k_B = 8.6\times10^{-5}$eV/K). **[NET June 2015]**

(a) 10^{10} K (b) 10^8 K (c) 10^6 K (d) 10^4 K

37. For a two levels system, the population of atoms in the upper and lower levels are 3×10^{18} and 0.7×10^{18}, respectively. If the coefficient of stimulated emission is 3.0×10^5 m^3/W-s^3 and the energy density is 9.0 J/m^3-Hz, the rate of stimulated emission will be **[NET Dec. 2015]**

(a) 6.3×10^{16} s^{-1} (b) 4.1×10^{16} s^{-1}
(c) 2.7×10^{16} s^{-1} (d) 1.8×10^{16} s^{-1}

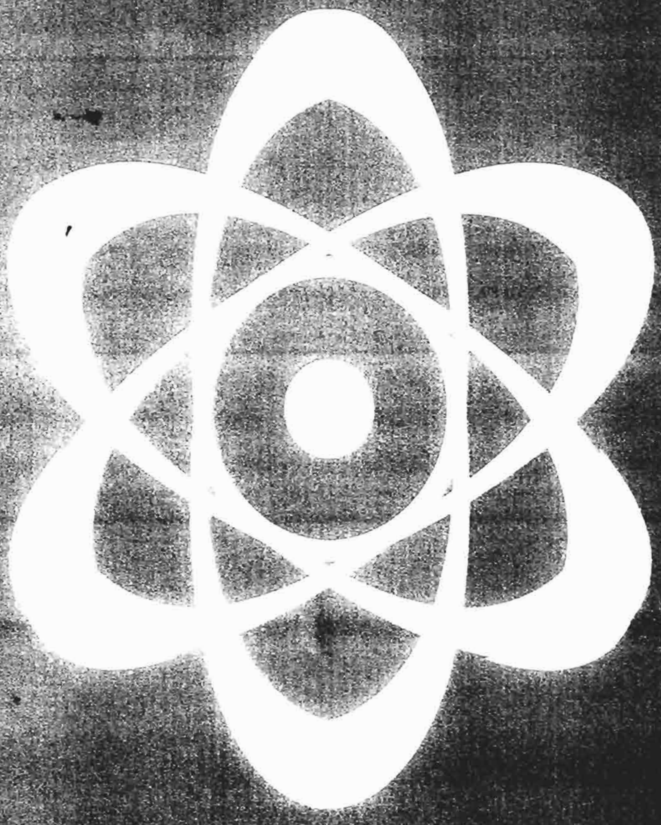
38. The separation between the energy levels of a two-level atom is 2 eV. Suppose that 4×10^{20} atoms are in the ground state and 7×10^{20} atoms are pumped into the excited state just before lasing starts. How much energy will be released in a single laser pulse ? **[NET June 2016]**

(a) 24.6 J (b) 22.4 J (c) 98 J (d) 48 J

CLASSICAL MECHANICS

CSIR-UGC-NET Dec. 2016 | GATE 2017

NET | GATE | TIFR | JEST



Previous Years' Questions

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62.

Tarakjyoti Kaur

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CLASSICAL MECHANICS

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CSIR-UGC-NET/JRF | GATE PHYSICS

Assignment-1: Classical Mechanics (General)

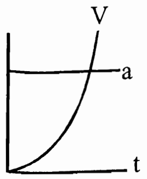
1. A ray of light is reflected from a mirror. If the mirror is slowly rotated with an angular velocity w, then the angle between the incident and the reflected rays changes at the rate
(a) –w (b) w (c) 2w (d) w/2

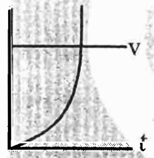
2. Two similar cars of mass m each, moving with a uniform speed v, make a head-on collision and come to rest. The damage suffered by the cars is directly proportional to
(a) v (b) v^2 (c) m^2v^2 (d) mv

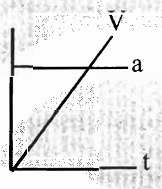
3. A person sitting on a chair cannot get up without bending forward because,
(a) Human body is not rigid. (b) Newtons third law is operative.
(c) Newtons second law is operative (d) Gravitational attraction is strong.

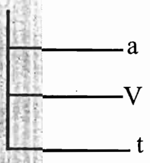
4. The work done in moving an object along a vector $2\hat{i}-3\hat{j}+4\hat{k}$, when a force $\vec{F}=\hat{i}-2\hat{j}-2\hat{k}$ is applied is:
(a) Zero (b) 29 (c) $\sqrt{38}$ (d) 3

5. An object is dropped from certain height. Among the following which graph correctly explain change in magnitude of acceleration and velocity before hitting the ground

(a) 

(b) 

(c) 

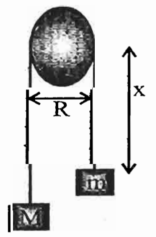
(d) 

6. In Millikan's oil-drop experiment an oil drop of radius r, mass m and charge $q \approx 6\pi\eta r(v_1+v_2)/E$ is moving upwards with a terminal velocity v_2 due to an applied electric field of magnitude E, where η is the coefficients of viscosity. The acceleration due to gravity is given by: **[JEST 2016]**
(a) $g=6\pi\eta r v_1/m$ (b) $g=3\pi\eta r v_1/m$ (c) $g=6\pi\eta r v_2/m$ (d) $g=3\pi\eta r v_2/m$

7. A plane is flying at height 10 km with speed of 1000 km/hr. What would be angular velocity of plane when it is observed by person standing on earth
(a) 1 rad/sec (b) 36 rad/sec (c) π rad/sec (d) π/2 rad/sec

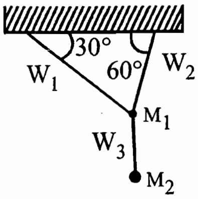
8. Two particles approach each other with different velocities. After collision, one of them is found to have momentum $\vec{p}$ in their centre of mass frame. In the same reference frame, the other particle must have momentum
(a) Zero (b) $\frac{-\vec{p}}{2}$ (c) $-\vec{p}$ (d) $-2\vec{p}$

9. A heavy uniform rope of length L and mass per unit length μ goes over a frictionless pulley of diameters R, and has two masses M and m attached to its two ends as shown. In terms of the distance x, the equilibrium position is given by



(a) $\frac{1}{2}\left(L-\frac{\mu L}{m+M}+\frac{\pi R}{2}\right)$ (b) $\frac{1}{2}\left(L+\frac{m+M}{\mu}\right)$ (c) $L-\frac{\pi R}{2}$ (d) $\frac{1}{2}\left(L-\frac{m-M}{\mu}-\frac{\pi R}{2}\right)$

10. Two masses M_1 and M_2 $(M_1 < M_2)$ are suspended from a perfectly rigid horizontal support by a system of three taut massless wires W_1, W_2 and W_3, as shown in the figure. All the three wires have identical cross-section and elastic properties and are know to be very strong. **[TIFR 2012]**



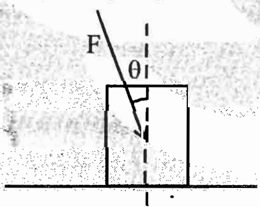
If the mass M_2 is increased gradually, but without limit, we should expect the wires to break in the following order :

(a) First W_2, then W_1 (b) First W_1, then W_2 (c) First W_2, then W_3 (d) First W_3

11. A high-velocity missile, travelling in a horizonal line with a kinetic energy of 3.0 Giga-Joules (GJ), explodes in flight and breaks into two pieces A and B of equal mass. One of these Pieces (A) flies of in a straight line perpendicular to the original direction in which the missile was moving and its kinetic energy is found to be 2.0 GJ. If gravity can be neglected for such high-velocity projectiles, it follows that the other piece (B) flew off in a direction at an angle with the original direction of

(a) 30° (b) 33° 24' (c) 45° (d) 60°

12. A block of negligible mass rests on a horizontal surface with coefficient of friction μ. A force F is applied downwards, at an angle θ to the vertical (see figure).



The maximum value of the angle at which the force cannot make the block slip is:

(a) $\theta = \tan^{-1}\mu$ (b) $\theta = \cot^{-1}\mu$ (c) $\theta = \sin^{-1}\mu$ (d) $\theta = \cos^{-1}\mu$

13. In an experiment it is found that the combination $\frac{F}{\rho v^2}$ increases with a spatial dimension R as R^α. Here F is a force, ρ is a mass density and v is a velocity. The value of α is

(a) 4 (b) 1 (c) 3 (d) 2

14. A rope of negligible mass goes around a frictionless pulley. A monkey is on one end of the rope, while at the other end hangs a mass that is of the same weight as the monkey. When the monkey starts to climb up the rope, then

(a) Both monkey and the mass move up, but the mass moves up at a rate exacty half that of the monkey
(b) The monkey remains at the same height, the mass moves up
(c) The monkey moves down, the mass moves up
(d) The monkey and the mass move up by the same distance.

15. A girl measures the period of simple pendulum inside a stationary lift and finds it to be T seconds. If the lift accelerates upward with an acceleration g/4, then the period of the pendulum will be

(a) T (b) T/4 (c) $2T/\sqrt{5}$ (d) $2T\sqrt{5}$

16. A particle of mass m is thrown upward with velocity v and there is retarding air resistance proportional to the square of the velocity with proportionality constant k. If the particle attains a maximum height after time t, and g is the gravitational acceleration, what is the velocity v? **[JEST]**

(a) $\sqrt{\frac{k}{g}}\tan(\sqrt{\frac{g}{k}}t)$ (b) $\sqrt{gk}\tan(\sqrt{\frac{g}{k}}t)$ (c) $\sqrt{\frac{g}{k}}\tan(\sqrt{gk}t)$ (d) $\sqrt{gk}\tan(\sqrt{gk}t)$

17. The free fall time of a test mass on an object of mass M from a height $2R$ to R is **[JEST]**

(a) $(\pi/2+1)\sqrt{\frac{R^3}{GM}}$ (b) $\sqrt{\frac{R^3}{GM}}$ (c) $(\pi/2)\sqrt{\frac{R^3}{GM}}$ (d) $\pi\sqrt{\frac{2R^3}{GM}}$

18. A small mass M hangs from a thin string and can swing like a pendulum. It is attached above the window of a car. When the car is at rest, the string hangs vertically. The angle made by the string with the vertical when the car has a constant acceleration $a = 1.2$ m/s^2 is approximately.

(a) 1° (b) 7° (c) 15° (d) 90°

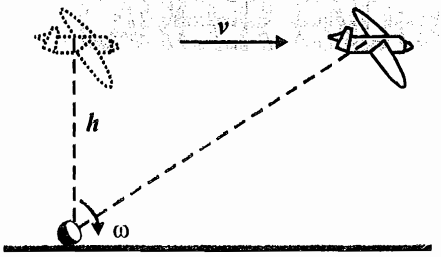
19. The period of a simple pendulum inside a stationary lift is T. If the lift accelerates downwards with an acceleration $g/4$, the period of the pendulum will be

(a) T (b) $T/4$ (c) $2T/\sqrt{3}$ (d) $2T/\sqrt{5}$

20. Spherical particles of a given material of density ρ are released from rest inside a liquid medium of lower density. The viscous drag force may be approximated by the Stoke's law, i.e, $F_d = 6\pi\eta R\text{v}$, where η is the viscosity of the medium, R the radius of a particle and v its instantaneous velocity. If $\tau(m)$ is the time taken by a particle of mass m to reach half its terminal velocity, then the ratio $\tau(8m)/\tau(m)$ is

(a) 8 (b) 1/8 (c) 4 (d) 1/4 **[NET]**

21. The directed beam from a small but powerful searchlight placed on the ground tracks a small plane flying horizontally at a fixed height 'h' above the ground with a uniform velocity v, as shown in the figure below.



[TIFR 2014]

If the searchlight starts rotating with an instantaneous angular velocity ω_0 at time $t = 0$ when the plane was directly overhead, then at a later time 't' its instantaneous angular velocity $\omega(t)$ is given by

(a) $\omega_0\exp(-\omega_0 t)$ (b) $\frac{\omega_0}{1+\tan\omega_0 t}$ (c) $\frac{\omega_0}{1+\omega_0^2 t^2}$ (d) $\frac{\omega_0}{1-\omega_0 t+\frac{1}{2}\omega_0^2 t^2}$

22. The conservation principles for energy, linear momentum and angular momentum arise from the necessity that **[TIFR 2014]**

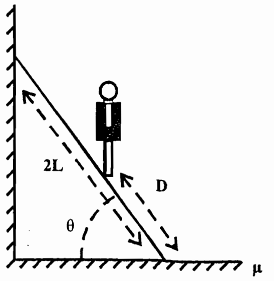
(a) The laws of physics should not involve infinite quantities.

(b) Internal forces on a body should cancel out, by Newton's (third) law of action and reaction

(c) Physical measurements should be independent of the origin and orientation of the coordinate system.

(d) The laws of physics should be independent of the state of rest or motion of the observer.

23. A uniform ladder of length 2L and mass 'm' leans against a wall in a vertical plane at an angle θ to the horizontal. The floor is rough, having a coefficient of static friction μ. A person of mass M stands on the ladder at a distance D from its base (see figure). If the wall is frictionless, the maximum distance (D_{max}) up the ladder that the person can reach before the ladder slips is **[TIFR 2014]**



[illegible]
[illegible]
Total Torque [illegible]

(a) $2\mu L\left(1+\frac{m}{M}\right)\tan\theta$

(b) $\left\{2\mu\left(1+\frac{m}{M}\right)\tan\theta-\frac{m}{M}\right\}L$

(c) $\mu L\tan\theta$

(d) $2\mu L\frac{m}{M}\tan\theta$

24. A body of mass m falls from rest at a height 'h' under gravity (acceleration due to gravity g) through a dense medium which provides a resistive force $F=-kv^2$, where k is a constant and v is the speed. It will hit the ground with a kinetic energy **[TIFR 2014]**

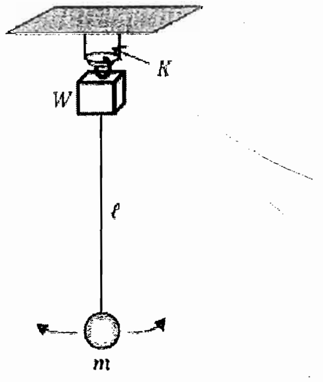
(a) $\frac{m^2g}{2k}\exp\left(-\frac{2kh}{m}\right)$

(b) $\frac{m^2g}{2k}\tanh\frac{2kh}{m}$

(c) $\frac{m^2g}{2k}\left\{1+\exp\left(-\frac{2kh}{m}\right)\right\}$

(d) $\frac{m^2g}{2k}\left\{1-\exp\left(-\frac{2kh}{m}\right)\right\}$

25. A weight W is suspended from a rigid support by a hard string with stiffness constant K. The spring is enclosed in a hard plastic sleeve, which prevents horizontal motion, but allows vertical oscillations (see figure). A simple pendulum of length 'l' with a bob of of mass m(mg <<W) is suspended from the weight W and is set oscillating in a horizontal line with a small amplitude. After some time has passed, the weight W is observed to be oscillating up and down with a large amplitude (but not hitting the sleeve). It follows that the stiffness constant K must be



(a) $K=\frac{4W}{\ell}$ (b) $K=\frac{2W}{\ell}$ (c) $K=\frac{W}{\ell}$ (d) $K=\frac{W}{2\ell}$ **[TIFR 2014]**

26. The time period of a simple pendulum under the influence of the acceleration due to gravity g is T. The bob is subjected to an additional acceleration of magnitude $\sqrt{3}g$ in the horizontal direction. Assuming small oscillations, the mean position and time period of oscillation, respectively, of the bob will be **[NET June 2014]**

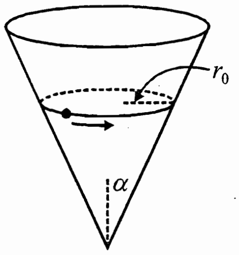
(a) 0° to the vertical and $\sqrt{3}T$

(b) 30° to the vertical and T/2

(c) 60° to the vertical and $T/\sqrt{2}$

(d) 0° to the vertical and $T/\sqrt{3}$

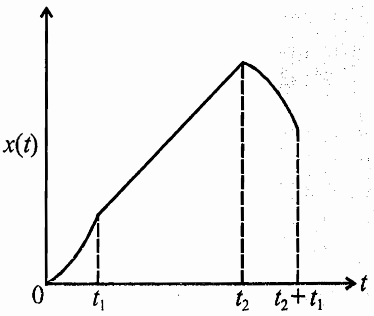
27. A particle slides on the inside surface of a frictionless cone. The cone is fixed with its tip on the ground and its axis vertical, as shown in the figure on the right. The semi-vertex angle of the cone is α. If the particle moves in a circle of radius r_0, without slipping downwards, the angular frequency w of this motion will be **[TIFR 2015]**



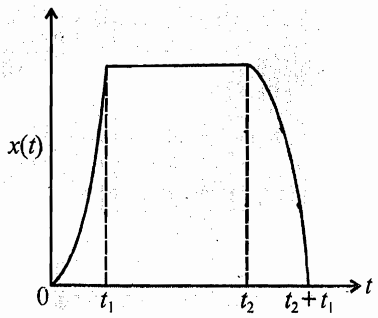
(a) $\sqrt{\dfrac{g}{r_0 \cos\alpha}}$ (b) $\sqrt{\dfrac{g}{r_0 \sin\alpha}}$ (c) $\sqrt{\dfrac{g}{r_0 \cot\alpha}}$ (d) $\sqrt{\dfrac{g}{r_0 \tan\alpha}}$

28. A car starts from rest and accelerates under a force F increasing linearly in time as $F = at$ where a is a constant. At time $t_1 > 0$, the force $\vec{F}$ is suddenly switched off. At a later time $t_2 > t_1$, brakes are applied resulting in a force F' whose magnitude increases linearly with time, $F' = -a(t - t_2)$ where a is the same constant as before. Which of the following graphs would best represent the change in the position of the car $x(t)$ with time ? **[TIFR 2015]**

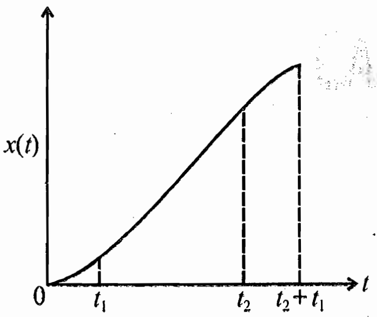
(a)



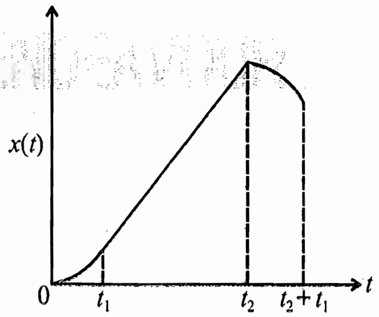
(b)



(c)

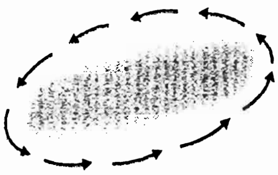


(d)

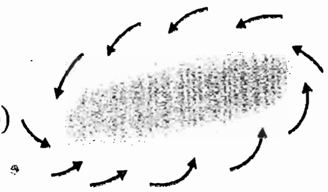


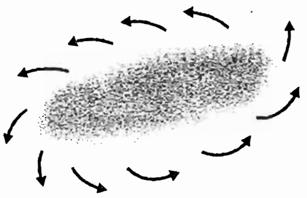
29. In the Earth's atmosphere, a localised low-pressure region (shaded in diagrams) develops somewhere in the southern hemisphere. Which one of the following diagrams represents the correct air flow pattern as observed from a satellite ? **[TIFR 2015]**

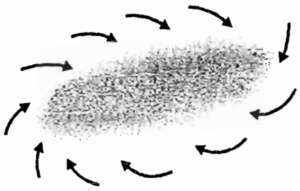
(a)



(b)



(c)


(d)


0. The radius of Earth is approximately 6400 km. The height h at which the acceleration due to Earth's gravity differs from g at the Earth's surface by approximately 1% is **[NET Dec. 2014]**

(a) 64 km (b) 48 km (c) 32 km (d) 16 km

1. An observer O is at the origin of an inertial frame of reference I and rotating with angular velocity $\omega\hat{z}$ relative to it. A satellite of mass M is in a circular orbit of radius R about the origin with angular velocity $-\omega\hat{z}$ as seen from I. The total force on the satellite in O's rest frame is: **[JEST]**

(a) $M\omega^2R\hat{r}$ (b) $-M\omega^2R\hat{r}$ (c) $4M\omega^2R\hat{r}$ (d) $-4M\omega^2R\hat{r}$

2. A particle of mass 0.01 kg falls freely in the earth's gravitational field with an initial velocity $v(0) = 10\,ms^{-1}$. If the air exerts a functional force of the form, $f = -kv$, then for $k = 0.05\ Nm^{-1}s$, the velocity (in ms^{-1}) at time $t = 0.2\ s$ is_______ (upto two decimal places) **[GATE 2015]**

(use $g = 10\,ms^{-2}$ and $e = 2.72$)

3. A bike stuntman rides inside a well of frictionless surface given by $z = a(x^2 + y^2)$, under the action of gravity acting in the negative z-direction. $\vec{g} = -g\hat{z}$. What speed should he maintain to be able to ride at a constant height z_0 without failing down ? **[JEST 2015]**

(a) $\sqrt{gz_0}$

(b) $\sqrt{3gz_0}$

(c) $\sqrt{2gz_0}$

(d) The biker will not be able to maintain a constant height, irrespective of speed.

4. A chain of mass M and length L is suspended vertically with its lower end touching a weighing scale. The chain is released and falls freely onto the scale. Neglecting the size of the individual links, what is the reading of the scale when a length x of the chain has fallen ? **[JEST 2015]**

(a) $\frac{Mgx}{L}$ (b) $\frac{2Mgx}{L}$ (c) $\frac{3Mgx}{L}$ (d) $\frac{4Mgx}{L}$

5. The distance of a star from the Earth is 4.25 light years, as measured from the Earth. A space ship travels from Earth to the star at a constant velocity in 4.25 years, according to the clock on the ship. The speed of the space ship in units of the speed of light is, **[JEST 2015]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{2}{3}$ (d) $\frac{1}{\sqrt{3}}$

36. A spring of force constant k is stretched by x. It takes twice as much work to stretch a second spring by $\frac{x}{2}$. The force constant of the second spring is, **[JEST 2015]**
(a) k (b) $2k$ (c) $4k$ (d) $8k$

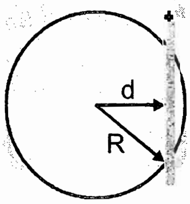
37. A particle of unit mass moves in the xy-plane in such a way that $\dot{x}(t) = y(t)$ and $\dot{y}(t) = x(t)$. We can conclude that it is in a conservative force-field which can be derived from the potential
(a) $\frac{1}{2}(x^2 + y^2)$ (b) $\frac{1}{2}(x^2 - y^2)$ (c) $x + y$ (d) $x - y$ **[NET June 2015]**

38. A ball is dropped vertically from a height H on to a plane surface and permitted to bounce repeatedly along a vertical line. After every bounce, its kinetic energy becomes a quarter of its kinetic energy before the bounce. The ball will come to rest after time **[TIFR 2016]**
(a) infinity (b) $(2H/g)^{1/2}$ (c) $2(2H/g)^{1/2}$ (d) $3(2H/g)^{1/2}$

39. In a moving car, the wheels will skid if the brakes are applied too suddenly. This is because
(a) the inertia of the car will carry it forward **[TIFR 2016]**
(b) the momentum of the car must be conserved
(c) the impulsive retarding force exceeds the limiting force of static friction
(d) the kinetic friction will suddenly get converted to static friction.

40. An aircraft, which weight 12,000 kg when unloaded, is on a relief mission, carrying 4,000 food packets weighing 1 kg each. The plane is gliding horizontally with its engines off at a uniform speed of 540 kmph when the first food packet is dropped. Assume that the horizontal air drag can be neglected and the aircraft keeps moving horizontally. If one food packet is dropped every second, then the distance between the last two packet drops will be **[TIFR 2016]**
(a) 1.5 km (b) 200 m (c) 150 m (d) 100 m

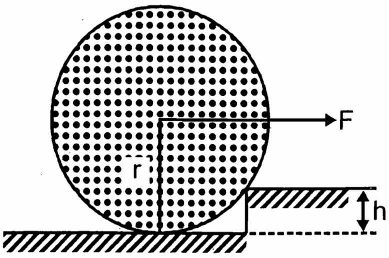
41. Imagine that a narrow tunnel is excavated through the Earth as shown in the diagram on the left and that the mass excavated to create the tunnel is extremely small compared to Earth's mass M. **[TIFR 2016]**



A person falls into the tunnel at one end, at time t = 0. Assuming that the tunnel is frictionless, the person will
(a) fall straight through, escaping Earth's gravity at time $2\pi\sqrt{R^3/GM}$
(b) describe simple harmonic motion with period $2\pi\,(d/R)\sqrt{R^3/GM}$
(c) describe simple harmonic motion with period $2\pi\sqrt{(R-d)^3/GM}$
(d) describe simple harmonic motion with period $2\pi\sqrt{R^3/GM}$

42. A uniform solid wheel of mass M and radius r is halted at a step of height h as shown in the figure. The minimum force F, applied horizontally at the centre of the wheel, necessary to raise the wheel over this step is

[TIFR 2016]



(a) $Mg\dfrac{\sqrt{h(2r-h)}}{r+h}$ (b) $Mg\dfrac{\sqrt{h(2r+h)}}{r-h}$ (c) $Mg\dfrac{\sqrt{h(r+h)}}{r-h}$ (d) $Mg\dfrac{\sqrt{h(2r-h)}}{r-h}$

43. The motion of a particle of mass m is described in a non-inertial frame of reference that is rotating with a uniform angular velocity ω. If r denotes the position of the particle in the non-inertial frame

(a) The centrifugal force on the particle is $-m\omega^2\vec{r}$

(b) The centrifugal force on the particle is $-m\vec{\omega}\times(\vec{\omega}\times\vec{r})$

(c) The Coriolis force on the particle is $-m\left(\vec{\omega}\times\dfrac{d\vec{r}}{dt}\right)$

(d) The Coriolis force on the particle $-2m\omega^2 r$

44. A particle of mass M moving in a straight line with speed υ collides with a stationary particle of the same mass. In the center of mass coordinate system, the first particle is deflected by 90°. The speed of the second particle, after collision, in the laboratory system will be:

(a) $v/\sqrt{2}$ (b) $\sqrt{2}v$ (c) v (d) $v/2$

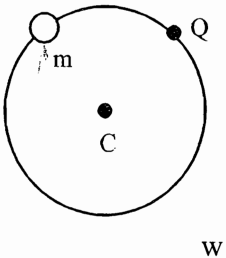
45. A rod of twisting couple C per unit twist connects two identical spheres of moment of inertia I each (about the length of the rod). This dumbell shapped arrangement is placed on a horizontal frictionless table and is given a twist causing the spheres to execute simple harmonic motion. The time period of oscillation is:

(a) $2\pi\sqrt{(I/2C)}$ (b) $2\pi\sqrt{(2I/C)}$ (c) $2\pi\sqrt{(4I/C)}$ (d) $2\pi\sqrt{(I/4C)}$

46. A circus motorcyclist rides on the inner surface of a large globe of radius R along a vertical longitude. In order that he stays on the innner surface, his minimum speed should be

(a) $2\sqrt{Rg}$ (b) $\sqrt{5Rg}$ (c) $\pi\sqrt{5Rg}$ (d) $2\pi\sqrt{Rg}$

47. A point mass m is moving along a circle of radius r and centred at a point C. Q represents a fixed point on the circle (see figure). If w_C, w_Q be the instantaneous angular velocity of the mass around the points C and Q respectively, then



(a) $w_Q = 2w_C$ (b) $w_Q = w_C$ (c) $w_Q = \dfrac{w_C}{2}$ (d) w_Q and w_C are independent

48. A horizontal circular platform rotates with a constant angular velocity Ω directed vertically upwards. A person seated at the centre shoots a bullet of mass 'm' horizontally with speed 'v'. The acceleration of the bullet, in the reference frame of the shooter, is **[NET June 2012]**

(a) $2v\Omega$ to his right (b) $2v\Omega$ to his left

(c) $v\Omega$ to his right (d) $v\Omega$ to his left

49. A jet of gas consists of molecules of mass m, speed v and number density n all moving co-linearly. This jet hits a wall at an angle θ to the normal. The pressure exerted on the wall by the jet assuming elastic collision will be

(a) $p = 2mnv^2\cos^2\theta$ (b) $p = 2mnv^2\cos\theta$

(c) $p = \sqrt{(3/2)}mnv\cos^2\theta$ (d) $p = mnv^2$

50. A particle of mass m is at the stable equilibrium position of its potential energy $V(x) = ax - bx^3$ where a, b are positive constants. The minimum velocity that has to be imparted to the particle to render its motion unstable is **[NET June 2013]**

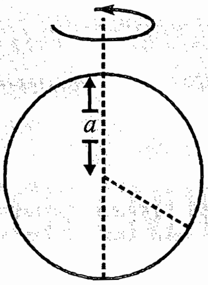
(a) $(64a^3/9m^2b)^{1/4}$ (b) $(64a^3/27m^2b)^{1/4}$

(c) $(16a^3/27m^2b)^{1/4}$ (d) $(3a^3/64m^2b)^{1/4}$

51. A ball of mass m, initially at rest, is dropped from a height of 5 meters. If the coefficient of restitution is 0.9, the speed of the ball just before it hits the floor the second time is approximately (take $g = 9.8$ m/s^2)

(a) 9.80 m/s (b) 9.10 m/s (c) 8.91 m/s (d) 7.02 m/s **[NET June 2016]**

52. A hoop of radius a rotates with constant angular velocity ω about the vertical axis as shown in the figure. A bead of mass m can slide on the hoop without friction. If $g < \omega^2 a$, at what angle θ apart from 0 and π is the bead stationary $\left(i.e., \frac{d\theta}{dt} = \frac{d^2\theta}{dt^2} = 0\right)$? **[JEST 2016]**



(a) $\tan\theta = \pi g/\omega^2 a$ (b) $\sin\theta = g/\omega^2 a$ (c) $\cos\theta = g/\omega^2 a$ (d) $\tan\theta = g/\pi\omega^2 a$

53. In a simple stellar model, the density ρ of a spherical star of mass M varies according to the distance r from the centre according to $\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right)$ where R is the radius of the star. The gravitational potential energy of this star (in terms of Newton's constant G_N) will be **[TIFR 2016]**

(a) $-G_N M^2/4\pi R$ (b) $-3G_N M^2/5R$ (c) $-5G_N M^2/7R$ (d) $-3G_N M^2/7R$

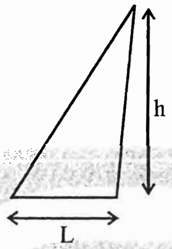
54. Four forces are given below in Cartesian and spherical polar coordinates. **[GATE 2015]**

(i) $\vec{F}_1 = K\exp\left(\frac{-r^2}{R^2}\right)\hat{r}$ (ii) $\vec{F}_2 = K\left(x^3\hat{y} - y^3\hat{z}\right)$

(iii) $\vec{F}_3 = K\left(x^3\hat{x} + y^3\hat{y}\right)$ (iv) $\vec{F}_4 = K\left(\frac{\hat{\phi}}{r}\right)$

where K is a constant. Identify the correct option.
(a) (iii) and (iv) are conservative but (i) and (ii) are not
(b) (i) and (ii) are conservative but (iii) and (iv) are not
(c) (ii) and (iii) are conservative but (i) and (iv) are not
(d) (i) and (iii) are conservative but (ii) and (iv) are not

55. A triangle of uniform mass density of base L and height h is shown below. The centre of mass of the triangle lies at this distance above the base:

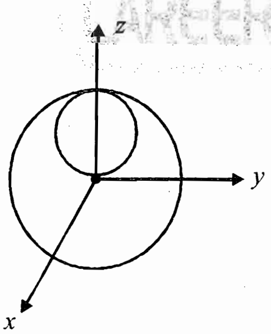


(a) $\frac{h}{2}$ (b) $\frac{2h}{2}$ (c) $\frac{h}{3}$ (d) $\frac{h}{6}$

56. The heaviest pebble of mass m that can be moved by a stream is known to be proportional to some powers of the speed of the stream v, the density (of water) ρ and the acceleration due to gravity g. Using dimensional analysis, we may conclude that m is proportional to

(a) $\frac{\rho v^4}{g}$ (b) $\frac{\rho v^5}{g^2}$ (c) $\frac{\rho v^6}{g^3}$ (d) $\frac{\rho v^3}{g^4}$ (e) $\rho v^2 g$

57. A spherical hole of diameter R has been scooped out of a solid sphere of uniform mass density and radius R. The hole is centred at $(x, y, z) = (0, 0, R/2)$ as shown in the figure



The centre of mass of the body is at

(a) $\left(0, 0, -\frac{R}{14}\right)$ (b) $\left(0, 0, \frac{R}{2}\right)$

(c) $\left(0, 0, \frac{R}{12}\right)$ (d) $\left(0, 0, -\frac{R}{8}\right)$ (e) $\left(0, 0, -\frac{R}{16}\right)$

58 Two particles of mass m_1 and m_2, interact through a force potential $V(r)$. At $t = 0$, their position and velocity vectors are given by $r_1 = (0,0,a), r_2 = (a,0,0)$, $v_1 = (b,2b,0)$ and $v_2 = (0,0,3b)$, where 'a' and 'b' are constants. If $m_2 = 2m_1$, which of the following vectors is perpendicular to the plane of motion?

(a) $2\hat{i} + 5\hat{j}$ (b) $2\hat{i} - \hat{j} - \hat{k}$

(c) $-\hat{i} + 2\hat{j} + \hat{k}$ (d) $\hat{i} - 3\hat{j} + 2\hat{k}$ (e) $\hat{i} + \hat{j} + \hat{k}$

59. A simple harmonic oscillator has a potential energy $V_0(x) = \frac{1}{2}kx^2$. If an additional potential energy term $V_1(x) = ax$ is added to it, then

(a) the motion is simple harmonic with a decreased frequency, and around a shifted equilibrium.
(b) the motion is no longer simple harmonic
(c) the motion is simple harmonic with an increased frequency, and around a shifted equilibrium
(d) the motion is simple harmonic with the same frequency, but around a shifted equilibrium.
(e) the motion is simple harmonic with an increased frequency around the origin.

60. A binary star is composed of two starts that orbit around their centre of mass under the influence of gravity. Consider such a system in which the two starts have identical mass. In the centre of mass frame, each star moves in a circular orbit with a speed of 200 km/s. If the orbital period is 15 days, what is the approximate mass of each start?

(a) 10^{32} kg (b) 10^{30} kg (c) 10^{34} kg (d) 10^{28} kg (e) 10^{26} kg

61. The total mass of a spherical star of radius R and uniform density is M. If it explodes into a gas cloud of much lower density, the gravitational potential energy will

(a) decrease by $\frac{GM^2}{5R}$ (b) increase by $\frac{2GM^2}{5R}$

(c) increase by $\frac{3GM^2}{5R}$ (d) increase by $\frac{4GM^2}{5R}$ (e) decrease by $\frac{GM^2}{R}$

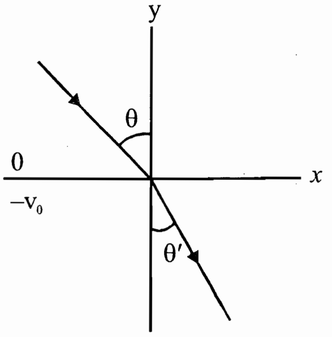
62. The position vector of a particle moving in two dimensions, as a function of time t, is given as $\vec{r}(t) = (\hat{i}t + \hat{j}\sqrt{t})e^{-t}$ for $t \geq 0$. Starting at $t = 0$, the distance of the particle from the origin reaches its maximum value at $t = t_m$, and at this instant, the velocity vector $\vec{v}$ makes an angle α with $\vec{r}$. The values of t_m and α are

(a) $t_m = \frac{1}{2}$ and $\alpha = \frac{\pi}{2}$ (b) $t_m = \frac{1}{\sqrt{2}}$ and $\alpha = \pi$

(c) $t_m = \frac{1}{2}$ and $\alpha = 0$ (d) $t_m = \frac{1}{\sqrt{2}}$ and $\alpha = \frac{\pi}{2}$

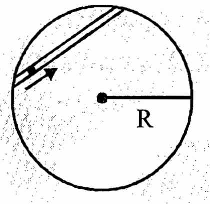
(e) $t_m = 1$ and $\alpha = \pi$

63. Consider the potential field $V(x,y)$ which is 0 and $-V_0\ (V_0>0)$ respectively in the regions of y greater and less than zero. Let θ and θ' be angles of incidence and refraction of the particle with the y-axis at the point of incidence as it crosses the x-axis. The ratio $\sin(\theta)/\sin(\theta')$ is given (in terms of $\Delta = V_0/E$) by



(a) $\sqrt{1+\frac{2V_0}{E}}$ (b) $\sqrt{1+\frac{V_0}{E}}$ (c) $1+\frac{V_0}{E}$ (d) $1+\frac{2V_0}{E}$

64. Consider the earth as a uniform (density) sphere of total mass 'M' and radius R. A small object slides along a tunnel connecting two points on the surface of the earth and is acted upon only by the gravitational force due to the earth. Using the value of the acceleration due to gravity as 9.81 metre/sec^2 and the radius of the earth R = 6.37×10^6 m, the time to slide between the two points is



(a) 21.1 min (b) 84.4 min (c) 42.2 min (d) 63.3 sec

65. A ball dropped from a height h can only attain the height 4h / 5 after bouncing off the floor. If the ball is dropped from a height of 1 m, the time it will take to come to rest is, approximately
[Ignore air resistance and the finite radius of the ball.]
(a) 1.9 s (b) 3.8 s (c) 8.0 s (d) 4.1 s

66. A small raindrop of mass m experiences a viscous drag force $F_d = bv$, proportional to its instantaneous speed 'v'. If it starts from rest at a height h, its speed after a time 't' is

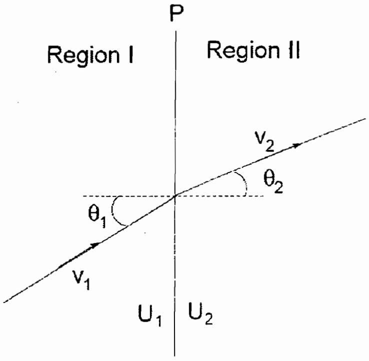
(a) $\upsilon(t)=\frac{mg}{b}\tanh\left(\frac{bt}{m}\right)$ (b) $\upsilon(t)=\frac{mg}{b}e^{-bt/m}$

(c) $\upsilon(t)=\frac{mg}{2b}(1-e^{-2bt/m})$ (d) $\upsilon(t)=\frac{mg}{b}(1-e^{-bt/m})$

67. A particle of mass 1kg is moving along the x-axis. There is no force on the particle except during the time interval between t = 0s and t = 5s, when a force of the form $F(t)=12t^2\ N/s^2$ acts on it. If the velocity of the particle at t = 0 is 500 m/s, its velocity at t = 10 s is:
(a) 800 m/s (b) 1000 m/s (c) 1700 m/s (d) 4500 m/s

68. Raindrops hit the ground with a terminal velocity that is achieved due to a balance between the gravitational force and viscous drag force of air. Assume a raindrop to be a perfectly spherical water droplet of radius R. The momentum that it transfer when it hits the ground is proportional to
(a) R (b) R^2 (c) R^3 (d) R^5.

69. A region of space is divided into two parts by a plane P, as shown in the figure below. A particle of mass m passes from Region I to Region II, where it has speed v_1 and v_2 respectively. There is a constant potential U_1 in Region I and U_2 in Region II. **[TIFR 2011]**



Let T_1 be the kinetic energy of the particles in Region I. If the trajectory of the particle is inclined to the normal to the plane P by angles θ_1 and θ_2, as shown in the figure then the ratio $\sin\theta_1/\sin\theta_2$ is given by

(a) $\sqrt{1-T_1/(U_1-U_2)}$ (b) $\sqrt{1+T_1/(U_1-U_2)}$ (c) $\sqrt{1-(U_1+U_2)/T_1}$

(d) $\sqrt{1+(U_1+U_2)/T_1}$ (e) $\sqrt{1-T_1/(U_1-U_2)}$ (f) $\sqrt{1+(U_1-U_2)/T_1}$

70. Two solid spheres S_1 and S_2 of the same uniform density fall from rest under gravity in a viscous medium and, after some time, each terminal velocities, v_1 and v_2 respectively. If the masses of S_1 and S_2 are m_1 and m_2 respectively, and $v_1 = 4\,v_2$, then the ratio m_1/m_2 is **[TIFR 2011]**

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) 4 (d) 8

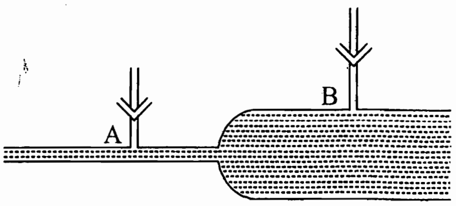
71. A high-velocity missile, travelling in a horizontal line with a kinetic energy of 3.0 Giga-Joules (GJ), explodes in flight and breaks into two pieces. A and B of equal mass. One of these pieces (A) flies off in a straight line perpendicular to the original direction in which the missile was moving and its kinetic energy is found to be 2.0 GJ. If gravity can be neglected for such high-velocity projectiles, it follows that the other piece (B) flew off in a direction at an angle with the original direction of **[TIFR 2012]**

(a) 30° (b) 33°24′ (c) 45° (d) 60°

72. Consider a spherical planet, rotating about an axis passing through its centre. The velocity of a point on its equator is v_{eq}. If the acceleration due to gravity g measured at the equator is half of the value of g measured at one of the poles, then the escape velocity for a particle shot upwards from that pole will be

(a) $v_{eq}/2$ (b) $v_{eq}/\sqrt{2}$ (c) $\sqrt{2}v_{eq}$ (d) $2v_{eq}$ **[TIFR 2012]**

73. An ideal liquid of density 1gm/cc is flowing at a rate of 10 gm/s through a tube with varying cross-section, as shown in the figure. **[TIFR 2012]**



Two pressure gauges attached at the points A and B (see figure) show readings of P_A and P_B respectively. If the radius of the tube at the points A and B is 0.2 cm and 1.0 cm respectively, then the difference in pressure $(P_B - P_A)$, in units of dyne cm^{-2}, is closest to

(a) 100 (b) 120 (c) 140 (d) 160

4. A stone is dropped vertically from the top of a tower of height 40m. At the same time a gun is aimed directly at the stone from the ground at a horizontal distance 30m from the base of the tower and fired. If the bullet from the gun is to hit the stone before it reaches the ground, the minimum velocity of the bullet must be, approximately, **[TIFR 2013]**

(a) $57.4\ \text{ms}^{-1}$ (b) $27.7\ \text{ms}^{-1}$ (c) $17.7\ \text{ms}^{-1}$ (d) $7.4\ \text{ms}^{-1}$

5. A particle with time-varying mass $m(t) = m_0\left(1-\frac{t}{\tau}\right)$, where m_0 and τ are positive constants, moves along the x-axis under the action of a constant positive force F for $0 \le t \le \tau$. If the particle is at rest at time $t = 0$, then at time $t = t$, its velocity v will be **[TIFR 2013]**

(a) $-\frac{\tau F}{m_0}\log\left(1-\frac{t}{\tau}\right)$ (b) $-\frac{Ft}{m_0}\log\frac{t}{\tau}$

(c) $\frac{Ft}{m_0}\left(1-\frac{t}{\tau}\right)^{-1}$ (d) $\frac{\tau F}{m_0}\left(1-\frac{t}{\tau}\right)$

6. Which of the following classic experiments provides unambiguous proof that the Earth is a non-inertial frame of reference with respect to the fixed stars? **[TIFR 2013]**

(a) Fizeau's rotating wheel experiment (b) Foucault's pendulum experiment

(c) Newton's coin-and-feather experiment (d) Michelson-Morley experiment

7. On a planet having the same mass and diameter as the Earth, it is observed that objects become weightless at the equator. Find the time period of rotation of this planet in minutes (as defined on the Earth). **[TIFR 2016]**

8. A particle moves in two dimensions on the ellipse $x^2 + 4y^2 = 8$. At a particular instant it is at the point $(x, y) = (2, 1)$ and the x-component of its velocity is 6 (in suitable units). Then the y-component of its velocity is **[NET June 2015]**

(a) -3 (b) -2 (c) 1 (d) 4

Answer Key: Classical Mechanics (General)

1. (c) **2. (d)** **3. (c)** **4. (a)** **5. (c)** **6. (a)** **7. (a)**

8. (c) **9. (d)** **10. (d)** **11. (a)** **12. (a)** **13. (d)** **14. (d)**

15. (d) **16. (c)** **17. (a)** **18. (b)** **19. (c)** **20. (c)** **21. (c)**

22. (c) **23. (b)** **24. (d)** **25. (a)** **26. (c)** **27. (d)**

28. (c) **29. (d)** **30. (c)** **31. (d)** **32. (4.93 to 4.98)** **33. (c)**

34. (c) **35. (b)** **36. (d)** **37. (a)** **38. (d)** **39. (a)** **40. (c)**

41. (d) **42. (d)** **43. (b)** **44. (a)** **45. (a)** **46. (a)** **47. (c)**

48. (a) **49. (a)** **50. (b)** **51. (c)** **52. (c)** **53. (c)** **54. (d)**

55. (c) **56. (c)** **57. (a)** **58. (a)** **59. (d)** **60. (a)** **61. (c)**

62. (d) **63. (b)** **64. (c)** **65. (c)** **66. (d)** **67. (b)** **68. (d)**

69. (f) **70. ( )** **71. (a)** **72. (d)** **73. (b)** **74. (c)** **75. (c)**

76. (b) **77. (85)** **78. (a)**

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Assignment-2: Classical Mechanics (Lagrangian Dynamics)

1. The Lagrangian of a particle of mass m moving in a central potential V(r) is

$$L=\frac{1}{2}m\left(\dot{r}^2+r\,\dot{\theta}^2+r^2\sin^2\theta\dot{\varphi}^2\right)-V\left(r\right)$$

(a) θ is a cyclic coordinate
(b) θ and φ are cyclic coordinates
(c) φ is a cyclic coordinate
(d) r is a cyclic coordinate

2. The Lagrangian of a particle of mass m moving in two dimension is $L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-\frac{k}{2}\left(x^2+y^2\right)$

If the particle has a finite angular momentum *l* about the origin, then we may conclude that it executes
(a) Oscillatory motion about the origin $r=0$
(b) Periodic motion with a constant value of r
(c) Oscillatory motion along the x-axis.
(d) Oscillatory motion along the y-axis.

3. A particle is moving on a sphere of fixed radius. The number of degrees of freedom for the particle will be
(a) 3
(b) 4
(c) 2
(d) 1

4. If a generalised co-ordinate q is ignorable or cyclic in a Lagrangian L, then the following is conserved.
(a) $\frac{\partial L}{\partial q}$
(b) $\frac{\partial L}{\partial t}$
(c) $\frac{\partial L}{\partial \dot{q}}$
(d) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$

5. The Lagrangian of a particle moving in a central field is given by $L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\phi}^2\right)-U\left(r\right)$

Which of the following is/are constant?
(a) Radial momentum
(b) Potential energy
(c) Azimuthal momentum
(d) Radial and azimuthal momentum

6. A system exhibits invariance under translation in space and time. The following quantities will remain conserved.
(a) Linear momentum and angular momentum.
(b) Angular momentum and energy.
(c) Linear momentum and energy.
(d) Linear momentum, angular momentum and energy.

7. One of the generalized co-ordinates Q_j of a system is cyclic. Then its
(a) Energy is not conserved.
(b) Conjugate momentum P_j is also cyclic.
(c) Conjugate momentum P_j is conserved.
(d) Conjugate momentum P_j is indeterminate.

8. A hollow cylinder of mass M and radius r rolls in the x direction from the point $x=0$ on a horizontal plane without friction. Its Lagrangian is:
(a) $\frac{1}{2}M\dot{x}^2$
(b) $\frac{1}{2}M\dot{x}^2+\frac{1}{2}Mr^2\dot{\theta}^2$
(c) $\frac{1}{2}M\dot{x}^2+\frac{1}{2}M\dot{r}^2\dot{\theta}^2$
(d) $\frac{1}{2}Mr\dot{\theta}^2$

Consider a six particle system with five particles arranged as regular pentagon with 6th one at the centre. If all the five particles are connected to the central one by rigid rods, then the number of degrees of freedom for the system are given by

(a) 13 (b) 6 (c) 5 (d) 18

The Lagrangian of a free particle having a rest m_0 and relativistic speed v is given by

(a) $\dfrac{-m_0c^2}{\sqrt{1-\dfrac{v^2}{c^2}}}$ (b) $m_0c^2\sqrt{c-v}$ (c) $m_0c\sqrt{c^2-v^2}$ (d) $-m_0c\sqrt{c^2-v^2}$

Ignorable coordinates of the system corresponding to the Lagrangian

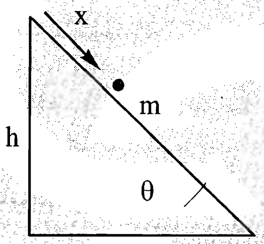
$L=\frac{1}{2}\left(\dot{\theta}^2+\dot{\phi}^2\sin^2\theta\right)+\frac{1}{2}\left(\dot{\psi}+\dot{\phi}\cos\theta\right)^2-k\cos\theta$, where θ, ϕ and ψ are generalized coordinates and k is a constant parameter, are

(a) θ and ϕ (b) ϕ and ψ (c) ψ and θ (d) θ, ϕ and ψ

Conservation of angular momentum is related to the invariance of Lagrangian under

(a) Rotation in space (b) Translation in space (c) Inversion in space (d) Translation in time

A particle of mass m is sliding on a frictionless inclined plane of angle of inclination θ with the horizontal and height h. The distance x traveled by the particle is measured from the top of the inclined plane and along the inclination. If the potential energy is zero at the bottom of the inclined plane, the Lagrangian of the particle is given by



(a) $L=\frac{1}{2}m\dot{x}^2-mg(x\sin\theta-h)$ (b) $L=\frac{1}{2}m\dot{x}^2-mg(h-x\sin\theta)$

(c) $L=\frac{1}{2}m\dot{x}^2-mg(h-x\cos\theta)$ (d) $L=\frac{1}{2}m\dot{x}^2-mg(x\cos\theta-h)$

4. The Lagrangian of a particle with respect to a stationary frame S is L. If the motion of the particle is studied from another inertial frame S' moving with a constant relative velocity with respect to S under the same potential, then

(a) both the Lagrangian and the equations of motion remain unchanged.

(b) the Lagrangian is modified by addition of a total time derivative of a scalar function and the equations of motion change.

(c) the Lagrangian is modified by addition of a total time derivative of a scalar function and the equations of motion remain unchanged

(d) the Lagrangian is modified by a constant factor and the equations of motion change.

5. The dynamics of a system with generalized coordinates (x, y, z) is described by a Lagrangian $L=L(\dot{x},\dot{y},\dot{z},x,y,t)$ where t is time. Which of the following statements is correct?

(a) The total energy and x-component of the linear momentum are conserved.

(b) The total energy is not conserved but y-component of the linear momentum is conserved.

(c) The total energy is conserved but z-component of linear momentum is not conserved.

(d) The total energy is not conserved but z-component of linear momentum is conserved.

16. The Lagrangian of the particle of mass m is: **[GATE 2007]**

$$L=\frac{m}{2}\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2\right]-\frac{V}{2}\left(x^2+y^2\right)+W\sin\omega t,$$

where V, W and ω are constants. The conserved quantities are:
(a) Energy and z-component of linear momentum only.
(b) Energy and z-component of angular momentum only.
(c) z-components of both linear and angular momenta only.
(d) Enregy and z-components of both linear and angular momenta.

17. Three particles of mass m each situated at $x_1(t)$, $x_2(t)$ and $x_3(t)$ respectively are connected by two springs of spring constant k and un-stereteched length ℓ. The system is free to oscillate only in one dimension along the straight line joining all the three particles. The Lagrangian of the system is:

(a) $L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}\left(x_1-x_2-\ell\right)^2+\frac{k}{2}\left(x_3-x_2-\ell\right)^2$

(b) $L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}\left(x_1-x_3-\ell\right)^2+\frac{k}{2}\left(x_3-x_2-\ell\right)^2$

(c) $L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}\left(x_1-x_2+\ell\right)^2-\frac{k}{2}\left(x_3-x_2+\ell\right)^2$

(d) $L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}\left(x_2-x_1-\ell\right)^2-\frac{k}{2}\left(x_3-x_2-\ell\right)^2$

Statement for Linked Q. 18 and Q.19.

A particle of mass m is constrained to move in a vertical plane along a trajectory given by $x = A\cos\theta$, $y = A\sin\theta$, where A is a constant: **[GATE]**

18. The Lagrangian of the particle is:

(a) $\frac{1}{2}mA^2\dot{\theta}^2-mgA\cos\theta$ (b) $\frac{1}{2}mA^2\dot{\theta}^2-mgA\sin\theta$

(c) $\frac{1}{2}mA^2\dot{\theta}^2$ (d) $\frac{1}{2}mA^2\dot{\theta}^2+mgA\cos\theta$

19. The equation of motion of the particle is:

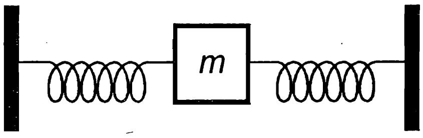
(a) $\ddot{\theta}+\frac{g}{A}\cos\theta=0$ (b) $\ddot{\theta}+\frac{g}{A}\sin\theta=0$ (c) $\ddot{\theta}=0$ (d) $\ddot{\theta}-\frac{g}{A}\sin\theta=0$

20. A particle of mass m moves in a potential $V(x)=\frac{1}{2}m\omega^2x^2+\frac{1}{2}m\mu v^2$ where x is the position coordinate, v is the speed, and ω and μ are constants. The canonical (conjugate) momentum of the particle is:
(a) $p = m(1+\mu)v$ (b) $p = mv$ (c) $p = m\mu v$ (d) $p = m(1-\mu)v$

21. The Lagrangian of a particle moving in a plane under the influence of a central potential is given by $L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2\right)-V(r)$. The generalized momenta corresponding to r and θ are given by:

(a) $m\dot{r}$ and $mr^2\dot{\theta}$ (b) $m\dot{r}$ and $mr\dot{\theta}$ (c) $m\dot{r}^2$ and $mr^2\dot{\theta}$ (d) $m\dot{r}^2$ and $mr^2\dot{\theta}^2$

22. A mass m is connected on either side with a spring each of spring constants k_1 and k_2. The free ends of springs are tied to rigid supports. The displacement of the mass is x from equilibrium position. Which one of the following is TRUE?



(a) The force acting on the mass is $(k_1 \cdot k_2)^{1/2} x$

(b) The angular momentum of the mass is zero about the equilibrium point and its Lagrangian is

$$\frac{1}{2}m\dot{x}^2 - \frac{1}{2}(k_1 + k_2)x^2.$$

(c) The total energy of the system is $\frac{1}{2}m\dot{x}^2$

(d) The angular momentum of the mass is $mx\dot{x}$ and the Lagrangian of the system is $\frac{m}{2}\dot{x}^2 + \frac{1}{2}(k_1 + k_2)x^2$.

23. The Lagrangian of a particle of mass m moving in a plane is given by $L = (½)\, m\, [v_x^2 + v_y^2] + a(xv_y - yv_x)$, where v_x and v_y are velocity components and a is a constant. The canonical momenta of the particle are given by:
(a) $p_x = m\, v_x$ and $p_y = m\, v_y$ (b) $p_x = m\, v_x + ay$ and $p_y = m\, v_y + a\, x$
(c) $p_x = m\, v_x - a\, y$ and $p_y = m\, v_y + a\, x$ (d) $p_x = m\, v_x - a\, y$ and $p_y = m\, v_y + a\, x$

24. Two particles of equal mass are connected by an inextensible string of length L. One of the masses is constrained to move on the surface of a horizontal table. The string passes through a small hole in the table and the other mass is hanging below the table. The only constraint is that the first mass moves on the surface of the table. The number of degrees of freedom of the mass-string system is:
(a) five (b) four (c) two (d) one

25. The homogeneity of time leads to the law of conservation of:
(a) Linear momentum (b) Angular momentum (c) Energy (d) Parity

26. The Lagrangian for the Kepler problem is given by:

$$L = \frac{1}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{\mu}{r}, (\mu > 0)$$

where (r, θ) denote the polar coordinates and the mass of the particle is unity. Then:
(a) $p_\theta = 2r^2\dot{\theta}$ (b) $p_r = 2\dot{r}$
(c) The angular momentum of the particle about the centre of attraction is a constant
(d) The total energy of the particle is time dependent.

27. with reference to a cylindrical polar coordinate system, a particle of charge q and momentum $\overline{p}$ is moving in an electric potential $\phi(r)$. Which of the following are conserved?
(a) p_r (b) p_θ (c) p_z (d) rp_θ

28. If $\frac{\partial L}{\partial q_n} = 0$, where L is the Lagrangian for a conservative system without constraints and q_n is a generalized coordinate, then the generalized momentum is:
(a) a cycle cooridnat (b) a constant of motion
(c) equal to $\frac{d}{dt}\left(\frac{\partial L}{\partial q_n}\right)$ (d) undefined

29. The potential energy of a classical particle moving in one dimension is kx^4, where k is a constant. If the particle moves from a point x_1 at time t_1 to a point x_2 at time t_2, the actual path followed by the particle is that which makes the following integral extremum.

(a) $\int_{t_1}^{t_2}\left(\frac{1}{2}mv^2 + kx^4\right)dt$ (b) $\int_{t_1}^{t_2}\left(\frac{1}{2}mv^2 - kx^4\right)dt$

(c) $\int_{t_1}^{t_2}\left(\frac{1}{2}mv^2 + 4kx^3\right)dt$ (d) $\int_{t_1}^{t_2}\left(\frac{1}{2}mv^2 - 4kx^3\right)dt$

30. Lagrangian of the Sun-Earth System is:

(a) $\frac{1}{2}m\dot{r} + \frac{1}{2}mr^2\dot{\theta}^2 - \frac{GMm}{r}$ (b) $\frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + \frac{GMm}{r}$

(c) $\frac{1}{2}m\dot{r}^2 - \frac{GMm}{r}$ (d) $\frac{1}{2}mr^2\dot{\theta}^2 + \frac{GMm}{r}$

Here r is the Sun-Earth distance, M and m are the masses of the Sun and Earth respectiely. $\dot{\theta}$ is the angular speed, and G the gravitational constant.

31. A particle is constrained to move along the inner surface of a hemisphere number of degrees of freedom of the particle is:

(a) one (b) two (c) three (d) four

32. The Lagrangian of a system is given by $L = \frac{1}{2}\dot{q}^2 + q\dot{q} - \frac{1}{2}q^2$. It describes the motion of **[GATE 2008]**

(a) A harmonic oscillator. (b) A damped harmonic oscillator.
(c) An anharmonic oscillator (d) A system with unbounded motion.

33. A cylinder of mass M and radius R is rolling down without slipping on an inclined plane of angle of inclination θ. The number of generalized coordinates required to describe the motion of this system. **[GATE 2008]**

(a) 1 (b) 2 (c) 4 (d) 6

34. The lagrangian of a particle of mass m moving in two dimension is $L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) - \frac{1}{2}k\left(x^2 + y^2\right)$

If the particle has a finite angular momentum *l* about the origin, then we may conclude that it executes

(a) Oscillatory motion about the origin $r = 0$ (b) Periodic motion with a constant value of r.
(c) Oscillatory motion along the x-axis. (d) Oscillatory motion along the y-axis.

35. A particle is moving under the action of a generalized potential, $V(q,\dot{q}) = \frac{(1+\dot{q})}{q^2}$. The magnitude of the generalized force is: **[GATE 2011]**

(a) $\frac{2(1+\dot{q})}{q^3}$ (b) $\frac{2(1-\dot{q})}{q^3}$ (c) $\frac{2}{q^3}$ (d) $\frac{\dot{q}}{q^3}$

36. The Lagrangian of a particle of charge e and mass m in applied electric and magnetic fields is given by $L = \frac{1}{2}m\vec{v}^2 + e\vec{A}.\vec{v} - e\phi$, where $\vec{A}$ and ϕ are the vector and scalar potentials corresponding to the magnetic and electric fields, respectively. Which of the following statements is correct? **[NET June 2011]]**

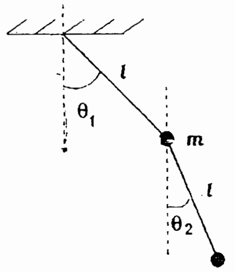
(a) The canonically conjugate momentum of the particle is given by $\vec{p} = m\vec{v}$

(b) The Hamiltonian of the particle is given by $H = \frac{\vec{p}^2}{2m} + \frac{e}{m}\vec{A}.\vec{p} + e\phi$

(c) L remains unchanged under a gauge transformation of the potentials.

(d) Under a gauge transformation of the potentials, L changes by the total time derivative of a function of $\vec{r}$ and t.

37. A double pendulum consists of two point masses m attached by massless strings of length l as shown in the figure: **[NET Dec. 2011]**



The kinetic energy of the pendulum is:

(a) $\frac{1}{2}m\ell^2\left[\dot\theta_1^2+\dot\theta_2^2\right]$ (b) $\frac{1}{2}m\ell^2\left[2\dot\theta_1^2+\dot\theta_2^2+2\dot\theta_1\dot\theta_2\cos\left(\dot\theta_1-\dot\theta_2\right)\right]$

(c) $\frac{1}{2}m\ell^2\left[\dot\theta_1^2+2\dot\theta_2^2+2\dot\theta_1\dot\theta_2\cos\left(\dot\theta_1-\dot\theta_2\right)\right]$ (d) $\frac{1}{2}m\ell^2\left[2\dot\theta_1^2+\dot\theta_2^2+2\dot\theta_1\dot\theta_2\cos\left(\dot\theta_1+\dot\theta_2\right)\right]$

38. A particle of mass 'm' moves inside a bowl. If the surface of the bowl is given by the equation $z=\frac{1}{2}a\left(x^2+y^2\right)$, where a is a constant, the Lagrangian of the particle is: **[NET Dec. 2011]**

(a) $\frac{1}{2}m\left(\dot r^2+r^2\dot\phi^2-gar^2\right)$ (b) $\frac{1}{2}m\left[\left(1+a^2r^2\right)\dot r^2+r^2\dot\phi^2\right]$

(c) $\frac{1}{2}m\left(\dot r^2+r^2\dot\theta^2+r^2\sin^2\theta\dot\phi^2-gar^2\right)$ (d) $\frac{1}{2}m\left[\left(1+a^2r^2\right)\dot r^2+r^2\dot\phi^2-gar^2\right]$

39. The Lagrangian for a symmetric top is given by $L=\frac{1}{2}I_1\left(\dot\theta^2+\dot\varphi^2\sin^2\theta\right)+\frac{1}{2}I_3\left(\dot\psi+\dot\varphi\cos\theta\right)^2-Mgl\cos\theta$

where θ,φ,ψ are the Euler angles. Then the following are conserved quantities:

(a) P_φ and P_ψ (b) P_θ and P_φ (c) P_θ and P_ψ (d) P_θ, P_φ and P_ψ

40. A system with d degrees of freedom can have at most k independent constants of motion. The value of k is equal to

(a) $d-1$ (b) $2d$ (c) $2d-1$ (d) d

41. The invariance of the Lagrangian $L=\frac{1}{2}m\left(\frac{dz}{dt}\right)\left(\frac{dz^*}{dt}\right)-V\left(zz^*\right)$ (where $z=re^{i\theta}=x+iy$ is a complex coordinate under the transformation $Z\to e^{i\alpha}z, z^*\to e^{-i\alpha}z^*$ (for a real number α), leads to the conservation of

(a) Angular momentum (b) linear momentum along x.
(c) linear momentum along y (d) energy

42. The Lagrangian of a 2-dimensional system is given by $L=\mu\left(\dot x_1^2+\dot x_2^2\right)-\gamma\left(x_1+x_2\right)$ where μ and γ are constants

NET

(a) The total energy E as well as $\left(\dot x_1+\dot x_2\right)$ are conserved

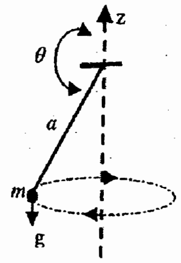
(b) The total energy E as well as $\left(\dot x_1-\dot x_2\right)$ are conserved

(c) Only the total energy E is conserved

(d) Only the sum $\left(\dot x_1+\dot x_2\right)$ is conserved.

43. A particle of mass m is attached to a fixed point 'O' by a weightless inextensible string of length a. It is rotating under the gravity as shown in the figure.

The Lagrangian of the particle is



$$L(\theta,\phi)=\frac{1}{2}ma^2\left(\dot{\theta}^2+\sin^2\theta\dot{\phi}^2\right)-mga\cos\theta$$

where θ and ϕ are the polar angles.

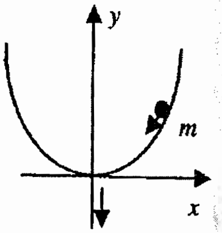
The Hamiltonian of the particle is:

(a) $H=\frac{1}{2ma^2}\left(p_\theta^2+\frac{p_\phi^2}{\sin^2\theta}\right)-mga\cos\theta$ (b) $H=\frac{1}{2ma^2}\left(p_\theta^2+\frac{p_\phi^2}{\sin^2\theta}\right)+mga\cos\theta$

(c) $H=\frac{1}{2ma^2}\left(p_\theta^2+p_\phi^2\right)-mga\cos\theta$ (d) $H=\frac{1}{2ma^2}\left(p_\theta^2+p_\phi^2\right)+mga\cos\theta$

Statement for Linked Answer Q.44 and Q. 45

A particle of mass m slides under the gravity without friction along parabolic path $y=ax^2$ as shown in the figure. Here a is a constant. **[GATE 2012]]**



44. The Lagrangian for this particle is given by,

(a) $L=\frac{1}{2}m\dot{x}^2-mgax^2$ (b) $L=\frac{1}{2}m\left(1+4a^2x^2\right)\dot{x}^2-mgax^2$

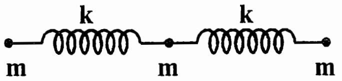
(c) $L=\frac{1}{2}m\dot{x}^2+mgax^2$ (d) $L=\frac{1}{2}m\left(1+4a^2x^2\right)\dot{x}^2+mgax^2$

45. The Lagrange's equation of motion of the particle is

(a) $\ddot{x}=2gax$ (b) $m\left(1+4a^2x^2\right)\ddot{x}=-2mgax-4ma^2x\,\dot{x}^2$

(c) $m\left(1+4a^2x^2\right)\ddot{x}=2mgax+4ma^2x\,\dot{x}^2$ (d) $\ddot{x}=-2gax$

46. Three particles of equal mass 'm' are connected by two identical massless springs of stiffness constant 'k' as shown in the figure. **[NET Dec. 2012]]**



If x_1, x_2 and x_3 denote the displacements of the masses from their respective equilibrium positions, the potential energy of the system is:

(a) $\frac{1}{2}k\left(x_1^2+x_2^2+x_3^2\right)$ (b) $\frac{1}{2}k\left[x_1^2+x_2^2+x_3^2-x_2\left(x_1+x_3\right)\right]$

(c) $\frac{1}{2}k\left[x_1^2+2x_2^2+x_3^2+2x_2\left(x_1+x_3\right)\right]$ (d) $\frac{1}{2}k\left[x_1^2+2x_2^2+x_3^2-2x_2\left(x_1+x_3\right)\right]$

The Lagrangian of a particle of mass m moving in one dimensions is given by **[NET June 2013]]**

$$L=\frac{1}{2}m\dot{x}^2-bx$$

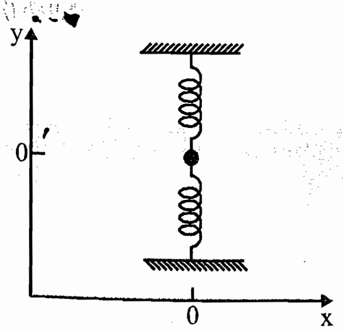
where b is a positive constant. The coordinate of the particle x(t) at time t is given by: (in the following c_1 and c_2 are constants)

(a) $-\frac{b}{2m}t^2+c_1t+c_2$ (b) c_1t+c_2

(c) $c_1\cos\left(\frac{bt}{m}\right)+c_2\sin\left(\frac{bt}{m}\right)$ (d) $c_1\cosh\left(\frac{bt}{m}\right)+c_2\sinh\left(\frac{bt}{m}\right)$

Consider a particle of mass *m* attached to two identical springs each of length *l* and spring constant k (see the figure below). The equilibrium configuration is the one where the springs are unstretched. There are no other external forces on the system. If the particle is given a small displacement along the x-axis, which of the following describes the equation of motion for small oscillations? **[NET Dec. 2013]**

spring force is [illegible]
quantity



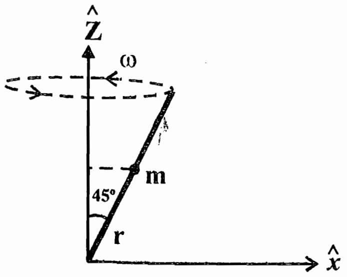
(a) $m\ddot{x}+\frac{kx^3}{l^2}=0$ (b) $m\ddot{x}+kx=0$ (c) $m\ddot{x}+2kx=0$ (d) $m\ddot{x}+\frac{kx^2}{l}=0$

The Lagrangian of a system with one degree of freedom *q* is given by $L=\alpha\dot{q}^2+\beta q^2$, where α and β are non-zero constants. If p_q denotes the canonical momentum conjugate to *q* then which one of the following statements is CORRECT?

(a) $p_q=2\beta q$ and it is a conserved quantity. (b) $p_q=2\beta q$ and it is not a conserved quantity.

(c) $p_q=2\alpha\dot{q}$ and it is a conserved quantity. (d) $p_q=2\alpha\dot{q}$ and it is not a conserved quantity.

A bead of mass 'm' can slide without friction along a massless rod kept at 45° with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed ω. At any instant, *r* is the distance of the bead from the origin. The momentum conjugate to 'r' is **[GATE 2014]**



(a) $m\dot{r}$ (b) $\frac{1}{\sqrt{2}}m\dot{r}$ (c) $\frac{1}{2}m\dot{r}$ (d) $\sqrt{2}m\dot{r}$

51. The equation of motion of a system described by the time-dependent Lagrangian $L = e^{\gamma t}\left[\frac{1}{2}m\dot{x}^2 - V(x)\right]$ is

(a) $m\ddot{x} + \gamma m\dot{x} + \frac{dV}{dx} = 0$ (b) $m\ddot{x} + \gamma m\dot{x} - \frac{dV}{dx} = 0$ **[NET Dec. 2014]**

(c) $m\ddot{x} - \gamma m\dot{x} + \frac{dV}{dx} = 0$ (d) $m\ddot{x} + \frac{dV}{dx} = 0$

52. The Lagrangian of a particle is given by $L = \dot{q}^2 - q\dot{q}$. Which of the following statements is true ?

(a) This is a free particle **[JEST 2015]**
(b) The particle is experiencing velocity dependent damping
(c) The particle is executing simple harmonic motion
(d) The particle is under constant acceleration

53. The Lagrangian of a particle moving in a plane is given in Cartesian coordinates as $L = \dot{x}\,\dot{y} - x^2 - y^2$. In polar coordinates the expression for the canonical momentum p_r (conjugate to the radial coordinate r) is **[NET Dec. 2015]**

(a) $\dot{r}\sin\theta + r\dot{\theta}\cos\theta$ (b) $\dot{r}\cos\theta + r\dot{\theta}\sin\theta$

(c) $2\dot{r}\cos 2\theta - r\dot{\theta}\sin 2\theta$ (d) $\dot{r}\sin 2\theta + r\dot{\theta}\cos 2\theta$

54. The number of degrees of freedom of a rigid body in d space-dimensions is **[NET June 2013]**

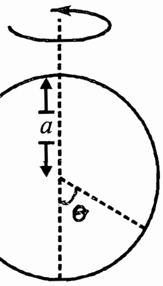
(a) 2d (b) 6 (c) d(d + 1)/2 (d) d!

55. The Lagrangian of a system moving in three dimensions is **[NET June 2016]**

$$L = \frac{1}{2}m\dot{x}_1^2 + m(\dot{x}_2^2 + \dot{x}_3^2) - \frac{1}{2}kx_1^2 - \frac{1}{2}k(x_2 + x_3)^2$$

The independent constant(s) of motion is/are
(a) energy alone
(b) only energy, one component of the linear momentum and one component of the angular momentum
(c) only energy and one component of the linear momentum
(d) only energy and one component of the angular momentum

56. A hoop of radius a rotates with constant angular velocity ω about the vertical axis as shown in the figure. A bead of mass m can slide on the hoop without friction. If $g < \omega^2 a$, at what angle θ apart from 0 and π is the bead stationary $\left(i.e., \frac{d\theta}{dt} = \frac{d^2\theta}{dt^2} = 0\right)$? **[JEST 2016]**



(a) $\tan\theta = \pi g/\omega^2 a$ (b) $\sin\theta = g/\omega^2 a$ (c) $\cos\theta = g/\omega^2 a$ (d) $\tan\theta = g/\pi\omega^2 a$

A particle of mass $m = 0.1$ kg is initially at rest at origin. It starts moving with a uniform acceleration $\vec{a} = 10\hat{i}$ ms^{-2} at $t = 0$. The action S of the particle, in units of J-s, at $t = 2s$ is _________. (Give your answer upto two decimal places). **[GATE 2016]**

The Lagrangian of a system is given by

$L = \frac{1}{2}ml^2\left[\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2\right] - mgl\cos\theta$, where m, l and g are constants. **[GATE 2016]**

Which of the following is conserved?

(a) $\dot{\varphi}\sin^2\theta$ (b) $\dot{\varphi}\sin\theta$ (c) $\frac{\dot{\varphi}}{\sin\theta}$ (d) $\frac{\dot{\varphi}}{\sin^2\theta}$

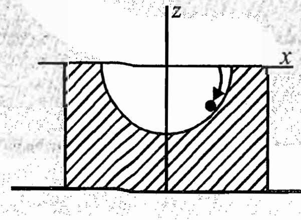
The dynamics of a particle of mass m is described in terms of three generalized co-ordinates ξ, η and φ. If the Lagrangian of the system is **[TIFR 2011]**

$$L = \frac{1}{8}m\left[(\xi + \eta)\left(\frac{\dot{\xi}^2}{\xi^2} + \frac{\dot{\eta}^2}{\eta^2}\right) + 4\xi\eta\dot{\varphi}^2\right] + \frac{1}{8}k(\xi + \eta)^2$$

where k is a constant, then a conserved quantity in the system will be

(a) $(m + k)\left(\dot{\xi} + \dot{\eta}\right)$ (b) $m\,\xi\eta\,\dot{\varphi}$ (c) $m\left(\frac{\dot{\xi}^2}{\eta^2} + \frac{\dot{\eta}^2}{\xi^2}\right)$ (d) $m(\xi + \eta)\left(\frac{\dot{\xi}}{\xi^2} + \frac{\dot{\eta}}{\eta^2}\right)$

A ball of mass m slides under gravity without friction inside a semicircular depression of radius a inside a fixed block placed on a horizontal surface, as shown in the figure. The equation of motion of the ball in the x-direction will be **[TIFR 2013]**



(a) $\ddot{x} = \frac{g}{a}x\sqrt{1 - \frac{x^2}{a^2}}$ (b) $\ddot{x} = \frac{g}{a}x$

(c) $\ddot{x} = -\frac{g}{a}x$ (d) $\ddot{x} = -\frac{3g}{a}x\sqrt{1 - \frac{x^2}{a^2}}$

1. The conservation principles for energy, linear momentum and angular momentum arise from the necessity that **[TIFR 2014]**

(a) The laws of physics should not involve infinite quantities.

(b) Internal forces on a body should cancel out, by Newton's (third) law of action and reaction

(c) Physical measurements should be independent of the origin and orientation of the coordinate system.

(d) The laws of physics should be independent of the state of rest or motion of the observer.

62. The Lagrangian of a free particle in spherical polar co-ordinates is given by $L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+r^2\dot{\phi}^2\sin^2\theta\right)$. The quantity that is conserved is **[GATE 2009]**

(a) $\frac{\partial L}{\partial \dot{r}}$ (b) $\frac{\partial L}{\partial \dot{\theta}}$ (c) $\frac{\partial L}{\partial \dot{\phi}}$ (d) $\frac{\partial L}{\partial \dot{\phi}}+\dot{r}\dot{\theta}$

63. A particle moves in a potential $V=x^2+y^2+\frac{z^2}{2}$. Which component (s) of the angular momentum is / are constant (s) of motion? **[NET Dec. 2013]**

(a) none (b) L_x, L_y and L_z (c) only L_x and L_y (d) only L_z

Answer Key: Classical Mechanics (Lagrangian Dynamics)

1. (c) 2. (b) 3. (c) 4. (c) 5. (c) 6. (c) 7. (c)
8. (b) 9. (b) 10. (d) 11. (b) 12. (a) 13. (b) 14. (c)
15. (d) 16. (c) 17. (d) 18. (b) 19. (a) 20. (d) 21. (a)
22. (b) 23. (b) 24. (c) 25. (c) 26. (c) 27. (b,c) 28. (b)
29. (b) 30. (b) 31. (b) 32. (a) 33. (b) 34. (b) 35. (a)
36. (d) 37. (b) 38. (d) 39. (a) 40. (b) 41. (a) 42. (b)
43. (b) 44. (b) 45. (b) 46. (d) 47. (a) 48. (d) 49. (d)
50. (a) 51. (a) 52. (a) 53. (d) 54. (c) 55. (a) 56. (c)
57. (26.65 to 26.68) 58. (a) 59. (b) 60. (d) 61. (c) 62. (c)
63. (d)

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Assignment-3: Classical Mechanics (Hamiltonian Dynamics)

1. A canonical transformation in classical Hamiltonian dynamics
(a) Cannot be made if there is more than one degree of freedom.
(b) Leaves the canonical Poisson brakcet relations unchanged.
(c) Can only be made for the cartesian components of the coordinates and momenta.
(d) Cannot be time-dependent

2. If P_x and L_z are constants of motion, then their Poisson bracket $[P_x, L_z]$ is :
(a) Also a constant of motion (b) Not a constant of motion.
(c) Zero (d) Undefined.

3. For a Hamiltonian $H = \frac{p^2}{2m} - \frac{e^2}{r} + k\left(x^2 + y^2\right)$, where $r^2 = x^2 + y^2 + z^2$ and k is a constant, the following quantities are constants of motion.
(a) E, L^2, L_z (b) E, L_z (c) E, L^2 (d) E only

4. The Hamiltonian of a charge particle of charge q, mass m, moving in an electromagnetic field $\left(\phi, \vec{A}\right)$ is given by
(a) $\frac{1}{2m}\left(\vec{P} - \frac{q}{c}\vec{A}\right)^2 + q\phi$ (b) $\frac{1}{2m}\left(\vec{P} - \frac{q}{c}\vec{A}\right)^2 - q\phi$
(c) $\frac{1}{2m}\left(\vec{P} + \frac{q}{c}\vec{A}\right)^2 - q\phi$ (d) $\frac{\vec{P}^2}{2m} + q\phi$

5. The Hamiltonian of a particle moving in a plane is given by

$$H = \frac{\overline{P}^2}{2m} + \frac{1}{2}m\omega^2 r^2 \sin^2\theta$$

where m is the mass, $\overline{p}$ the momentum, ω a constant, and $(r\ \theta)$ the radial and polar coordinates. The following quantity/quantities will be conserved.
(a) Energy (b) Angular momentum
(c) Energy and angular momentum (d) Energy and linear momentum.

6. The value of Poisson brakcet $\left[x^2, p_x^2\right]$ is :
(a) $4x^2$ (b) $4p_x^2$ (c) $4xp_x$ (d) xp_x

7. If the Lagrangian of a system has only one term which has a linear dependence on velocity, then its Hamiltonian
(a) Also has only one term depending on velocity.
(b) Is zero.
(c) Is not defined.
(d) Is independent of velocity and is constant.

8. A particle of mass m and coordinate q has the Lagrangian **[NET June 2014]**

$$L=\frac{1}{2}m\dot{q}^2-\frac{\lambda}{2}q\dot{q}^2$$

where λ is a constant. The Hamiltonian for the system is given by

(a) $\frac{p^2}{2m}+\frac{\lambda qp^2}{2m^2}$ (b) $\frac{p^2}{2(m-\lambda q)}$ (c) $\frac{p^2}{2m}+\frac{\lambda qp^2}{2(m-\lambda q)^2}$ (d) $\frac{pq}{2}$

9. The action variable J for a simple harmonic oscillator having hamiltonian $H=\frac{p^2}{2m}+\frac{1}{2}kq^2$, is related to its energy E by

(a) $J=\frac{1}{2\pi}\sqrt{\frac{k}{m}}E$ (b) $J=\sqrt{\frac{m}{k}}E$ (c) $J=\sqrt{\frac{k}{m}}E$ (d) $J=2\pi\sqrt{\frac{m}{k}}E$

10. A particle is being acted upon by the inverse square force $\vec{F}=-\frac{k}{r^2}\hat{r}$. The Hamiltonian of the system is:

(a) $H=\frac{\vec{p}.\vec{p}}{2m}-\frac{k}{r}$ (b) $H=\frac{\vec{p}.\vec{p}}{2m}+\frac{k}{r}$ (c) $H=\frac{\vec{p}.\vec{p}}{2m}-\frac{k}{r^2}$ (d) $H=\frac{\vec{p}.\vec{p}}{2m}+\frac{k}{r^2}$

11. The Hamiltonian of a system as a function of the canonical conjugate cordinate q and momentum p is given by H(q,p). The Hamilton's canonical equations are given by

(a) $p=\frac{\partial H}{\partial q}, q=\frac{\partial H}{\partial p}$ (b) $p=-\frac{\partial H}{\partial q}, q=\frac{\partial H}{\partial p}$

(c) $\dot{p}=-\frac{\partial H}{\partial q}, \dot{q}=\frac{\partial H}{\partial p}$ (d) $\dot{p}=\frac{\partial H}{\partial q}, \dot{q}=-\frac{\partial H}{\partial q}$

12. Two sets of transformation of conjugate position and momentum are given as (i) Q = p, P = – q and (ii) Q = p + iq, P = p–iq. Which of the following statements is correct?
(a) Both (i) and (ii) represent canonical transformations.
(b) (i) represent a canonical transformation but (ii) does not represents a canonical transformation.
(c) (i) does not represent a canonical transformation but (ii) does not representsa canonical transformation.
(d) Neither (i) nor (ii) represents any canonical transformation.

13. The Hamiltonian of a particle is $H=\frac{p^2}{2m}+pq$, where q is the generalized coordinate and p is the corresponding canonical momentum. The Lagrangian is: **[GATE 2007]**

(a) $\frac{m}{2}\left(\frac{dq}{dt}+q\right)^2$ (b) $-\frac{m}{2}\left(\frac{dq}{dt}-q\right)^2$ (c) $\frac{m}{2}\left[\left(\frac{dq}{dt}\right)^2+q\frac{dq}{dt}-q^2\right]$ (d) $\frac{m}{2}\left[\left(\frac{dq}{dt}\right)^2-q\frac{dq}{dt}-q^2\right]$

14. A particle of charge q, mass m and linear momentum $\vec{p}$ enters an electromagnetic field of vector potential $\vec{A}$ and scalar potential ϕ. The Hamiltonian of the particle is:

(a) $\frac{p^2}{2m}+q\phi+\frac{A^2}{2m}$ (b) $\frac{1}{2m}\left(\vec{p}-\frac{q}{c}\vec{A}\right)^2+q\phi$ (c) $\frac{1}{2m}\left(\vec{p}-\frac{q}{c}\vec{A}\right)^2+\vec{p}\bullet\vec{A}$ (d) $\frac{p^2}{2m}q\phi-\vec{p}\bullet\vec{A}$

15. For the given transformations (i) Q = p, P = –q and (ii) Q = p, P = q, where p and q are canonically conjugate variables, which one of the following statements is true?
(a) Both (i) and (ii) are canonical (b) Only (i) is canonical
(c) Only (ii) is canonical (d) Neither (i) nor (ii) is conical

16. The Hamiltonian corresponding to the Langrangian $L = a\dot{x}^2 + b\dot{y}^2 - kxy$ is:

(a) $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} + kxy$ (b) $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} - kxy$ (c) $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} + kxy$ (d) $\frac{p_x^2 + p_y^2}{4ab} + kxy$

17. The value of the Poisson bracket $\left[\vec{a}.\vec{r}, \vec{b}.\vec{p}\right]$, where $\vec{a}$ and $\vec{b}$ are constant vectors, is:

(a) $\vec{a}\,\vec{b}$ (b) $\vec{a} - \vec{b}$ (c) $\vec{a} + \vec{b}$ (d) $\vec{a}.\vec{b}$

18. Hamilton canonical equations of motion for a conservative system are:

(a) $-\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $-\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$ (b) $\frac{dp_i}{dt} = \frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

(c) $-\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$ (d) $\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $-\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

where q_i, p_i and H are generalized cooridnate, generalized momentum and Hamiltonian respectively.

19. A linear transformation of a generalized coordinate q and the corresponding momentum p to Q and P given by

$Q = q + p;\ P = q + \alpha p$

is canonical if the value of the constant α is:

(a) −1 (b) 0 (c) +1 (d) +2

20. q_1 and q_2 are generalised coordipate and p_1, p_2 are the corresponding generalised moments. The Poisson braket {X, Y} of $X = q_1^2 + q_2^2$ and $Y = 2p_1 + p_2$ is:

(a) $\left(q_1^2 + q_2^2\right)p_1$ (b) $3\left(q_1^2 + q_2^2\right)$ (c) $4q_1 + 2q_2$ (d) 0

21. For a simple harmonic oscillator the Lagrangian is given by $L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2$. If $A(p,d) = \frac{p + iq}{\sqrt{2}}$ and H(p,d) is the Hamiltonian of the system, the Poisson bracket $\{A(p,q), H(p,q)\}$ is given by **[GATE 2008]**

(a) iA(p,q) (b) $A^*(p,q)$ (c) $-iA^*(p,q)$ (d) −iA(p,q)

Statement for Linked Q.22 and Q.23.

The Lagrangian for a simple pendulum is given by: $L = \frac{1}{2}m\ell^2\dot{\theta}^2 - mg\ell(1 - \cos\theta)$ **[GATE 2010]**

22. Hamilton's equations are then given by

(a) $\dot{p}_\theta = -mg\ell\sin\theta;\quad \dot{\theta} = \frac{p_\theta}{m\ell^2}$ (b) $\dot{p}_\theta = mg\ell\sin\theta;\quad \dot{\theta} = \frac{p_\theta}{m\ell^2}$

(c) $\dot{p}_\theta = -mg\ddot{\theta};\quad \dot{\theta} = \frac{p_\theta}{m}$ (d) $\dot{p}_\theta = -\left(\frac{g}{\ell}\right)\theta;\quad \dot{\theta} = \frac{p_\theta}{m\ell}$

23. The Poisson bracket between θ and $\dot{\theta}$ is:

(a) $\{\theta, \dot{\theta}\} = 1$ (b) $\{\theta, \dot{\theta}\} = \frac{1}{m\ell^2}$ (c) $\{\theta, \dot{\theta}\} = \frac{1}{m}$ (d) $\{\theta, \dot{\theta}\} = \frac{g}{\ell}$

24. Given the Lagrangian $L = -\sqrt{1 - \left(\frac{dx}{dt}\right)^2}$

where 'x' is a co-ordinate and t is the time, the Hamiltonian is

(a) $\frac{\left(\frac{dx}{dt}\right)^2}{\sqrt{1 - \left(\frac{dx}{dt}\right)^2}}$ (b) $\sqrt{1 - \left(\frac{dx}{dt}\right)^2}$ (c) $\frac{1}{\sqrt{1 - \left(\frac{dx}{dt}\right)^2}}$ (d) $\left(\frac{dx}{dt}\right)^2$

25. Let (p,q) and (P,Q) be two pairs of canonical variables. The transformation **[GATE 2011]**

$$Q=q^{\alpha}\cos(\beta p);\ P=q^{\alpha}\sin(\beta p)$$

is canonical for

(a) $\alpha=2,\beta=1/2$ (b) $\alpha=2,\beta=2$ (c) $\alpha=1,\beta=1$ (d) $\alpha=1/2,\beta=2$

26. The Hamiltonian of a particle of unit mass moving in the xy-plane is given to be:

$H=xp_x-yp_y-\frac{1}{2}x^2+\frac{1}{2}y^2$ in suitable units. The initial values are given to be (x(0), y(0)) = (1, 1) and $(p_x(0),p_y(0))=\left(\frac{1}{2},-\frac{1}{2}\right)$. During the motion, the curves traced out by the particles in the xy-plane and the p_xp_y-plane are **[NET June 2011]]**

(a) Both straight lines (b) A straight line and a hyperbola respectively

(c) A hyperbola an ellipse, respectively (d) Both hyperbolas.

27. A Dynamical system with two degrees of freedom, has generalised coordinates q_1 and q_2, and kinetic energy

$$T=\lambda\dot{q}_1\dot{q}_2$$

[TIFR 2012]

If the potential energy is $V(q_1,q_2)=0$, the correct form of the Hamiltonian for this system is:

(a) p_1p_2/λ (b) $\lambda\dot{q}_1\dot{q}_2$ (c) $(p_2\dot{q}_1+p_2\dot{q}_2)/2$ (d) $(p_1q_2+p_2q_1)/2$

28. For what value of α will the transformation $q\rightarrow Q=q^{\alpha}\cos 2p$ and $p\rightarrow P=q^{\alpha}\sin 2p$ be cononical

(a) $\alpha=2$ (b) $\alpha=1/2$ (c) $\alpha=-1$ (d) $\alpha=1$

29. The Lagrangian of a relativistic particle of mass m and velocity v is $L=-mc^2\sqrt{1-\frac{v^2}{c^2}}$. Its Hamiltonian H is

(a) $H=mc^2\sqrt{1-\frac{v^2}{c^2}}$ (b) $H=\frac{mc^2}{\sqrt{1-v^4/c^2}}$ (c) $H=\sqrt{m^2c^2+p^2c^2}$ (d) $H=\frac{m^2c^4}{\sqrt{m^2c^4+p^2c^2}}$

30. A spring made of a given material has the restoring force $F=-kx^3$. The total energy of a point of mass m at the end of this spring is :

(a) $E=\frac{1}{2}m\dot{x}^2+\frac{1}{4}kx^4$ (b) $E=\frac{1}{2}m\dot{x}^2-\frac{1}{4}kx^4$ (c) $E=\frac{1}{2}m\dot{x}^2+kx^3$ (d) $E=\frac{1}{2}m\dot{x}^2+\frac{1}{2}kx^2$

31. The phase space orbit of a body sliding down a frictionless inclined plane in uniform gravity is

(a) a straight line (b) an ellipse (c) a parabola (d) a hyperbola

32. Consider a canonical transformation $q\rightarrow Q=-q$ and $p\rightarrow P=-p$. The generating function for this transformation is given by

(a) $F_3=-pQ$ (b) $F_1=-qQ$ (c) $F_2=-qP$ (d) $F_4=-pP$

33. What is the Poisson bracket between the position x and x-component of the angular momentum L_x

(a) x (b) y (c) z (d) 0

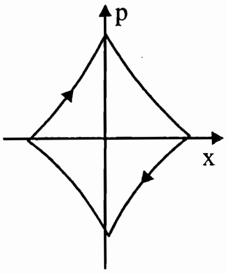
The Hamiltonian of a system is independent of the coordinate q and is given by $H = ap^4$ where a is a constant. The corresponding Lagrangian is then given by

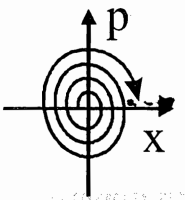
(a) $L = m\dot{q}^2 - ap^4$ (b) $L = \frac{1}{2}m\dot{q}^2 - ap^4$ (c) $L = a\dot{q}^4$ (d) $L = \frac{3\dot{q}}{4}\left(\frac{\dot{q}}{4a}\right)^{1/3}$

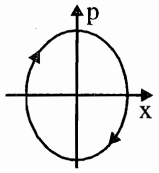
If the coordinate q and the momentum p form a canonical pair (q,p). which one of the sets given below also forms a canonical pair ?

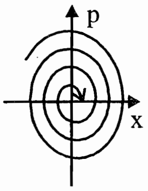
(a) $(q,-p)$ (b) (q^2,p^2) (c) $(p,-q)$ (d) $(q^2,-p^2)$

The bob of a simple pendulum, which undergoes small oscillations, is immersed in water. Which of the following figures best represents the phase space diagram for the pendulum? **[NET June 2012]**

(a) 

(b) 

(c) 

(d) 

(a) FIGURE (b) FIGURE (c) FIGURE (d) FIGURE

If the Lagrangian of a particle moving in one dimensions is given by $L = \frac{\dot{x}^2}{2x} - V(x)$, the Hamiltonian is:

(a) $\frac{1}{2}xp^2 + V(x)$ (b) $\frac{\dot{x}^2}{2x} + V(x)$ (c) $\frac{1}{2}\dot{x}^2 + V(x)$ (d) $\frac{p^2}{2x} + V(x)$ **[NET June 2012]**

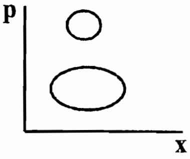
The Poisson bracket $\{|r|,|p|\}$ has the value **[NET]**

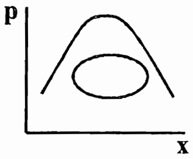
(a) $\vec{r}\cdot\vec{p}$ (b) $\hat{r}.\hat{p}$ (c) 3 (d) 1

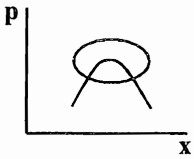
The Hamiltonian of a simple pendulum consisting of a mass 'm' attached to a massless string of length l is $H = \frac{p_\theta^2}{2m\ell^2} + mg\ell(1-\cos\theta)$. If L denotes the Lagrangian, the value of $\frac{dL}{dt}$ is : **[NET]**

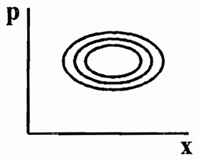
(a) $-\frac{2g}{\ell}p_\theta \sin\theta$ (b) $-\frac{g}{\ell}p_\theta \sin 2\theta$ (c) $\frac{g}{\ell}p_\theta \cos\theta$ (d) $\ell p_\theta^2 \cos\theta$

Which of the following set of phase-space trajectories which one is not possible for a particle obeying Hamilton's equations of motion (for a time-independent Hamiltonian)? **[NET Dec. 2012]**

(a) 

(b) 

(c) 

(d) 

The Hamiltonian of a relativistic particle of rest mass m and momentum p is given by $H = \sqrt{p^2 + m^2} + V(x)$, in units in which the speed of light $c = 1$. The corresponding Lagrangian is **[NET Dec. 2013]**

(a) $L = m\sqrt{1+\dot{x}^2} - V(x)$ (b) $L = -m\sqrt{1-\dot{x}^2} - V(x)$

(c) $L = \sqrt{1+m\dot{x}^2} - V(x)$ (d) $L = \frac{1}{2}m\dot{x}^2 - V(x)$

42. A system is governed by the Hamiltonian $H = \frac{1}{2}(p_x - ay)^2 + \frac{1}{2}(p_y - bx)^2$ **[NET June 2013]**

where a and b are constants and p_x, p_y are momenta conjugate to x and y respectively. For what values of a and b will the quantities $(p_x - 3y)$ and $(p_y + 2x)$ be conserved?

(a) $a = -3,\ b = 2$ (b) $a = 3,\ b = -2$ (c) $a = 2,\ b = -3$ (d) $a = -2,\ b = 3$

43. The coordinates and momenta $x_i, p_i\ (i = 1,2,3)$ of a particle satisfy the canonical Poissoa bracket relations $\{x_i, p_j\} = \delta_{ij}$. If $C_1 = x_2p_3 + x_3p_2$ and $C_2 = x_1p_2 - x_2p_1$ are constants of motion, and if $C_3 = \{C_1, C_2\} = x_1p_3 + x_3p_1$, then **[NET June 2014]**

(a) $\{C_2, C_3\} = C_1$ and $\{C_3, C_1\} = C_2$ (b) $\{C_2, C_3\} = -C_1$ and $\{C_3, C_1\} = -C_2$

(c) $\{C_2, C_3\} = -C_1$ and $\{C_3, C_1\} = C_2$ (d) $\{C_2, C_3\} = C_1$ and $\{C_3, C_1\} = -C_2$

44. A canonical transformation relates the old coordinates (q, p) to the new ones (Q, P) by the relations $Q = q^2$ and $P = p/2q$. The corresonding time-independent generating function is **[NET June 2014]**

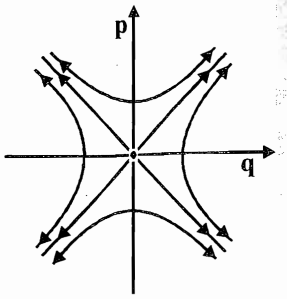
(a) $\frac{P}{q^2}$ (b) q^2P (c) q^2/P (d) qP^2

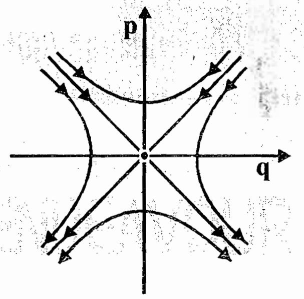
45. The Hamilton's canonical equations of motion in terms of Poisson Brackets are **[GATE 2014]**

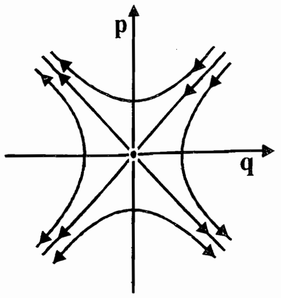
(a) $\dot{q} = \{q, H\};\ \dot{p} = \{p, H\}$ (b) $\dot{q} = \{H, q\};\ \dot{p} = \{H, q\}$

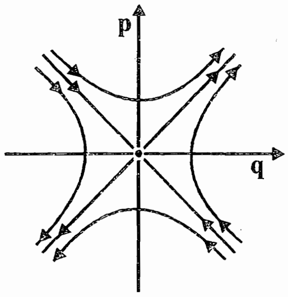
(c) $\dot{q} = \{H, p\};\ \dot{p} = \{H, q\}$ (d) $\dot{q} = \{p, H\};\ \dot{p} = \{q, H\}$

46. The Hamiltonian of a particle of mass 'm' is given by $H = \frac{p^2}{2m} - \frac{\alpha q^2}{2}$. Which of the folloiwng figures describes the motion of the particle in phase space? **[GATE 2014]**

(a)


(b)


(c)


(d)


47. Given that the linear transformation of a generalized coordinate 'q' and the corresponding momentump,

$$Q = q + 4ap$$
$$p = q + 2p$$

[GATE 2014]

is canonical, the value of the constant 'a' is ___________

48. In a system with two degrees of freedom, if (p, q) are the canonical coordinates, then which of the following transformations to (P, Q) is canonical? **[TIFR]**

(a) $P=\frac{1}{2}\left(p^2+q^2\right), Q=\tan^{-1}\frac{2q}{p}$ (b) $P=\frac{1}{2}\left(p^2+q^2\right), Q=\cot^{-1}\frac{p}{q}$

(c) $P=\frac{1}{2}\left(p^2+q^2\right), Q=\sin^{-1}\frac{q}{2p}$ (d) $P=\frac{1}{2}\left(p^2+q^2\right), Q=\cos^{-1}\frac{p}{q}$

49. The expression $\sum_{i,j,k=1}^{3}\in_{ijk}\left\{x_i,\{p_j,L_k\}\right\}$ (where $\in_{ijk}$ is the Levi-Civita symbol, $\vec{x},\vec{p},\vec{L}$ are the position, momentum and angular momentum respectively, and $\{A, B\}$ represents the Poisson bracket of A and B) simplifies to **[NET Dec. 2014]**

(a) 0 (b) 6 (c) $\vec{x}\cdot(\vec{p}\times\vec{L})$ (d) $\vec{x}\times\vec{p}$

50. A mechanical system is described by the Hamiltonian $H(q,p)=\frac{p^2}{2m}+\frac{1}{2}m\omega^2q^2$. As a result of the canonical transformation generated by $F(q,Q)=-\frac{Q}{q}$, the Hamiltonian in the new coordinate Q and momentum P becomes **[NET Dec. 2014]**

(a) $\frac{1}{2m}Q^2P^2+\frac{m\omega^2}{2}Q^2$ (b) $\frac{1}{2m}Q^2P^2+\frac{m\omega^2}{2}P^2$

(c) $\frac{1}{2m}P^2+\frac{m\omega^2}{2}Q^2$ (d) $\frac{1}{2m}Q^2P^4+\frac{m\omega^2}{2}P^{-2}$

51. The Lagrangian for a particle of mass m at a position $\vec{r}$ moving with a velocity $\vec{v}$ is given by $L=\frac{m}{2}\vec{v}^2+C\vec{r}\cdot\vec{v}-V(r)$, where $V(r)$ is a potential and C is a constant. If $\vec{p}_c$ is the canonical momentum, then its Hamiltonian is given by **[GATE 2015]**

(a) $\frac{1}{2m}(\vec{p}_c+C\vec{r})^2+V(r)$ (b) $\frac{1}{2m}(\vec{p}_c-C\vec{r})^2+V(r)$

(c) $\frac{p_c^2}{2m}+V(r)$ (d) $\frac{1}{2m}p_c^2+C^2r^2+V(r)$

52. The Hamiltonian for a system of two particles of masses m_1 and m_2 at $\vec{r}_1$ and $\vec{r}_2$ having velocities $\vec{v}_1$ and $\vec{v}_2$ is given by $H=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2+\frac{C}{(\vec{r}_1-\vec{r}_2)^2}\hat{z}\cdot(\vec{r}_1\times\vec{r}_2)$, where C is a constant. Which one of the following statements is correct? **[GATE 2015]**

(a) The total energy and total momentum are conserved
(b) Only the total energy is conserved
(c) The total energy and the z-component of the total angular momentum are conserved
(d) The total energy and total angular momentum are conserved

53. If the Lagrangian of a dynamical system in two dimensions is $L = \frac{1}{2}m\dot{x}^2 + m\dot{x}\dot{y}$, then its Hamiltonian is

(a) $H = \frac{1}{m}p_x p_y + \frac{1}{2m}p_y^2$ (b) $H = \frac{1}{m}p_x p_y + \frac{1}{2m}p_x^2$ **[NET June 2015]**

(c) $H = \frac{1}{m}p_x p_y - \frac{1}{2m}p_y^2$ (d) $H = \frac{1}{m}p_x p_y - \frac{1}{2m}p_x^2$

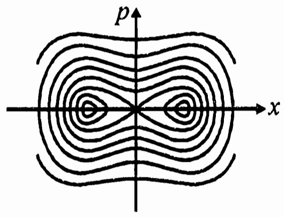
54. Let q and p be the canonical coordinate and momentum of a dynamical system. Which of the following transformations is canonical? **[NET June 2015]**

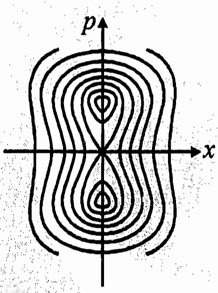
$$\text{A} : Q_1 = \frac{1}{\sqrt{2}}q^2 \text{ and } P_1 = \frac{1}{\sqrt{2}}p^2$$

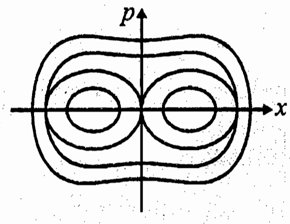
$$\text{B} : Q_2 = \frac{1}{\sqrt{2}}(p+q) \text{ and } P_2 = \frac{1}{\sqrt{2}}(p-q)$$

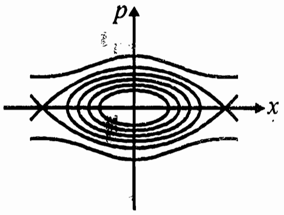
(a) neither A nor B (b) both A and B (c) only A (d) only B

55. Which of the following figures is a schematic representation of the phase space trajectories (i.e., contours of constant energy) of a particle moving in a one-dimensional potential $V(x) = -\frac{1}{2}x^2 + \frac{1}{4}x^4$? **[NET June 2015]**

(a)


(b)


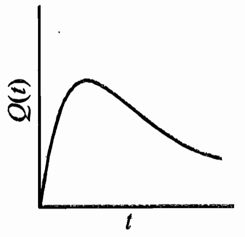
(c)


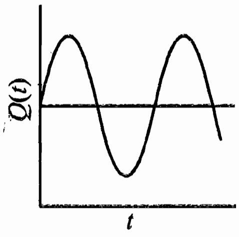
(d)


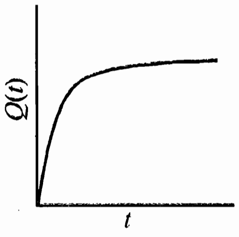
56. Let A, B and C be functions of phase space variables (coordinates and momenta of a mechanical system). If {,} represents the Poisson bracket, the value of {A,{B,C}} – {{A,B},C} is given by **[NET Dec. 2013]**

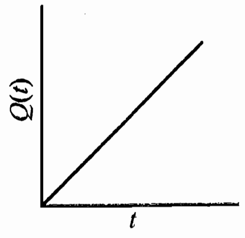
(a) 0 (b) {B,{C,A}} (c) {A,{C,B}} (d) {{C,A},B}

57. A canonical transformation $(p, q) \to (P, Q)$ is performed on the Hamiltonian $H = \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2 q^2$ via the generating function $F = \frac{1}{2}m\omega q^2 \cot Q$. If $Q(0) = 0$, which of the following graphs shows schematically the dependence of $Q(t)$ on t? **[NET Dec. 2015]**

(a)


(b)


(c)


(d)


The Hamiltonian of a system with generalized coordinate and momentum (q, p) is $H = p^2q^2$. A solution of the Hamiltonian equation of motion is (in the following A and B are constants) **[NET June 2016]**

(a) $p = Be^{-2At}, q = \frac{A}{B}e^{2At}$ (b) $p = Ae^{-2At}, q = \frac{A}{B}e^{-2At}$

(c) $p = Ae^{At}, q = \frac{A}{B}e^{-At}$ (d) $p = 2Ae^{-A^2t}, q = \frac{A}{B}e^{A^2t}$

A canonical transformation $(q, p) \to (Q, P)$ is made through the generating function $F(q, P) = q^2P$ on the Hamiltonian $H(q, p) = \frac{p^2}{2\alpha q^2} + \frac{\beta}{4}q^4$, where α and β are constants. The equations of motion for (Q, P) are

(a) $\dot{Q} = \frac{P}{\alpha}$ and $\dot{P} = -\beta Q$ (b) $\dot{Q} = \frac{4P}{\alpha}$ and $\dot{P} = \frac{-\beta Q}{2}$ **[NET June 2016]**

(c) $\dot{Q} = \frac{P}{\alpha}$ and $\dot{P} = -\frac{2P^2}{Q} - \beta Q$ (d) $\dot{Q} = \frac{2P}{\alpha}$ and $\dot{P} = -\beta Q$

If a central force acting on a particle of mass m is given by **[TIFR 2013]**

$$F(r) = -\frac{k}{r^2}$$

where r is the distance of the particle from the origin and k is a positive constant, the Hamiltonian for the system, in spherical polar coordinates, will have the form

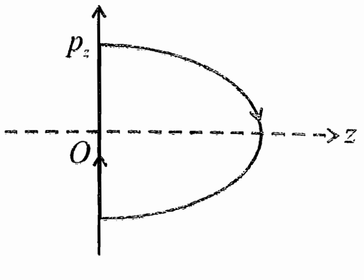
(a) $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\varphi}^2) + \frac{k}{r}$ (b) $\frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + \frac{p_\varphi^2\mathrm{cosec}^2\theta}{2mr^2} + \frac{k}{r}$

(c) $\frac{1}{2mr^2}(r^2p_r^2 + p_\theta^2 + p_\varphi^2 + 2mkr)$ (d) $\frac{p_r^2}{2m} + \frac{p_\theta^2}{2m} + \frac{p_\varphi^2}{2m} + \frac{k}{r}$

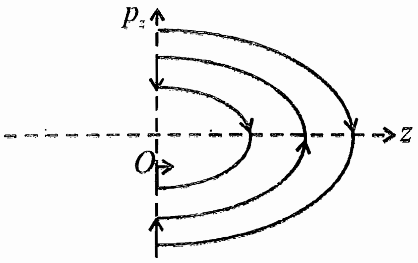
An inelastic ball of mass m has been thrown vertically upwards from the ground at $z = 0$. The initial kinetic energy of the ball is E. The phase trajectory of the ball after successive bouncing on the ground is .

[GATE 2008]

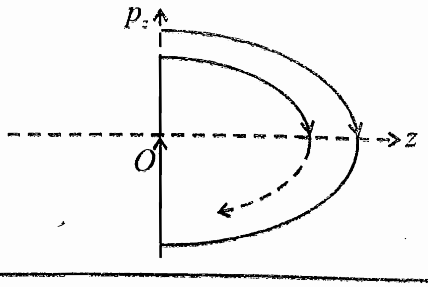
(a)



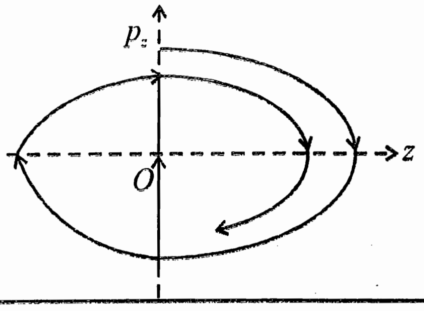
(b)



(c)



(d)



62. If p and q are the position and momentum variables, which one of the following is NOT a canonical transformation? **[GATE 2009]**

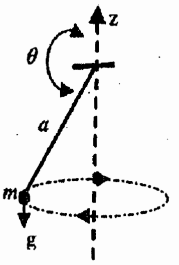
(a) $Q=\alpha q$ and $P=\frac{1}{\alpha}p$, for $\alpha \neq 0$

(b) $Q=\alpha q+\beta p$ and $P=\beta q+\alpha p$ for α, β real and $\alpha^2-\beta^2=1$

(c) $Q=p$ and $P=q$

(d) $Q=p$ and $P=-q$

63. A particle of mass m is attached to a fixed point 'O' by a weightless inextensible string of length a. It is rotating under the gravity as shown in the figure. **[GATE 2012]**



The Lagrangian of the particle is

$$L(\theta,\phi)=\frac{1}{2}ma^2\left(\dot{\theta}^2+\sin^2\theta\dot{\phi}^2\right)-mga\cos\theta$$

where θ and ϕ are the polar angles.

The Hamiltonian of the particle is:

(a) $H=\frac{1}{2ma^2}\left(p_\theta^2+\frac{p_\phi^2}{\sin^2\theta}\right)-mga\cos\theta$ (b) $H=\frac{1}{2ma^2}\left(p_\theta^2+\frac{p_\phi^2}{\sin^2\theta}\right)+mga\cos\theta$

(c) $H=\frac{1}{2ma^2}\left(p_\theta^2+p_\phi^2\right)-mga\cos\theta$ (d) $H=\frac{1}{2ma^2}\left(p_\theta^2+p_\phi^2\right)+mga\cos\theta$

64. The Hamiltonian of a system with n degrees of freedom is given by **[NET June 2011]**

$H(q_1,\ldots\ldots\ldots,q_n;p_1,\ldots\ldots\ldots\ldots,p_n;t)$,

with an explicit dependence on the time t. Which of the following is correct?

(a) Different phase trajectories cannot intersect each other.

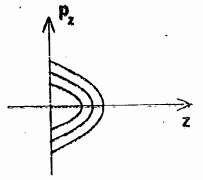
(b) H always represents the total energy of the system and is a constant of the motion.

(c) The equations $\dot{q}_i=\partial H/\partial p_i$, $\dot{p}_i=-\partial H/\partial q_i$ are not valid since H has explicit time dependence.

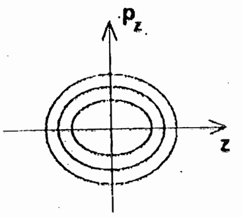
(d) Any initial volume element in phase space remains unchanged in magnitude under time evolution.

65. The trajectory on the zp_z-plane (phase-space trajectory) of a ball bouncing perfectly elastically off a hard surface at $z=0$ is given by approximately by (neglect friction) : **[NET Dec. 2011]**

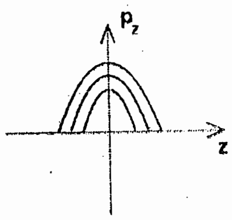
(a)



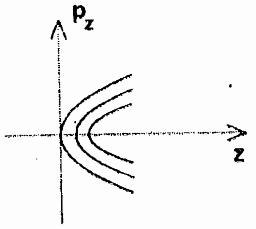
(b)



(c)



(d)



6. If L_i are the components of the angular momentum $\vec{L}$, then the quantity $\sum i=1,2,3\left[\left[\vec{L},L_i\right],L_i\right]$ equals

(a) $\vec{L}$ (b) $-2\vec{L}$ (c) $3\vec{L}$ (d) $-\vec{L}$ **[NET June 2015]**

Answer Key: Classical Mechanics (Hamiltonian Dynamics)

1. (b)	**2. (a)**	**3. (b)**	**4. (a)**	**5. (a)**	**6. (c)**	**7. (b)**
8. (b)	**9. (d)**	**10. (a)**	**11. (c)**	**12. (b)**	**13. (b)**	**14. (b)**
15. (b)	**16. (c)**	**17. (d)**	**18. (d)**	**19. (d)**	**20. (c)**	**21. (a)**
22. (a)	**23. (b)**	**24. (c)**	**25. (d)**	**26. (d)**	**27. (a)**	**28. (b)**
29. (c)	**30. (a)**	**31. (c)**	**32. (c)**	**33. (d)**	**34. (d)**	**35. (c)**
36. (d)	**37. (a)**	**38. (b)**	**39. (a)**	**40. (c)**	**41. (b)**	**42. (d)**
43. (d)	**44. (b)**	**45. (a)**	**46. (d)**	**47. (0.24 to 0.26)**		**48. (b)**
49. (b)	**50. (d)**	**51. (b)**	**52. (c)**	**53. (c)**	**54. (d)**	**55. (a)**
56. (d)	**57. (d)**	**58. (a)**	**59. (b)**	**60. (a)**	**61. (c)**	**62. (c)**
63. (b)	**64. (d)**	**65. (a)**	**66. (b)**			

CSIR-UGC-NET/JRF | GATE PHYSICS

Assignment-4: Classical Mechanics (Special Theory of Relativity)

1. A relativistic particle of rest mass m_0 is moving with a speed v. The value of v at which its kinetic energy is equal to its rest energy is:

(a) $v=\frac{c}{2}$ (b) $v=\frac{c}{4}$ (c) $v=\frac{\sqrt{3}}{2}c$ (d) $v=2\left(\sqrt{2}-1\right)c$

2. A meter rod is moving along the positive x axis with velocity 0.6C. Its length as measured by a stationary observer in meters, is:

(a) 0.6 (b) 0.8 (c) 1.0 (d) 1.2

3. An event is space-like if the two world points are separated such that

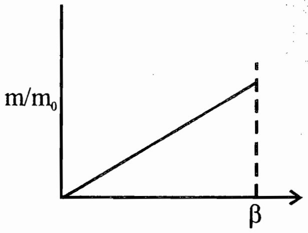
(a) $|\vec{r}_1-\vec{r}_2|^2\le c^2\left(t_1-t_2\right)^2$ (b) $|\vec{r}_1-\vec{r}_2|^2\le c^2\left(t_1+t_2\right)^2$

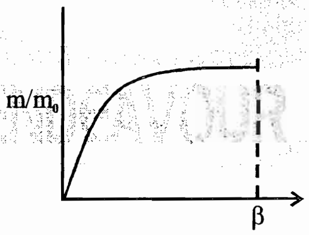
(c) $|\vec{r}_1-\vec{r}_2|^2> c^2\left(t_1-t_2\right)^2$ (d) $|\vec{r}_1-\vec{r}_2|^2> c^2\left(t_1-t_2\right)^2$

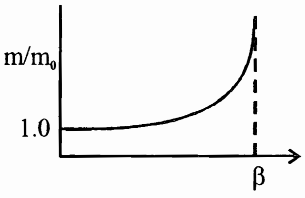
4. A particle moves with a relativistic speed such that its energy is twice the rest mass energy. The speed of the particle is:

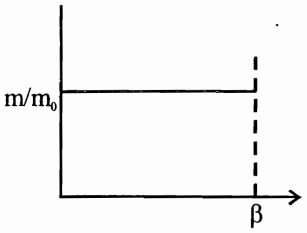
(a) $v=\frac{c}{2}$ (b) $v=\frac{3}{4}c$ (c) $v=\frac{\sqrt{3}}{2}c$ (d) $v=c$

5. For a particle moving with relativistic speed, the ratio of its relativistic mass to its rest mass $\left(m/m_0\right)$ varies with the parameter $\beta=v/c$ as

(a)


(b)


(c)


(d)


6. A particle of mass M at rest decays into two identical particles of mass m each such that 2m is less than M. The speed of each particle after decay

(a) $v=0$ (b) $v=c\sqrt{1+\left(2m/M\right)^2}$ (c) $v=c\sqrt{M/m}$ (d) $v=c\sqrt{1-\left(2m/M\right)^2}$

Statement for Linked Q.7 and Q.8. **[GATE 2007]**

In the laboratory fram, a particle P of rest mass m0 is moving in the positive x direction with a speed of $\frac{5c}{19}$. It approaches an identical particle Q moving in the negative x direction with a speed of $\frac{2c}{5}$.

The speed of the particle P in the rest frame of the particle Q is:

(a) $\frac{7c}{95}$ (b) $\frac{13c}{85}$ (c) $\frac{3c}{5}$ (d) $\frac{63c}{95}$

The energy of the particle P in the rest frame of the particle Q is:

(a) $\frac{1}{2}m_0c^2$ (b) $\frac{5}{4}m_0c^2$ (c) $\frac{19}{13}m_0c^2$ (d) $\frac{11}{9}m_0c^2$

The mass m of a moving particle is $\frac{2m_0}{\sqrt{3}}$, where m_0 is its rest mass. The linear momentum of the particle is:

(a) $2m_0c$ (b) $\frac{2m_0c}{\sqrt{3}}$ (c) $\frac{m_0c}{\sqrt{3}}$ (d) m_0c

0. A rod of length ℓ_0 makes an angle θ_0 with the y-axis in its rest frame, while the rest frame moves to the right along the x-axis with relativistic speed v with respect to the lab frame. If $\gamma = (1 - v^2/c^2)^{-1/2}$, the angle q in the lab frame is:

(a) $\theta = \tan^{-1}(\gamma\tan\theta_0)$ (b) $\theta = \tan^{-1}(\gamma\cot\theta_0)$ (c) $\theta = \tan^{-1}\left(\frac{1}{\gamma}\tan\theta_0\right)$ (d) $\theta = \tan^{-1}\left(\frac{1}{\gamma}\cot\theta_0\right)$

11. An electron gains energy so that it mass becomes $2m_0$. Its speed is:

(a) $\frac{\sqrt{3}}{2}c$ (b) $\frac{3}{4}c$ (c) $\frac{3}{2}c$ (d) $\sqrt{\frac{3}{2}}c$

12. The speed of a particle whose kinetic energy is equal to its rest mass energy is given by (c is the speed of light in vacuum)

(a) c/3 (b) $\sqrt{2}c/3$ (c) c/2 (d) $\frac{\sqrt{3}}{2}c$

13. Which of the following equations is relatively invariant? (α, β, δ and d are constants of suitable dimensions)

(a) $\frac{\partial\phi(x,t)}{\partial t} = \alpha\frac{\partial^2\phi(x,t)}{\partial x^2}$ (b) $\frac{\partial^2\phi(x,t)}{\partial t^2} = \beta\frac{\partial^2\phi(x,t)}{`\partial x^2}$

(c) $\frac{\partial^2\phi(x,t)}{\partial t^2} = \gamma\frac{\partial^2\phi(x,t)}{\partial x}$ (d) $\frac{\partial\phi(x,t)}{\partial t} = \delta\frac{\partial^3\phi(x,t)}{\partial x^3}$

14. A relativeistic particle of mass m and velocity c/2 $\hat{z}$ is moving towards a wall. The wall is moving with a velocity c/3 $\hat{z}$. The velocity of the particle after it suffers an elastic collision is $\upsilon\,\hat{z}$ with υ equal to:

(a) c/2 (b) c/5 (c) c/7 (d) c/15

(All the velocities refer to the laboratory frame of reference)

15. The momentum of an electron (mass m) which has the same kinetic energy as its rest mass energy is (c is velocity of light)

(a) $\sqrt{3}$ mc (b) $\sqrt{2}$ mc (c) mc (d) $\text{mc}/\sqrt{2}$

16. In a system of units in which the velocity of light c = 1, which of the following is a Lorentz transformation?

(a) $x' = 4x, y = y', z' = z, t' = 0.25t$
(b) $x' = x - 0.5t, y' = y, z' = z, t' = t + x$
(c) $x' = 1.25x - 0.75t, y' = y$
$z' = z, t' = 0.75t - 1.25x$
(d) $x' = 1.25x - 0.75t, y' = y$
$z' = z, t' = 1.25t - 0.75x$

17. Kinetic energy of a relativistic particle of rest mass m moving with speed v is:

(a) $\frac{1}{2}mv^2$ (b) $\frac{mc^2}{\sqrt{1-v^2/c^2}}$ (c) $\frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2$ (d) $\frac{1}{2}m\left(v^2 - c^2\right)$

18. A particle has rest mass m_0, and momentum m_0c, where c is the velocity of light. The total energy and the velocity of the particle are respectively

(a) $\sqrt{2}m_0c^2$ and $c/2$ (b) $2m_0c^2$ and $c/\sqrt{2}$ (c) $\sqrt{2}m_0c^2$ and $c/\sqrt{2}$ (d) $2m_0c^2$ and $c/2$

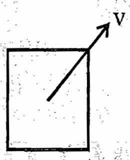
19. Two particles, each of rest mass m collide head-on and stick together. Before collision, the speed of each mass was 0.6 times the speed of light in free space. The mass of final entity is: **[GATE 2011]**

(a) 5m/4 (b) 2m (c) 5m/2 (d) 25m/8

20. A constant force F is applied to a relativistic particle of rest mass m. If the particle starts from rest at t = 0, its speed after a time t is **[NET Dec. 2011]**

(a) Ft/m (b) $c\tanh\left(\frac{Ft}{mc}\right)$ (c) $c\left(1 - e^{-Ft/mc}\right)$ (d) $\frac{Fct}{\sqrt{F^2t^2 + m^2c^2}}$

21. A uniform cube of side l (in its rest frame) is moving at a constant velocity v along the direction shown in the figure with respect to a stationary observer.



The volume of the cube as seen by her is

(a) $\ell^3\sqrt{1-\frac{v^2}{c^2}}$ (b) $\ell^3\left(1-\frac{v^2}{c^2}\right)$ (c) $\ell^3\left(1-\frac{v^2}{c^2}\right)^{3/2}$ (d) ℓ^3

22. A pair of identical relativistic particles with mass m each and momenta $p\hat{x}$ and $-p\hat{x}$ respectively collide. After collision, they are transformed to a pair of new particles with mass M each. If the magnitudes of the final state momenta are P_1 & P_2 respectively, then

(a) $P_1 = P_2 = p + (m - M)c$
(b) $P_1^2 = P_2^2 = p^2 + \left(M^2 - m^2\right)c^2$
(c) $P_1^2 = P_2^2 = p^2 + \left(m^2 - M^2\right)c^2$
(d) $P_1^2 = p^2 + \left(m^2 - M^2\right)c^2$ and $P_2^2 = p^2 + \left(M^2 - m^2\right)c^2$

23. A rod of proper length ℓ_0 oriented parallel to the x-axis moves with speed 2c/3 along the x-axis in the S-frame, where c is the speed of the light in free space. The observer is also moving along the x-axis with speed c/2 with respect to the S-frame. The length of the rod as measured by the observer is

(a) 0.35 ℓ_0 (b) 0.48 ℓ_0 (c) 0.87 ℓ_0 (d) 0.97 ℓ_0

24. Two events, separated by a (spatial) distance 9×10^9 m, are simultaneous in one inertial frame. The time interval between these two events in a frame moving with a constant speed 0.8c (where the speed of light $c = 3\times10^8$ m/s) is: **[NET June 2012]**

(a) 60 s (b) 40 s (c) 20 s (d) 0 s

25. Let v, p and E denotes the speed, the magnitude of the momentum, and the energy of a free particle of rest mass 'm'. Then **[NET Dec. 2012]**

(a) $\frac{dE}{dp} = \text{constant}$ (b) $p = mv$ (c) $v = \frac{cp}{\sqrt{p^2 + m^2c^2}}$ (d) $E = mc^2$

26. Although mass-energy equivalence of special relativity allows conversion of photon to an electron-positron pair, such a process cannot occur in free space because:

(a) The mass is not conserved (b) The energy is not conserved
(c) The momentum is not conserved (d) The charge is not conserved

27. An electron is accelerated from rest by 10.2 million volts. The percent increase in its mass is:

(a) 20,000 (b) 2,000 (c) 200 (d) 20

28. A circle of radius 5m lies at rest in x – y plane in the laboratory. For an observer moving with a uniform velocity υ along the y direciton, the circle appears to be an ellipse with an equation $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

The speed of the observer in terms of the velocity of light c is:

(a) 9c/25 (b) 3c/5 (c) 4c/5 (d) 16c/25

29. What is the proper time interval between the occurence of two events if in one inertial frame the events are separated by 7.5×10^8 m and occur 6.5 s apart? **[NET June 2012]**

(a) 6.50 s (b) 6.00 s (c) 5.75 s (d) 5.00 s

30. The relativistic form of Newton's second law of motion is **[GATE 2013]**

(a) $F = \frac{mc}{\sqrt{c^2 - v^2}}\frac{dv}{dt}$ (b) $F = \frac{m\sqrt{c^2 - v^2}}{c}\frac{dv}{dt}$ (c) $F = \frac{mc^2}{c^2 - v^2}\frac{dv}{dt}$ (d) $F = m\frac{c^2 - v^2}{c^2}\frac{dv}{dt}$

31. A light beam is propagating through a block of glass with index of refraction *n*. If the glass is moving at constant velocity *v* in the same direction as the beam, the velocity of the light in the glass block as measured by an observer in the laboratory is approximately

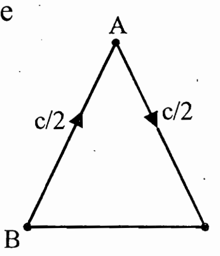
(a) $u = \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right)$ (b) $u = \frac{c}{n} - v\left(1 - \frac{1}{n^2}\right)$

(c) $u = \frac{c}{n} + v\left(1 + \frac{1}{n^2}\right)$ (d) $u = \frac{c}{n}$

32. The velocity of a particle at which the kinetic energy is equal to its rest energy is (in terms of *c*, the speed of light in vacuum)

(a) $\sqrt{3}c/2$ (b) $3c/4$ (c) $\sqrt{3/5}c$ (d) $c/\sqrt{2}$

33. In the laboratory frame, two observers A and B are moving along the sides of an equilateral triangle with equal speeds c/2, as shown in the figure **[TIFR]**



The speed of B as measured by A will be

(a) $\frac{\sqrt{3}}{2}c$ (b) $\frac{4}{3\sqrt{3}}c$ (c) $\frac{\sqrt{13}}{7}c$ (d) $\frac{\sqrt{5}}{3}c$

34. An electron is moving with a velocity of $0.85c$ in the same direction as that of a moving photon. The relative velocity of the electron with respect to photon is **[GATE]**

(a) c (b) $-c$ (c) $0.15c$ (d) $-0.15c$

35. If a Higgs boson of mass m_H with a speed $\beta = \frac{v}{c}$ decays into a pair photons, then the invariant mass of the photon pair is **[NET]**

[**Note:** The invariant mass of a system of two particles, with four-momenta $\mathbf{p}_1$ and p_2 is $(p_1 + p_2)^2$]

(a) βm_H (b) m_H (c) $m_H/\sqrt{1-\beta^2}$ (d) $\beta m_H/\sqrt{1-\beta^2}$

36. A light source is switched on and of at a constant frequency f. An observer moving with a velocity u with respect to the light source will observe the freuqency of the switching to be **[NET June 2014]**

(a) $f\left(1-\frac{u^2}{c^2}\right)^{-1}$ (b) $f\left(1-\frac{u^2}{c^2}\right)^{-1/2}$ (c) $f\left(1-\frac{u^2}{c^2}\right)$ (d) $f\left(1-\frac{u^2}{c^2}\right)^{1/2}$

37. Which one of the following quantities is invariant under Lorentz transformation? **[GATE 2014]**

(a) Charge density (b) Charge (c) Current (d) Electric field

38. If the half-life of an elementary particle moving with speed 0.9c in the laboratory frame is 5×10^{-8}s, then the proper half-life is ______ $\times10^{-8}$ s. ($c = 3\times10^8$ m/s).

39. A light beam is propagating through a medium with index of refraction 1.5. If the medium is moving at constant velocity $0.7c$ in the same direction as the beam, what is the velocity of light in the medium as measured by an observer in the laboratory ? (c = velocity of light in vacuum)

(a) $0.93c$ (b) $0.98c$ (c) $0.96c$ (d) $0.9c$

40. A collimated beam of pions originate from an accelerator and propagates in vacuum along a long straight beam pipe. The intensity of this beam was measured in the laboratory after a distance of 75 m and found to have dropped to one-fourth of its intensity at the point of origin. If the proper half-life of a pion is 1.77×10^{-8}s, the speed of the pions in the beam, as measured in the laboratory, must be

(a) $0.99c$ (b) $0.98c$ (c) $0.97c$ (d) $0.96c$

41. According to the special theory of relativity, the speed v of a free particle of mass m and total energy E is:

[NET Dec. 2014]

(a) $v = c\sqrt{1-\frac{mc^2}{E}}$ (b) $v = \sqrt{\frac{2E}{m}}\left(1+\frac{mc^2}{E}\right)$ (c) $v = c\sqrt{1-\left(\frac{mc^2}{R}\right)^2}$ (d) $v = c\left(1+\frac{mc^2}{E}\right)$

42. In an inertial farme S, two events A and B take place at $(ct_A = 0, \vec{r}_A = 0)$ and $(ct_B = 0, \vec{r}_B = 2\hat{y})$, respectively. The times at which these events take place in a frame $S¢$ moving with a velocity $0.6\,c\,\hat{y}$ with respect to S are given by **[GATE 2015]**

(a) $ct'_A = 0;\ ct'_B = -3/2$ (b) $ct'_A = 0;\ ct'_B = 0$

(c) $ct'_A = 0;\ ct'_B = 3/2$ (d) $ct'_A = 0;\ ct'_B = 1/2$

43. A particle with rest mass M is at rest and decays into two particles of equal rest masses $\frac{3}{10}M$ which move along the z-axis. Their velocities are given by **[GATE 2015]**

(a) $\vec{v}_1 = \vec{v}_2 = (0.8c)\hat{z}$ (b) $\vec{v}_1 = -\vec{v}_2 = (0.8c)\hat{z}$

(c) $\vec{v}_1 = -\vec{v}_2 = (0.6c)\hat{z}$ (d) $\vec{v}_1 = (0.6c)\hat{z};\ \vec{v}_2 = (-0.8c)\hat{z}$

44. Consider three inertial frames of reference A, B, and C. The frame B moves with a velocity $c/2$ with respect to A and C moves with a velocity $c/10$ with respect to B in the same direction. The velocity of C as measured in A is **[NET June 2015]**

(a) $\frac{3c}{7}$ (b) $\frac{4c}{7}$ (c) $\frac{c}{7}$ (d) $\frac{\sqrt{3}c}{7}$

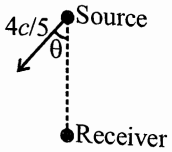
45. In an observer's rest frame, a particle is moving towards the observer with an energy E and momentum p. If c denotes the velocity of light in vacuum, the energy of the particle in another frame moving in the same direction as the particle with a constant velocity v is

(a) $\frac{(E+vp)}{\sqrt{1-(v/c)^2}}$ (b) $\frac{(E-vp)}{\sqrt{1-(v/c)^2}}$ (c) $\frac{(E+vp)}{[1-(v/c)^2]^2}$ (d) $\frac{(E-vp)}{[1-(v/c)^2]^2}$

46. Consider a particle of mass m moving with a speed v. If T_R denotes the relativistic kinetic energy and T_N its non-relativistic approximation, then the value of $(T_R - T_N)/T_R$ for $v = 0.01c$, is **[NET Dec. 2015]**

(a) 1.25×10^{-5} (b) 5.0×10^{-5} (c) 7.5×10^{-5} (d) 1.0×10^{-4}

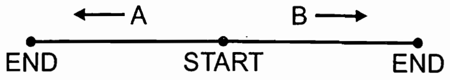
47. A distant source, emitting radiation of frequency ω, moves with a velocity $4c/5$ in a certain direction with respect to a receiver (as shown in the figure). **[NET Dec. 2015]**



The upper cut-off frequency of the receiver is $3\omega/2$. Let θ be the angle as shown. For the receiver to detect the radiation, θ should at least be

(a) $\cos^{-1}\left(\frac{1}{2}\right)$ (b) $\cos^{-1}\left(\frac{3}{4}\right)$ (c) $\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (d) $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

48. In a futuristic scenario, two spaceships, A and B, are running a race, where they start from the same point (marked START) but fly in opposite directions at constant speeds close to the speed of light. An observer fixed at the starting point observes that they both cross the points marked END, which are equidistant from the starting point, at the same time. Afterwards this observer receives messages from both spaceships.



Which of the following could be true? **[TIFR 2016]**

(a) Both A and B agree that A won the race

(b) A and B both claim to have won the race

(c) Both A and B agree that they crossed the end point simultaneously

(d) A thinks B won the race while B thinks A won the race

49. Let (x, t) and (x', t') be the coordinate systems used by the observers O and O', respectively. Observer O' moves with a velocity $\mathrm{v} = \beta c$ along their common positive x-axis. If $x_+ = x + ct$ and $x_- = x - ct$ are the linear combinations of the coordinates, the Lorentz transformation relating O and O' takes the form

(a) $x'_+ = \dfrac{x_- - \beta x_+}{\sqrt{1-\beta^2}}$ and $x'_- = \dfrac{x_+ - \beta x_-}{\sqrt{1-\beta^2}}$,

(b) $x'_+ = \sqrt{\dfrac{1+\beta}{1-\beta}}\, x_+$ and $x'_- = \sqrt{\dfrac{1-\beta}{1+\beta}}\, x_-$ **[NET June 2016]**

(c) $x'_+ = \dfrac{x_+ - \beta x_-}{\sqrt{1-\beta^2}}$ and $x'_- = \dfrac{x_- - \beta x_+}{\sqrt{1-\beta^2}}$,

(d) $x'_+ = \sqrt{\dfrac{1-\beta}{1+\beta}}\, x_+$ and $x'_- = \sqrt{\dfrac{1+\beta}{1-\beta}}\, x_-$

50. For a particle of energy E and momentum $\boldsymbol{p}$ (in a frame F), the rapidity y is defined as $y = \dfrac{1}{2}\ln\left(\dfrac{E + p_3 c}{E - p_3 c}\right)$. In a frame F' moving with velocity $\boldsymbol{v} = (0, 0, \beta c)$ with respect to F, the rapidity y' will be

(a) $y' = y + \dfrac{1}{2}\ln(1-\beta^2)$

(b) $y' = y - \dfrac{1}{2}\ln\left(\dfrac{1+\beta}{1-\beta}\right)$ **[NET June 2016]**

(c) $y' = y + \ln\left(\dfrac{1+\beta}{1-\beta}\right)$

(d) $y' = y + 2\ln\left(\dfrac{1+\beta}{1-\beta}\right)$

51. Light takes approximately 8 minutes to travel from the Sun to the Earth. Suppose in the frame of the Sun an event occurs at $t = 0$ at the Sun and another event occurs on Earth at $t = 1$ minute. The velocity of the inertial frame in which both these events are simultaneous is: **[JEST 2016]**

(a) $c/8$ with the velocity vector pointing from Earth to Sun

(b) $c/8$ with the velocity vector pointing from Sun to Earth

(c) The events can never be simultaneous - no such frame exists

(d) $c\sqrt{1-\left(\frac{1}{8}\right)^2}$ with velocity vector pointing from Sun to Earth

52. The kinetic energy of a particle of rest mass m_0 is equal to its rest mass energy. Its momentum in units of $m_0 c$, where c is the speed of light in vacuum, is ________. (Give your answer upto two decimal places)

[GATE 2016]

53. In an inertial frame of reference S, an observer finds two events occurring at the same time at co-ordinates $x_1 = 0$ and $x_2 = d$. A different inertial frame S' moves with velocity v with respect to S along the positive x-axis. An observer in S' also notices these two events and finds them to occur at times t_1' and t_2' and at positions x_1' and x_2', respectively. If $\Delta t' = t_2' - t_1'$, $\Delta x' = x_2' - x_1'$ and $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, which of the following statements is true?

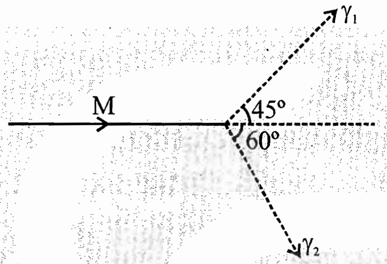
(a) $\Delta t' = 0, \Delta x' = \gamma d$

(b) $\Delta t' = 0, \Delta x' = d/\gamma$ **[GATE 2016]**

(c) $\Delta t' = -\gamma v d/c^2, \Delta x' = \gamma d$

(d) $\Delta t' = -\gamma v d/c^2, \Delta x' = d/\gamma$

54. A particle of rest mass M is moving along the positive x-direction. It decays into two photons γ_1 and γ_2 as shown in the figure. The energy of γ_1 is 1GeV and the energy of γ_2 is 0.82 GeV. The value of M (in units of GeV/c^2) is ______________. (Give your answer upto two decimal places) **[GATE 2016]**



55. The recoil momemtum of an atom is p_A when it emits an infrared photon of wavelength 1500 nm, and it is p_B when it emits a photon of visible wavelength 500 nm. The ratio $\frac{p_A}{p_B}$ is. **[GATE 2014]**

(a) 1 : 1 (b) $1:\sqrt{3}$ (c) 1 : 3 (d) 3 : 2

56. The kinetic energy of a free relativistic particle is defined as $E - m_0c^2$, where E and m_0 are respectively its total energy and rest mass. Let 'v' be the speed of the particle when its kinetic energy is half of its rest mass energy. Then the ratio v/c is

(a) $\frac{\sqrt{5}}{3}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$

57. The kinetic energy of a free relativistic particle is mc^2, where E and m are its total energy and rest mass respectively. Let v0 be the speed at which the kinetic energy equals the rest mass energy of the particle. Then

(a) $v_0 = c/\sqrt{2}$ (b) $v_0 = \sqrt{3}c/2$ (c) $v_0 = c$ (d) $v_0 > c$ (so this can never happen)

58. For the set of all Lorentz transformations with velocities along the x-axis, consider the two statements given below:

P : If L is Lorentz transformation then, L^{-1} is also a Lorentz transformation

Q : If L_1 and L_2 are Lorentz transformation then, L_1L_2 is necessarily a Lorentz transformation

Choose the correct option **[GATE 2010]**

(a) P is true and Q is false (b) both P and Q are true

(c) Both P and Q are false (d) P is false and Q is true

59. A π^0 meson at rest decays into two photons, which move along the x-axis. They are both detected simultaneously after a time, t = 10 s. In an inertial frame moving with a velocity V = 0.6 c in the direction of one of the photons, the time interval between the two detections is **[GATE 2010]**

(a) 15s (b) 0 s (c) 10 s (d) 20 s

60. Consider the decay process $\tau^- \rightarrow \pi^- + \nu_\tau$ in the rest frame of the τ^-. The masses of τ^-, π^- and ν_τ are M_τ, M_π and zero respectively. **[NET June 2011]**

(A) The energy of π^- is:

(a) $\frac{\left(M_\tau^2 - M_\pi^2\right)c^2}{2M_\tau}$ (b) $\frac{\left(M_r^2 + M_\pi^2\right)c^2}{2M_\tau}$ (c) $\left(M_r - M_\pi\right)c^2$ (d) $\sqrt{M_\tau M_\pi}\, c^2$

(B) The velocity is π^- is:

(a) $\frac{\left(M_\tau^2 - M_\pi^2\right)c}{M_\tau^2 + M_\pi^2}$ (b) $\frac{\left(M_\tau^2 - M_\pi^2\right)c}{M_\tau^2 - M_\pi^2}$ (c) $\frac{M_\pi c}{M_\tau}$ (d) $\frac{M_\tau c}{M_\pi}$

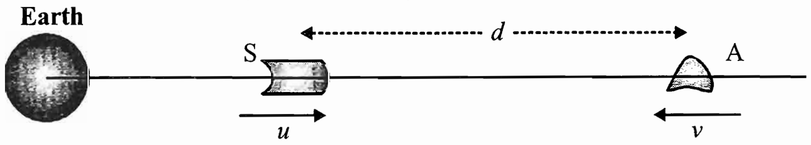
61. The area of a disc in its rest frame S is equal to 1 (in some units). The disc will appear distorted to an observer O moving with a speed u with respect to S along the plane of the disc. The area of the disc measured in the rest frame of the observer O is (c is the speed of light in vacuum) **[NET June 2013]**

(a) $\left(1-\frac{u^2}{c^2}\right)^{1/2}$ (b) $\left(1-\frac{u^2}{c^2}\right)^{-1/2}$ (c) $\left(1-\frac{u^2}{c^2}\right)$ (d) $\left(1-\frac{u^2}{c^2}\right)^{-1}$

62. A rod of length L carries a total charge Q distributed uniformly. If this is observed in a frame moving with a speed v along the rod, the charge per unit length (as measured by the moving observer) is

(a) $\frac{Q}{L}\left(1-\frac{v^2}{c^2}\right)$ (b) $\frac{Q}{L}\sqrt{1-\frac{v^2}{c^2}}$ (c) $\frac{Q}{L\sqrt{1-\frac{v^2}{c^2}}}$ (d) $\frac{Q}{L\left(1-\frac{v^2}{c^2}\right)}$ **[NET June 2015]**

63. A spaceship S blasts off from the Earth. After some time, Earth station informs the crew that they have settled into a constant velocity $0.28c$ radially outward from the Earth, but unfortunately they are on a head-on collision course with an asteroid A at a distance of 15 light-minutes coming in towards the Earth along the same radius (see figure below). **[TIFR 2013]**



Instruments on-board the spaceship immediately estimate the speed of the asteroid to have a constant value $0.24c$. It follows that the maximum time (in minutes) available to the crew to evacuate the ship before the collision is

(a) 60 (b) 30 (c) 29 (d) 63

Answer key (Special Theory of Relativity)

1. (a) 2. (b) 3. (c) 4. (c) 5. (c) 6. (d) 7. (c)

8. (b) 9. (c) 10. (a) 11. (a) 12. (d) 13. (b) 14. (c)

15. (a) 16. (d) 17. (c) 18. (c) 19. (c) 20. (d) 21. (a)
22. (c) 23. (d) 24. (b) 25. (c) 26. (c) 27. (d) 28. (c)

29. (b) 30. (a) 31. (a) 32. (a) 33. (d) 34. (b) 35. (b)

36. (d) 37. (b) 38. (2.1 to 2.3) 39. (a) 40. (a) 41. (c) 42. (a)

43. (b) 44. (b) 45. (b) 46. (c) 47. (b) 48. (b) 49. (d)

50. (b) 51. (b) 52. (1.72 to 1.74) 53. (c) 54. (1.40 to 1.45)

55. ( ) 56. (a) 57. (b) 58. (b) 59. (a) 60_A (b) 60_B(a)

61. (a) 62. (c)

CAREER ENDEAVOUR

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Assignment-5: Classical Mechanics (Central Forces)

1. In Kepler's problem of planetary motion, the orbits depend on the various values of the eccentricity (e) and energy (E). What values of e and E belong to a hyperbolic orbit?

(a) $E > 0, e = 1$ (b) $E < 0, e = 1$ (c) $E > 0, e = 0$ (d) $E > 0, e > 1$

2. The vector force corresponding to a potential energy $V = kr^3$ is given by

(a) $-3k\,r\,\vec{r}$ (b) $-3k\,r^2\,\vec{r}$ (c) $3k\,r^2\,\vec{r}$ (d) $3k\,r\,\vec{r}$

3. A particle of mass m moves in a central potential $V(r) = Kr^3$ in a circle of radius 'a' its kinetic energy is:

(a) $\frac{3Km\,a^3}{2}$ (b) $\frac{Km\,a^3}{2}$ (c) $\frac{3Km\,a^2}{2}$ (d) $\frac{Km\,a^2}{2}$

4. Motion of a particle in spherically symmetric attractive potential is:

(a) Always bounded though not in circular orbits

(b) Always in circular orbits and bounded

(c) May be bounded or unbounded depending upon the energy and angular momentum of the particle.

(d) Not bounded at all.

5. The central forces, which support closed orbits for bound particles, are given by the following, where k>0.

(a) $f(r) = \frac{k}{r^{(3-n)}}$, n = 1 and 4 (b) $f(r) = -\frac{k}{r^{3-n}}$, n = 1 to 4

(c) $f(r) = -\frac{k}{r^{3-n}}$, n = 1 and 4 (d) $f(r) = \frac{k}{r^{1-n}}$, n = 1 and 4

6. Two heavenly bodies of masses 6 and 4 units move under the influence of that mutual central force separating out the centre of mass motion. The resultant one-body motion corresponds to a particle of mass

(a) 10 units (b) 2.4 units (c) 6 units (d) 4 units.

7. Potential energy $V = kr^n$ then the relation between K.E. & P.E. is

(a) $\langle T\rangle = \langle V\rangle$ (b) $\langle T\rangle = \frac{n}{2}\langle V\rangle$ (c) $\langle T\rangle = \frac{3}{2}\langle V\rangle$ (d) $\langle T\rangle = 2\langle V\rangle$

8. If a planet sweeps in equal area in equal time then the following orbit is represented.

(a) Circular orbit (b) Elliptical (c) Hyperbola (d) Parabola.

9. A particle moves under a central force field with the total energy E and the angular momentum $\vec{L}$. For this particle,

(a) E is conserved but $\vec{L}$ is not conserved. (b) E is not conserved but $\vec{L}$ is conserved.

(c) both E and $\vec{L}$ are conserved. (d) neither E nor $\vec{L}$ is conserved.

10. A space station moving in a circular orbit around the Earth goes into a new bound orbit by firing its engine radially outwards. The orbit is:
(a) A large circle (b) A smaller circle (c) An ellipse (d) A parabola

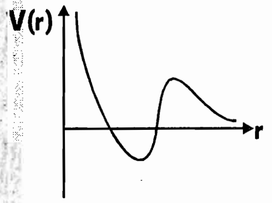
11. A particle is moving in an inverse square force field. If the total energy of the particle is positive, then trajectory of the particle is:
(a) Circular (b) Elliptical (c) Parabolic (d) Hyperbolic

12. A particle is moving in a spherically symmetric potential $V(r) = \alpha r^2$, where α is a positive constant. In a stationary state, the expectation value of the kinetic energy $\langle T\rangle$ of the particle is:
(a) $\langle T\rangle = \langle V\rangle$ (b) $\langle T\rangle = 2\langle V\rangle$ (c) $\langle T\rangle = 3\langle V\rangle$ (d) $\langle T\rangle = 4\langle V\rangle$

13. Consider two particles with position vectors $\bar{r}_1$ and $\bar{r}_2$. The force exerted by particle 2 on particle 1 is, $\bar{f}(\bar{r}_1, \bar{r}_2) = (\dot{r}_2 - \dot{r}_1)(\bar{r}_2 - \bar{r}_1)$. The force is:
(a) Central and conservative (b) Non-central and conservative
(c) Central and non-conservative (d) Non-central and non-conservative.

14. A particle moves in a central force field $\bar{f} = -kr^n \hat{r}$, where k is a constant, r, the distance of the particle from the origin and $\hat{r}$ is the unit vector in the direction of position vector $\bar{r}$. Closed stable orbits are possible for:
(a) $n = 1$ and $n = 2$ (b) $n = 1$ and $n = -1$ (c) $n = 2$ and $n = -2$ (d) $n = 1$ and $n = -2$

15. A particle moves in the central potential V(r) shown in the figure below
Which of the following statements is true?
(a) Both a stable circular orbit and an unstable circular orbit are possible.
(b) Only a stable circular orbit is possible.
(c) Only an unstable circular orbit is possible.
(d) No circular orbit is possible.

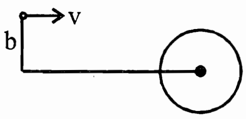


16. A particle of mass m is located at a distance z along the axis of a uniform disk of mass M and radius R. The gravitational force felt by the mass m is given by

(a) $\dfrac{2GMm}{R^2}\left(\dfrac{z}{(z^2+R^2)^{1/2}}-1\right)$ (b) $\dfrac{2GMm}{R^2}\left(\dfrac{z}{(z^2-R^2)^{1/2}}-1\right)$

(c) $\dfrac{2GMm}{R^2}\left(\dfrac{z}{(z^2+R^2)^{1/2}}+1\right)$ (d) $\dfrac{2GMm}{R^2}\left(\dfrac{z}{(z^2+R^2)^{1/2}}\right)$

17. A small asteroid is approaching a massive star with a speed v, from a very large distance, at an impact parameter b as shown below.



If the mass of the star is M and its radius is R, then what is the minimum value of b such that the asteroid will miss the star?

(a) $R\sqrt{1+2GM/v^2R}$ (b) $R/\sqrt{1-2GM/v^2R}$
(c) R (d) R/2

18. Two gravitating bodies A and B with masses m_A and m_B, respectively, are moving in circular orbit. Assume that $m_B >> m_A$ and let the radius of the orbit of body A be R_A. If the body A is losing mass adiabatically, its orbital radius R_A is proportional to **[NET June 2011]**

(a) $1/m_A$ (b) $1/m_A^2$ (c) m_A (d) m_A^2

19. Consider a spherical planet, rotating about an axis passing through its centre. The velocity of a point on its equator is v_{eq}. If the acceleration due to gravity 'g' measured at the equator is half value of 'g' measured at one of the poles, then the escape velocity for a particle shot upwards from that pole will be

(a) $v_{eq}/2$ (b) $v_{eq}/\sqrt{2}$ (c) $\sqrt{2}\,v_{eq}$ (d) $2v_{eq}$

20. Two particles of identical mass move in circular orbits under a central potential $V(r)=\frac{1}{2}kr^2$. Let ℓ_1 and ℓ_2 be the angular momenta and r_1, r_2 be the radii of the orbits respectively. If $\ell_1/\ell_2 = 2$, the value of r_1/r_2 is:

(a) $\sqrt{2}$ (b) $1/\sqrt{2}$ (c) 2 (d) $1/2$ **[NET Dec. 2011]**

21. Consider a particle with a finite angular momentum l moving in a central field defined by a linear restoring force. The minimum allowed energy E_{min} scales as a function of l as

(a) $E_{min} \propto \frac{1}{\ell}$ (b) $E_{min} \propto \sqrt{\ell}$ (c) $E_{min} \propto \ell$ (d) $E_{min} \propto \frac{1}{\sqrt{\ell}}$

22. With what angular velocity ω should a spherical planet of mass M and radius R rotate so that the effective gravitational acceleration on its surface at the equator is zero?

(a) $\omega=\sqrt{\frac{GM}{R^3}}$ (b) $\omega=\frac{GM}{R^3}$ (c) $\omega=\frac{\sqrt{GM}}{R}$ (d) $\omega=\frac{\sqrt{GM}}{R^2}$

23. In a central force potential $V(r)=-k/r$, the equation for the orbit of a particle of mass m and angular momentum l is given by $r(\theta)=\frac{\ell^2/mk}{1+\sqrt{1+\frac{2E\ell^2}{mk^2}}\cos\theta}$. If R is the radius of a circular orbit, the distance 'd' at minimum approach for parabolic orbit is

(a) $d=2R$ (b) $d=\frac{R}{2}$ (c) $d=\frac{R}{\sqrt{2}}$ (d) $d=\frac{R}{1+\sqrt{2}}$

24. An object moving in an unknown potential filed $V(\vec{r})$ executes a closed orbit confined to a plane. It sweeps out the equal areas in equal time intervals. From this, one can conclude that

(a) $V(\vec{r})$ is a central potential (b) The orbit must be elliptical

(c) $V(\vec{r})\alpha\frac{1}{|\vec{r}|}$ (d) $V(\vec{r})=V(\theta,\varphi)$

25. For a signal particle under a central force which varies as r^2, the average kinetic energy T and average potential energy V obey the relation

(a) T=V (b) $T=\frac{3}{2}V$ (c) $T=\frac{1}{2}V$ (d) T=2V

26. In a central force field the trajectory of a particle of mass m and angular momentum L in plane polar coordinates is given by $\frac{1}{r}=\frac{m}{L^2}(1+\varepsilon\cos\theta)$ **[GATE 2012]**

Where, ε is the eccentricity of the particle's motion. Which one of the following choices for ε gives rise to a parabolic trajectory?

(a) $\varepsilon=0$ (b) $\varepsilon=1$ (c) $0<\varepsilon<1$ (d) $\varepsilon>1$

27. A Planet orbits a massive star in a highly elliptical orbit. i.e., the total orbital energy E is close to zero. The initial distance of closest approach is R_0. energy is dissipated through tidal motions until the orbit is circularized with a final radius of R_f. Assume that orbital angular momentum is conserved during the circularization process. Then

(a) $R_f=R_0/2$ (b) $R_f=R_0$ (c) $R_f=\sqrt{2}R_0$ (d) $R_f=2R_0$

28. A binary system consists of two stars of equal mass m orbiting each in a circular orbit under the influence of gravitational forces. The period of the orbit is τ. At t = 0, the motion is stopped and the stars are allowed to fall towards each other. After what time t, expressed in terms of τ do they collide?

The following integral may be useful $\left(x=r^{1/2}\right)$, $\int\frac{x^2dx}{\sqrt{a-x^2}}=-\frac{x}{2}\sqrt{a-x^2}+\frac{a}{2}\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)$

(a) $\sqrt{2}\tau$ (b) $\frac{\tau}{\sqrt{2}}$ (c) $\frac{\tau}{2\sqrt{2}}$ (d) $\frac{\tau}{4\sqrt{2}}$

29. A binary star system consists of two stars S_1 and S_2, with masses m and 2m respectively separated by a distannce 'r'. If both S_1 and S_2 individually follow circular orbits around the centre of the mass with intantaneous speeds v_1 and v_2 respectively, the ratio of speeds v_1/v_2 is: **[NET Dec. 2012]**

(a) $\sqrt{2}$ (b) 1 (c) 1/2 (d) 2

30. A planet of mass 'm' moves in the gravitational field of the Sun (mass M). If the semi-major and semi-minor axes of the orbit are 'a' and 'b' respectively, the angular momentum of the planet is: **[NET Dec. 2012]**

(a) $\sqrt{\frac{m^2Gb^2}{a}}$ (b) $\sqrt{2GMm^2(a-b)}$ (c) $\sqrt{\frac{2GMm^2ab}{(a-b)}}$ (d) $\sqrt{\frac{2GMm^2ab}{(a+b)}}$

31. A planet of mass m moves in the inverse square central force field of the Sun of mass M. If the semi-major and semi-minor axes of the orbit are a and b, respectively, the total energy of the planet is: **[NET Dec. 2011]**

(a) $-\frac{GMm}{2a}$ (b) $-GMm\left(\frac{1}{a}+\frac{1}{b}\right)$ (c) $-\frac{GMm}{a}\left(\frac{1}{b}-\frac{1}{a}\right)$ (d) $-GMm\frac{(a-b)}{(a+b)^2}$

32. A planet of mass m and an angular momentum L moves in a circular orbit in a potential, V(r) = –k/r, where k is a constant. If it is slightly perturbed radially, the angular frequency of radial oscillations is

(a) $mk^2/\sqrt{2}L^3$ (b) mk^2/L^3 (c) $\sqrt{2}mk^2/L^3$ (d) $\sqrt{3}mk^2/L^3$ **[NET June 2013]**

33. If, in a Kepler potential, the pericentre distance of a particle in a parabolic orbit is r_p while the radius of the circular orbit with the same angular momentum is r_c, then

(a) $r_c=2r_p$ (b) $r_c=r_p$ (c) $2r_c=r_p$ (d) $r_c=\sqrt{2}r_p$

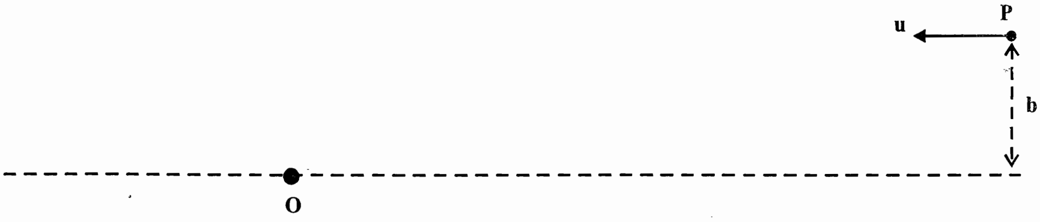
34. A planet of mass 'm' moves in a circular orbit of radius r_0 in the gravitational potential $V(r) = -\frac{k}{r}$ where 'k' is a positive constant. The orbital angular momentum of the planet is **[GATE 2014]**

(a) $2r_0 km$ (b) $\sqrt{2r_0 km}$ (c) $r_0 km$ (d) $\sqrt{r_0 km}$

35. A particle P of mass m moves under the influence of a central potential, centred at the origin O, of the form

$$V(r) = -\frac{k}{3r^3}$$ **[TIFR 2014]]**

where 'k' is a positive constant.



If the particle P comes in from infinity with initial velocity 'u' and impact parameter b(see figure), then the largest value of b for which the particle gets captured by the potential is

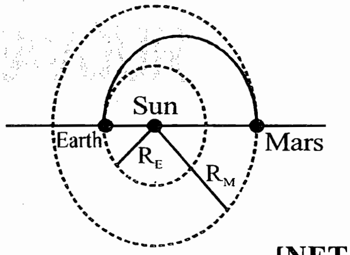
(a) $\left(\frac{3k^2}{m^2u^4}\right)^{1/6}$ (b) $\left(\frac{k}{3mu}\right)^{1/3}$ (c) $\left(\frac{2k^2}{m^2u^4}\right)^{1/6}$ (d) $\left(\frac{2k}{3mu}\right)^{1/3}$

36. A particle moves under the influence of a central potential in an orbit $r = k\theta^4$, where k is a constant and r is the distance from the origin. It follows that the angle θ varies with time t as

(a) $\theta \propto t^{1/9}$ (b) $\theta \propto t^{1/8}$ (c) $\theta \propto t^{1/7}$ (d) $\theta \propto t^{1/6}$

37. The probe *Mangalyaan* was sent recently to explore the planet Mars. The inter-planetary parts of the trajectory is approximately a half-ellipse with the Earth (at the time of launch), Sun and Mars (at the time the probe reaches the destination) forming the major axis. Assuming that the orbits of Earth and Mars are approximately circular with radii R_E and R_M, respectively, the velocity (with respect to the Sun) of the probe during its voyage when it is at a distance $r(R_E \ll r \ll R_M)$ from the Sun, neglecting the effect of Earth and Mars, is

(a) $\sqrt{2GM\frac{(R_E+R_M)}{r(R_E+R_M-r)}}$ (b) $\sqrt{2GM\frac{(R_E+R_M-r)}{r(R_E+R_M)}}$



(c) $\sqrt{2GM\frac{R_E}{rR_M}}$ (d) $\sqrt{\frac{2GM}{r}}$ **[NET Dec. 2014]**

38. A classical particle with total energy E moves under the influence of a potential $V(x,y) = 3x^3 + 2x^2y + 2xy^2 + y^3$. The average potential energy, calculated over a long time is equal to, **[JEST 2015]**

(a) $\frac{2E}{3}$ (b) $\frac{E}{3}$ (c) $\frac{E}{5}$ (d) $\frac{2E}{5}$

39. How is your weight affected if the Earth suddenly doubles in radius, mass remaining the same? **[JEST 2015]**

(a) Increases by a factor of 4 (b) Increases by a factor of 2
(c) Decreases by a factor of 4 (d) Decreases by a factor of 2

. A particle moves in three-dimensional space in a central potential $V(r) = kr^4$, where k is a constant. The angular frequency ω for a circular orbit depends on its radius R as **[NET Dec. 2015]**

(a) $\omega \propto R$ (b) $\omega \propto R^{-1}$

(c) $\omega \propto R^{1/4}$ (d) $\omega \propto R^{-2/3}$

. The central force which results in the orbit $r = a(1+\cos\theta)$ for a particle is proportional to : **[JEST 2016]**

(a) r (b) r^2 (c) r^{-2} (d) none of these

2. A particle moving under the influence of a central force $\vec{F}(\vec{r}) = -k\vec{r}$ (where $\vec{r}$ is the position vector of the particle and k is a positive constant) has non-zero angular momentum. Which of the following curves is a possible orbit for this particle? **[GATE 2016]**

(a) A straight line segment passing through the origin (b) An ellipse with its center at the origin

(c) An ellipse with one of the foci at the origin (d) A parabola with its vertex at the origin

3. A satellite is moving in a circular orbit around the Earth. If T, V and E are its average kinetic, average potential and total energies, respectively, then which one of the following options is correct ? **[GATE 2015]**

(a) $V = -2T; E = -T$ (b) $V = -T; E = 0$

(c) $V = \dfrac{-T}{2}; E = \dfrac{T}{2}$ (d) $V = \dfrac{-3T}{2}; E = \dfrac{-T}{2}$

4. A small meteor approaches the Earth. When it is at a large distance, it have velocity v_∞ and impact parameter b. If R_e is the radius of the Earth and v_0 is the escape velocity, the condition for the meteor to strike the Earth is **[TIFR 2010]**

(a) $b < R_e\sqrt{1-\left(\dfrac{v_0}{v_\infty}\right)^2}$ (b) $b > R_e\sqrt{1+\left(\dfrac{v_0}{v_\infty}\right)^2}$

(c) $b < R_e\sqrt{1+\left(\dfrac{v_0}{v_\infty}\right)^2}$ (d) $b = R_e\left(\dfrac{v_0}{v_\infty}\right)$

5. Two planets A and B move around the Sun in elliptic orbits with time periods T_A and T_B respectively. If the eccentricity of the orbit of B is ε and its distance of closest approach to the Sun is R, then the maximum possible distance between the planets is **[TIFR 2013]**

[Eccentricity of an ellipse: $\varepsilon = \dfrac{r_{\max} - r_{\min}}{r_{\max} + r_{\min}}$]

(a) $\dfrac{1+\varepsilon^2}{1-\varepsilon^2}\left(1+\dfrac{T_A^{3/2}}{T_B^{3/2}}\right)R$ (b) $\dfrac{1+\varepsilon}{1-\varepsilon}\left(1+\dfrac{T_A^{2/3}}{T_B^{2/3}}\right)R$

(c) $\sqrt{\dfrac{1+\varepsilon}{1-\varepsilon}\left(1+\dfrac{T_A^3}{T_B^3}\right)}R$ (d) $\sqrt{\dfrac{1+\varepsilon^2}{1-\varepsilon^2}}\left(1+\dfrac{T_A^{2/3}}{T_B^{2/3}}\right)R$

16. The maximum and minimum speeds of a comet that orbits the Sun are 80 and 10 km/s respectively. The ratio of the aphelion distance of the comet to the radius of the Earth's orbit is ____________. (Assume that Earth moves in a circular orbit of radius 1.5×10^8 km with a speed of 30 km/s.)

47. Consider a particle of mass m following a trajectory given by $x = x_0 \cos \omega_1 t$ and $y = y_0 \sin \omega_2 t$, where x_0, y_0, ω_1 and ω_2 are constants of appropriate dimensions. The force on the particle is

(a) central only if $\omega_1 = \omega_2$

(b) central only if $x_0 = y_0$ and $\omega_1 = \omega_2$

(c) always central

(d) central only if $x_0 = y_0$ and $\omega_1 \neq \omega_2$

Answer Key: Classical Mechanics (Central Forces)

1. (d)	2. (a)	3. (a)	4. (c)	5. (a)	6. (b)	7. (b)
8. (b)	9. (c)	10. (c)	11. (d)	12. (a)	13. (d)	14. (d)
15. (a)	16. (a)	17. (a)	18. (b)	19. (d)	20. (a)	21. (c)
22. (a)	23. (b)	24. (a)	25. (b)	26. (b)	27. (d)	28. (d)
29. (d)	30. (a)	31. (a)	32. (b)	33. (a)	34. (d)	35. (a)
36. (a)	37. (b)	38. (d)	39. (c)	40. (a)	41. (d)	42. (b)
43. (a)	44. ( )	45. (c)	46. (2)	47. (a)		

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Assignment-6: Classical Mechanics (Small Oscillations)

1. The total energy E of a particle of mass m executing small oscillations about the origin along on the x-direction is given by, $E = \frac{1}{2}mv^2 + V_0 \cosh\left(\frac{x}{L}\right)$, where V_0 and L are positive constants. The time period T of oscillation is

(a) $T = \frac{1}{2\pi}\sqrt{\frac{m}{V_0}}$ (b) $T = 2\pi\sqrt{\frac{L}{m}}$ (c) $T = \pi L\sqrt{\frac{m}{E}}$ (d) $T = 2\pi\sqrt{\frac{mL^2}{V_0}}$

2. A particle moves in one dimension in a potential $V(x) = x^2/2 + x^3/3$, in suitable units. If E is the total energy of the particle, then the motion is:

(a) Always bounded if $0 \le E \le \frac{1}{6}$ (b) Always unbounded if $0 \le E \le \frac{1}{6}$

(c) Always bounded if $E < 0$

(d) Bounded if $0 \le E \le \frac{1}{6}$ and the initial position satisfies $-1 < x(0) < \frac{1}{2}$

3. Two simple pendula each of length L and mass m are suspended from a horizontal support. If they are coupled by a massless spring of spring constnat k, then the higher frequency of oscillation of the system is:

(a) $\omega = \sqrt{\frac{g}{2L} + \frac{2k}{m}}$ (b) $\omega = \sqrt{\frac{2g}{L} + \frac{2k}{m}}$ (c) $\omega = \sqrt{\frac{g}{L} + \frac{k}{m}}$ (d) $\omega = \sqrt{\frac{g}{L} + \frac{2k}{m}}$

4. In the free vibration of a linear triatomic molecule, the central atom takes part in the oscillatory motion for
(a) All three modes (b) Two modes
(c) One mode only (d) Does not take part at all

5. In a normal mode of vibration of a molecule, all atoms move
(a) With the same frequency (b) With the same amplitude
(c) In the same direction (d) Opposite to each other.

6. A column of mercury of total length L partially fills a U tube. If the mercury is set into small oscillation, its frequency is given by

(a) $2\pi\sqrt{\frac{2g}{L}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2g}{L}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{g}{L}}$ (d) $2\pi\sqrt{\frac{g}{L}}$

7. The equation of motion of a simple pendulum is
(a) Linear for small and large amplitude oscillations.
(b) Nonlinear for small and large amplitude oscillations.
(c) Linear for small amplitude and nonlinear for large amplitude oscillations.
(d) Non linear for small amplitude and linear for large amplitude oscillations.

A linear symmetric triatomic molecule consisting of masses 1, 2 and 1 units connected by springs of unit force constant admits normal modes of following nonzero normal frequencies:

(a) 1, 1 (b) 1, 2 (c) $1, \sqrt{2}$ (d) $1, \sqrt{3}$

For a system of 12 degrees of freedom, the number of nonzero frequency normal modes are

(a) 12 (b) 10 (c) 9 (d) 6

Consider two simple pendulums each consisting of a point mass m attached to a string of length ℓ. The masses are coupled by a spring of spring constant k. The kinetic energy T and the potential energy V of the system are given by $T = m\ell^2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $V = m\ell^2\begin{pmatrix} \frac{g}{\ell}+\frac{k}{m} & -\frac{k}{m} \\ -\frac{k}{m} & \frac{g}{\ell}+\frac{k}{m} \end{pmatrix}$

The frequencies of small oscillation of this system are

(a) $\omega_1 = \sqrt{g/\ell}, \omega_2 = \sqrt{g/\ell + 2k/m}$ (b) $\omega_1 = \sqrt{2k/m}, \omega_2 = \sqrt{g/\ell + 2k/m}$

(c) $\omega_1 = \sqrt{g/\ell}, \omega_2 = \sqrt{k/m}$ (d) $\omega_1 = \sqrt{g/\ell}, \omega_2 = \sqrt{g/\ell + k/m}$

Statement for Linked Answer Q.11 and Q.12:

The Lagrangian of two coupled oscillators of mass m each is:

$$L = \frac{1}{2}m\left(\dot{x}_1^2 + \dot{x}_2^2\right) - \frac{1}{2}m\omega_0^2\left(x_1^2 + x_2^2\right) + m\omega_0^2\mu x_1 x_2$$

1. The equations of motion are:

(a) $\ddot{x}_1 + \omega_0^2 x_1 = \omega_0^2\mu x_1, \ddot{x}_2 + \omega_0^2 x_2 = \omega_0^2\mu x_2$ (b) $\ddot{x}_1 + \omega_0^2 x_1 = \omega_0^2\mu x_2, \ddot{x}_2 + \omega_0^2 x_2 = \omega_0^2\mu x_1$

(c) $\ddot{x}_1 + \omega_0^2 x_1 = \omega_0^2\mu x_1, \ddot{x}_2 + \omega_0^2 x_2 = \omega_0^2\mu x_2$ (d) $\ddot{x}_1 + \omega_0^2 x_1 = \omega_0^2\mu x_1, \ddot{x}_2 + \omega_0^2 x_2 = \omega_0^2\mu x_1$

2. The normal modes of the system are:

(a) $\omega_0\sqrt{\mu^2 - 1}, \omega_0\sqrt{\mu^2 + 1}$ (b) $\omega_0\sqrt{1 - \mu^2}, \omega_0\sqrt{1 + \mu^2}$

(c) $\omega_0\sqrt{\mu - 1}, \omega_0\sqrt{\mu + 1}$ (d) $\omega_0\sqrt{1 - \mu}, \omega_0\sqrt{1 + \mu}$

13. A particle of mass m is moving in a potential of the form $V(x, y, z) = (1/2)\, m\omega^2\,(3x^2 + 3y^2 + 2z^2 + 2xy)$. The oscillation frequencies of the three normal modes of the particle are given by:

(a) $\omega, \sqrt{3}\omega$ and $\sqrt{3}\omega$ (b) $\sqrt{2}\omega, \sqrt{3}\omega$ and $\sqrt{3}\omega$ (c) $\sqrt{2}\omega, \sqrt{2}\omega$ and 2ω (d) $\sqrt{2}\omega, 2\omega$ and 2ω

14. The Lagrangian for a three particle system is given by $L = \frac{1}{2}\left(\dot{\eta}_1^2 + \dot{\eta}_2^2 + \dot{\eta}_3^2\right) - a^2\left(\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_1\eta_3\right)$,

where a is real. Then one of the normal cooridnates has a frequency ω given by:

(a) $\omega^2 = a^2$ (b) $\omega^2 = \frac{a^2}{2}$ (c) $\omega^2 = 2a^2$ (d) $\omega^2 = \sqrt{2}a^2$

15. The motion of a simple pendulum undergoing large oscillations can be described as:

(a) harmonic, non-conservative (b) harmonic, consrevative

(c) anharmonic, non-conservative (d) anharmonic, conservative

16. A loop of radius R is pivoted at a point on the circumference. The period of small oscillations in the plane of the hoop is

(a) $2\pi\sqrt{\frac{2R}{g}}$ (b) $2\pi\sqrt{\frac{R}{4g}}$ (c) $2\pi\sqrt{\frac{R}{g}}$ (d) $2\pi\sqrt{\frac{9R}{7g}}$

17. A simple pendulum is in the form of a unit mass at the end of a massless rigid rod of unit length. If θ is the angle with the downward vertical, the kinetic energy T of the pendulum is given by

(a) $T = 2g\sin^2 \theta/2$ (b) $T = 2g\cos^2 \theta/2$ (c) $T = g\sin^2 \theta$ (d) $T = g\cos^2 \theta$

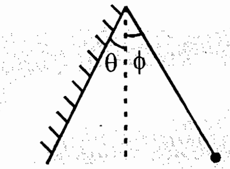
18. A particle moves in one dimension in a potential $V(x) = x^2/2 + x^3/3$, in suitable units. If E is the total energy of the particle, then the motion is

(a) Always bounded if $0 \le E \le \frac{1}{6}$ (b) Always unbounded if $0 \le E \le \frac{1}{6}$

(c) Always bounded if $E < 0$

(d) Bounded if $0 \le E \le \frac{1}{6}$ and the initial position satisfies $-1 < x(0) < \frac{1}{2}$

19. A simple pendulum of length l is suspended from a hook mounted on a slanted wall. The wall makes a small angle θ with the vertical. The pendulum is displaced from the vertical by a small angle $\phi(\phi > \theta)$ and released. Assuming that the collision of the bob is elastic, the time period of oscillation is:



(a) $2\pi\sqrt{\frac{\ell}{g}}$ (b) $2\sqrt{\frac{\ell}{g}}\left[\pi/2 + \sin^{-1}\frac{\theta}{\phi}\right]$ (c) $2\pi\sqrt{\frac{\ell}{g}\frac{\theta}{\phi}}$ (d) $2\pi\sqrt{\frac{\ell}{g}}\left[\pi/2 + \cos^{-1}\frac{\theta}{\phi}\right]$

20. A point particle of mass m carrying an electric charge q is attached to a spring of stiffness constant k. A constant electric field E along the direction of the spring is switched on for a time interval T (where $T \ll \sqrt{m/k}$). Neglecting radiation loss, the amplitude of oscillation after the field is switched off is:

(a) qE/k (b) qET^2/m (c) $qE\sqrt{m}/Tk^{3/2}$ (d) $qET/\sqrt{mk}$ **[NET]**

21. Consider a simple pendulum executing small oscillations (in vacuum) with a time period T_0. If a charge +q is pasted onto the pendulum bob, then the resultant (modified) pendulum

(a) continues to execute non- sinusoidal small oscillations with the same time period T_0.

(b) executes damped oscillation (c) executes sinusoidal oscillatoions with $T > T_0$.

(d) executes sinusoidal oscillations with $T < T_0$.

22 Two object each of mass m are connected by a spring of spring constant k and are capable of moving in a line. The normal modes have the frequencies.

(a) $\sqrt{\frac{2k}{m}}$ and 0 (b) $\sqrt{\frac{k}{m}}$ and 0 (c) $\sqrt{\frac{2k}{m}}$ and $\sqrt{\frac{k}{m}}$ (d) $\sqrt{\frac{k}{2m}}$ and 0

Consider a mass m attached to a spring. When the spring is extended from its equilibrium position, the restoring force is given by $F=-k_1x-k_2x^3$. Where k_1 and k_2 are positive constants. The spring is now extended slightly and let go. Which of the following is true?

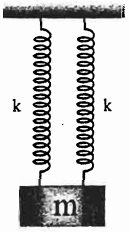
(a) The mass executes sinusoidal oscillations

(b) The mass executes periodic, but non sinusoidal oscillations

(c) The mass executes a damped oscillatory motion

(d) the total energy of the system decreases with time.

Consider a bar of mass m attached to two identical massless springs (of spring constant k) in parallel, as shown in the figure, The bar would oscillate with a frequency.



(a) $\sqrt{k/m}$ (b) $\sqrt{\frac{2k}{m}}$ (c) $\sqrt{\frac{k}{2m}}$ (d) $2\sqrt{k/m}$

A particle of unit mass moves along the x-axis under the influence of a potential, $V(x)=x(x-2)^2$. The particle is found to be in stable equilibrium at the point $x=2$. The time period of oscillation of the particle is

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π **[GATE 2012]**

For small angular displacements (i.e, $\sin\theta\approx\theta$) a simple pendulum oscillates harmonically. For larger displacements. the motion

(a) becomes aperiodic (b) remains periodic with the same period

(c) remains periodic with a higher period (d) remains period with a lower period.

m=1, go this [illegible]

7. Consider the motion of a classical particle in a one dimensional double-well potential $V(x)=\frac{1}{4}(x^2-2)^2$. If the particle is displaced infinitesimally from the minimum the positive x-axis (and friction is neglected), then **[NET June 2012]**

(a) the particle will execute simple harmonic motion in the right well with an angular frequency $\omega=\sqrt{2}$

(b) the particle will execute simple harmonic motion in the right well with an angular frequency $\omega=2$

(c) the particle will switch between the right and left wells

(d) the particle will approach the bottom of the right well and settle there

8. A solid cylinder of height H, radius R and density ρ, floats vertically on the surface of a liquid of density ρ_0. The cylinder will be set into oscillatory motion when a small instantaneous downward force is applied. The frequency of oscillation is

(a) $\frac{\rho g}{\rho_0 H}$ (b) $\frac{\rho}{\rho_0}\sqrt{\frac{g}{H}}$ (c) $\sqrt{\frac{\rho g}{\rho_0 H}}$ (d) $\sqrt{\frac{\rho_0 g}{\rho H}}$

29. Consider small oscillations of a triatomic molecule. Which of the following is true?

(a) There cannot be any mode of zero frequency.

(b) There may not be any mode of zero frequency.

(c) There must be mode of zero frequency

(d) There must be a mode of imaginary frequency.

30. A particle of mass m is moving in the potential $V(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4$ where a, b are positive constants. The frequency of small oscillations about a point of stable equilibrium is **[NET Dec. 2014]**

(a) $\sqrt{a/m}$ (b) $\sqrt{2a/m}$ (c) $\sqrt{3a/m}$ (d) $\sqrt{6a/m}$

31. A particle of mass m moves in the one-dimensional potential $V(x) = \frac{\alpha}{3}x^3 + \frac{\beta}{4}x^4$ where $\alpha, \beta > 0$. One of the equilibrium points is $x = 0$. The angular frequency of small oscillations about the other equilibrium point is **[NET June 2015]**

(a) $\frac{2\alpha}{\sqrt{3m\beta}}$ (b) $\frac{\alpha}{\sqrt{m\beta}}$ (c) $\frac{\alpha}{\sqrt{12m\beta}}$ (d) $\frac{\alpha}{\sqrt{24m\beta}}$

32. A particle of mass 'm' is in a potential given by $V(r) = -\frac{a}{r} + \frac{ar_0^2}{3r^3}$ where 'a' and 'r_0' are positive constants. When distrubed slightly from its stable equilibrium position, it undergoes a simple harmonic oscillation. The time period of oscillation is **[GATE 2014]**

(a) $2\pi\sqrt{\frac{mr_0^3}{2a}}$ (b) $2\pi\sqrt{\frac{mr_0^3}{a}}$ (c) $2\pi\sqrt{\frac{2mr_0^3}{a}}$ (d) $4\pi\sqrt{\frac{mr_0^3}{a}}$

33. The Lagrangian of a system is given by $L = \frac{1}{2}m\dot{q}_1^2 + 2m\dot{q}_2^2 - k\left(\frac{5}{4}q_1^2 + 2q_2^2 - 2q_1q_2\right)$ where m and k are positive constants. The frequencies of its normal modes are **[NET Dec. 2015]**

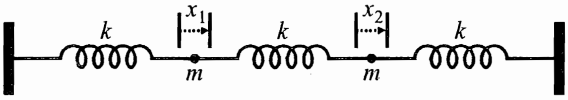
(a) $\sqrt{\frac{k}{2m}}, \sqrt{\frac{3k}{m}}$

(b) $\sqrt{\frac{k}{2m}(13 \pm \sqrt{73})}$

(c) $\sqrt{\frac{5k}{2m}}, \sqrt{\frac{k}{m}}$

(d) $\sqrt{\frac{k}{2m}}, \sqrt{\frac{6k}{m}}$

34. For the coupled system shown in the figure, the normal coordinates are $x_1 + x_2$ and $x_1 - x_2$, corresponding to the normal frequencies ω_0 and $\sqrt{3}\omega_0$, respectively. **[JEST 2016]**



At $t = 0$, the displacements are $x_1 = A, x_2 = 0$, and the velocities are $v_1 = v_2 = 0$. The displacement of the second particle at time t is given by:

(a) $x_2(t) = \frac{A}{2}\left(\cos(\omega_0 t) + \cos\left(\sqrt{3}\omega_0 t\right)\right)$

(b) $x_2(t) = \frac{A}{2}\left(\cos(\omega_0 t) - \cos\left(\sqrt{3}\omega_0 t\right)\right)$

(c) $x_2(t) = \frac{A}{2}\left(\sin(\omega_0 t) - \sin\left(\sqrt{3}\omega_0 t\right)\right)$

(d) $x_2(t) = \frac{A}{2}\left(\sin(\omega_0 t) - \frac{1}{\sqrt{3}}\sin\left(\sqrt{3}\omega_0 t\right)\right)$

A particle of mass m moves in one dimension under the influence of a potential energy **[TIFR 2013]**

$$V(x)=-a\left(\frac{x}{\ell}\right)^2+b\left(\frac{x}{\ell}\right)^4$$

where a and b are positive constants and ℓ is a characteristic length. The frequency of small oscillations about a point of stable equilibrium is

(a) $\frac{1}{2\pi\ell}\sqrt{\frac{b}{m}}$ (b) $\frac{1}{\pi\ell}\sqrt{\frac{a}{m}}$ (c) $\frac{1}{\pi\ell}\sqrt{\frac{a^2}{mb}}$ (d) $\frac{2b}{\pi\ell}\sqrt{\frac{1}{ma}}$

A particle of unit mass moves in a potential $V(x)=ax^2+\frac{b}{x^2}$, where a and b are positive constants. The angular frequency of small oscillations about the minimum of the potential is: **[NET June 2011]**

(a) $\sqrt{8b}$ (b) $\sqrt{8a}$ (c) $\sqrt{8a/b}$ (d) $\sqrt{8b/a}$

The Lagrangian of a diatomic molecule is given by $L=\frac{m}{2}\left(\dot{x}_1^2+\dot{x}_2^2\right)-\frac{k}{2}x_1x_2$, where m is the mass of each of the atoms and x_1 and x_2 are the displacements of atoms measured from the equilibrium position and $k>0$. The norrmal frequencies are **[GATE 2009]**

(a) $\left(\frac{k}{m}\right)^{1/2}$ (b) $\left(\frac{k}{m}\right)^{1/4}$ (c) $\left(\frac{k}{2m}\right)^{1/4}$ (d) $\left(\frac{k}{2m}\right)^{1/2}$

A particle is placed in a region with the potential $V(x)=\frac{1}{2}kx^2-\frac{\lambda}{3}x^3$, where $k,\lambda>0$ then,

(a) $x=0$ and $x=\frac{k}{\lambda}$ are points of stable equilibrium **[GATE 2010]**

(b) $x=0$ is a point of stable equilbrium and $x=\frac{k}{\lambda}$ is a point of unstable equilibrium

(c) $x=0$ and $x=\frac{k}{\lambda}$ are points of unstable equilibrium

(d) there are no points of stable or unstable equilibrium

Answer Key: Classical Mechanics (Small Oscillation)

1. (d)	2. (b)	3. (d)	4. (c)	5. (b)	6. (b)	7. (a)
8. (c)	9. (d)	10. (a)	11. (b)	12. (d)	13. (c)	14. (d)
15. (d)	16. (a)	17. (a)	18. (d)	19. (b)	20. (d)	21. (a)
22. (a)	23. (b)	24. (b)	25. (b)	26. (b)	27. (b)	28. (d)
29. (c)	30. (b)	31. (b)	32. (a)	33. (a)	34. (b)	35. (b)
36. (b)	37. (d)	38. (b)				

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Assignment-7: Classical Mechanics (Rigid Body)

An asymmetric rigid body has three distinct principal moments of inertia, with $I_x < I_y < I_z$. If we consider rotation with uniform angular velocity about the x, y and z axes, respectively, then the motion is:
(a) Stable about the y-axis, but not about the x and z axes.
(b) Stable about the x and y axes, but not about the z-axis.
(c) Stable about the y and z axes, but not about the x-axis.
(d) Stable about the x and z axes, but not about the y-axis.

A particle having angular momentum $\vec{L}$ is placed in a rotating frame having angular velocity $\vec{\omega}$. It will have lowest energy when
(a) L is parallel to $\vec{\omega}$ (b) L is anti-parallel to $\vec{\omega}$
(c) L and $\vec{\omega}$ are perpendicular to each other. (d) L oscillates from parallel to anti parallel.

An N-particle system is being acted upon by external forces. The applied torque is zero but the total external force is not. Let $\overline{P}$ be the total linear momentum and $\overline{L}$ be the total angular momentum. Then which of the following statement is true?
(a) There is no conserved quantity (b) $\overline{p}$ is conserved.
(c) $\overline{L}$ is conserved. (d) Both $\overline{P}$ and $\overline{L}$ are conserved.

A particle of mass m is free to slide along a thin rod. The rod rotates with a constant angular velocity ω about a fixed axis passing through one end of the rod. The angle between the rod and the axis of rotation remains fixed. The number of generalized coordinates required to describe the motion is
(a) 1 (b) 2 (c) 3 (d) 4

For torque free motion of a symmetric body of moment of inertia tensor $\overleftrightarrow{I}$ and angular velocity $\vec{\omega}$ can be described by the Euler's equations given in

(a) $I_{12}\dot{\omega}_1 = (I_{11} - I_{33})\omega_1\omega_3$
$I_{13}\dot{\omega}_2 = (I_{33} - I_{22})\omega_3\omega_2$
$I_{23}\dot{\omega}_3 = (I_{22} - I_{11})\omega_1\omega_2$

(b) $I_{11}\dot{\omega}_1 = (I_{12} - I_{13})\omega_2\omega_3$
$I_{22}\dot{\omega}_2 = (I_{32} - I_{21})\omega_3\omega_1$
$I_{33}\dot{\omega}_3 = (I_{31} - I_{23})\omega_1\omega_3$

(c) $I_{11}\dot{\omega}_1 = (I_{22} - I_{33})\omega_2\omega_3$
$I_{22}\dot{\omega}_2 = (I_{33} - I_{11})\omega_3\omega_1$
$I_{33}\dot{\omega}_3 = (I_{11} - I_{22})\omega_1\omega_2$

(d) $I_{11}\dot{\omega}_1 = (I_{12} - I_{33})\omega_2\omega_3$
$I_{22}\dot{\omega}_2 = (I_{23} - I_{11})\omega_3\omega_1$
$I_{33}\dot{\omega}_3 = (I_{13} - I_{22})\omega_1\omega_2$

A system of four particles is in x-y plane. Of these, two particles each of mass m are located at (1, 1) and (1, –1). The remaining two particles each of mass 2m are located at (1, 1) and (–1, –1). The xy-component of the moment of inertia tensor of this system of particles is:
(a) 10 m (b) –10 m (c) 2m (d) –2m

7. The moment of inertia tensor of a rigid body is given by $I = \begin{pmatrix} 8 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 8 \end{pmatrix}$. The magnitude of the moment of iner

about an axis $\hat{n} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$ is :

(a) 6 (b) 5 (c) 2 (d) 8/3

8. A rigid body is rotating about its centre of mass, fixed at the origin, with an angular velocity $\vec{\omega}$ and angul acceleration $\vec{\alpha}$. If the torque acting on it is $\vec{\tau}$ and its angular momentum is $\vec{L}$, the rate of change of its kinet energy is

(a) $\frac{1}{2}\vec{\tau}.\vec{\omega}$ (b) $\frac{1}{2}\vec{L}.\vec{\omega}$ (c) $\frac{1}{2}\left(\vec{\tau}.\vec{\omega} + \vec{L}.\vec{\alpha}\right)$ (d) $\frac{1}{2}\vec{L}.\vec{\alpha}$

9. The moment of inertia of a thin rectangular plate of length *a*, width *b*, mass *m* about an axis passing through th centre and perpendicular to the plate is

(a) $\frac{1}{12}m\left(a^2+b^2\right)$ (b) $\frac{1}{6}m\left(a^2+b^2\right)$ (c) $\frac{1}{12}mab$ (d) $\frac{1}{6}mab$

10. In the absence of an applied torque a rigid body with three distinct principal moments of inertia given b I_1, I_2 and I_3 is rotating freely about a fixed point inside the body. The Euler equations for the components of i angular velocity $(\omega_1, \omega_2, \omega_3)$ are $\dot{\omega}_1 = \frac{I_2 - I_3}{I_1}\omega_2\,\omega_3,\ \dot{\omega}_2 = \frac{I_3 - I_1}{I_2}\omega_1\,\omega_3,\ \dot{\omega}_3 = \frac{I_1 - I_2}{I_3}\omega_1\,\omega_2$

A. The equilibrium points in $(\omega_1, \omega_2, \omega_3)$ space are

(a) $(1,-1,0),(-1,0,1)$ and $(0,-1,1)$ (b) $(1,1,0),(1,0,1)$ and $(0,1,1)$

(c) $(1,0,0),(0,1,0)$ and $(0,0,1)$ (d) $(1,1,1),(-1,-1,-1)$ and $(0,0,0)$

B. The constants of motion are

(a) $\omega_1^2 + \omega_2^2 + \omega_3^2$ and $I_1\omega_1 + I_2\omega_2 + I_3\omega_3$ (b) $I_1\omega_1^2 + I_2w_2^2 + I_3\omega_3^2$ and $I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2$

(c) $I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2$ and $\omega_1 + \omega_2 + \omega_3$ (d) $\omega_1^2 + \omega_2^2 + \omega_3^2$ and $I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2$

11. An annulus of mass M made of a material of uniform density has inner and outer radii 'a' and 'b' respectiv Its principal momnet of inertia along the axis of symmetry perpendicular to the plane of the annulus is :

(a) $\frac{1}{2}M\frac{\left(b^4+a^4\right)}{\left(b^2-a^2\right)}$ (b) $\frac{1}{2}M\pi\left(b^2-a^2\right)$ (c) $\frac{1}{2}M\left(b^2-a^2\right)$ (d) $\frac{1}{2}M\left(b^2+a^2\right)$

12. The principal moments of inertia of a rigid body of an arbitrary shape are given by, I_1, I_2 and I_3. which o following represents a valid set of values for I_i (in some arbitrary units)?

(a) $I_1 = -3, I_2 = 4, I_3 = 1$ (b) $I_1 = 3,\ I_2 = 3,\ I_3 = 8$

(c) $I_1 = 3,\ I_2 = 7,\ I_3 = 4$ (d) $I_1 = 0,\ I_2 = 0,\ I_3 = 1$

Two uniform thin rods of equal length L, and masses M_1 and M_2 are joined together along the length. The moment of inertia of the combined rod of length 2L about an axis passing through the mid-point perpendicular to the length of the rod is,

(a) $(M_1 + M_2)\frac{L^2}{12}$ (b) $(M_1 + M_2)\frac{L^2}{6}$ (c) $(M_1 + M_2)\frac{L^2}{3}$ (d) $(M_1 + M_2)\frac{L^2}{2}$

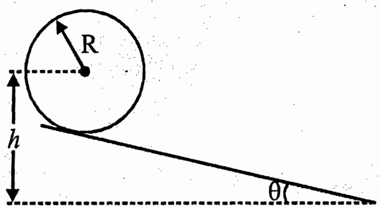
Two bodies of equal mass 'm' are connected by a massless rigid rod of length 'ℓ' lying in the xy-plane with the centre of the rod at the origin. If this system is rotating about the z-axis with a frequency ω, its angular momentum is

(a) $m\ell^2\omega/4$ (b) $m\ell^2\omega/2$ (c) $m\ell^2\omega$ (d) $2m\ell^2\omega$

A uniform circular disk of radius R and mass M is rotating with angular speed ω about an axis, passing through its center and inclined at an angle 60 degrees with respect to its symmetry axis. The magnitude of the angular momentum of the disk is, **[GATE]**

(a) $\frac{\sqrt{3}}{4}\omega MR^2$ (b) $\frac{\sqrt{3}}{8}\omega MR^2$ (c) $\frac{\sqrt{7}}{8}\omega MR^2$ (d) $\frac{\sqrt{7}}{4}\omega MR^2$

A ring of mass m and radius R rolls (without slipping) down an inclined plane starting from rest. If the centre of the ring is initially at a height h, the angular velocity when the ring reaches the base is



(a) $\sqrt{g/(h-R)}\tan\theta$ (b) $\sqrt{g/(h-R)}$ **[NET]**

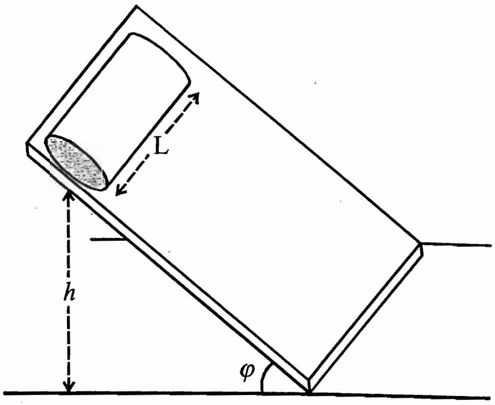
(c) $\sqrt{g(h-R)/R^2}$ (d) $\sqrt{2g/(h-R)}$

7. A pendulum consists of a ring of mass M and radius R suspended by a massless rigid rod of length l attached to its rim. When the pendulum oscillates in the plane of the ring, the time period of oscillation is

(a) $2\pi\sqrt{\frac{l+R}{g}}$ (b) $\frac{2\pi}{\sqrt{g}}(l^2+R^2)^{1/4}$ **[NET]**

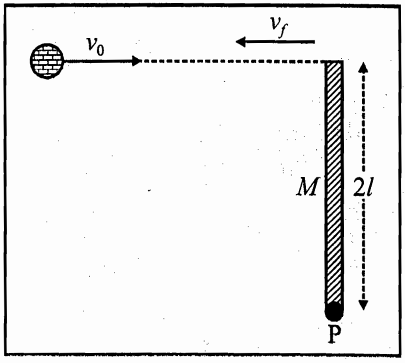
(c) $2\pi\sqrt{\frac{2R^2+2Rl+l^2}{g(R+l)}}$ (d) $\frac{2\pi}{\sqrt{g}}(2R^2+2Rl+l^2)^{1/4}$

8. Two cylinders A and B of the same length L and outer radius R were placed at the same height h on an inclined plane at an angle φ with the horizontal (see figure). Starting from rest, each cylinder was allowed to roll down the plane without slipping. It was found that A reached the end of the inclined plane earlier than B. Which of the following possibilities could be true ? **[TIFR]**



(a) A is hollow and made of copper; B is hollow and made of copper; B is heavier than A.
(b) A is solid and made of copper; B is solid and made of aluminium.
(c) A is hollow and made of aluminium; B is solid and made of aluminium.
(d) A is solid and made of copper; B is hollow and made of copper.

19. A thin uniform rod of length $2l$ and mass M is pivoted at one end P on a horizontal plane (see figure). A ball of mass $m \ll M$ and speed v_0 strikes the free end of the rod perpendicularly and bounces back with velocity v_f along the original line of motion as shown in the figure. If the collision is perfectly elastic the magnitude of v_f is



[TIFR]

(a) $\dfrac{M-4m}{M+4m}v_0$ (b) $\dfrac{M+4m}{M-4m}v_0$ (c) $\dfrac{M-3m}{M+3m}v_0$ (d) $\dfrac{M+3m}{M-3m}v_0$

20. Two masses, m each, are placed at the points $(x, y) = (a, a)$ and $(-a, -a)$, and two masses, $2m$ each, are placed at the points $(a, -a)$ and $(-a, a)$. The principal moments of inertia of the system are

(a) $2ma^2, 4ma^2$ (b) $4ma^2, 8ma^2$ **[NET Dec. 2015]**

(c) $4ma^2, 4ma^2$ (d) $8ma^2, 8ma^2$

21. A cylindrical shell of mass m has an outer radius b and an inner radius a. The moment of inertia of the shell about the axis of the cylinder is: **[JEST 2016]**

(a) $\frac{1}{2}m(b^2-a^2)$ (b) $\frac{1}{2}m(b^2+a^2)$ (c) $m(b^2+a^2)$ (d) $m(b^2-a^2)$

22. Consider a solid cylinder of uniform mass density and a very thin cylindrical shell of uniform mass per unit area. They have the same radius, length and total mass. If they are both rolling on a plane without slipping with the same angular velocity, the ratio of the total kinetic energy of the solid cylinder to that of the cylindrical shell is

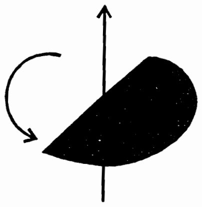
(a) 3 : 4 (b) 1 : 2 (c) 1 : 1 (d) 2 : 1 (e) 2 : 3

The principal axes of the ellipse given by the equation $9x^2 + 6y^2 - 4xy = 4$ are along the directions of the vectors.

(a) $2\hat{i} + 3\hat{j}$ and $3\hat{i} - 2\hat{j}$ (b) $2\hat{i} + \hat{j}$ and $\hat{i} - 2\hat{j}$

(c) $3\hat{i} + 2\hat{j}$ and $2\hat{i} - \hat{j}$ (d) $\hat{i} + 2\hat{j}$ and $2\hat{i} - \hat{j}$ (e) $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$

. A half-disc of mass M and radius R rotates about an axis that passes through the centre of the straight side and is perpendicular to its plane (see figure). The moment of inertia about the axis of rotation is:



(a) $MR^2/8$ (b) $MR^2/4$ (c) $MR^2/2$ (d) $2MR^2$

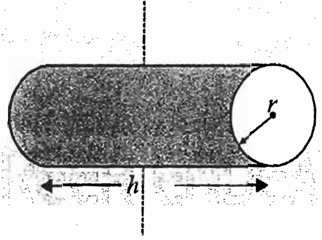
5. Three identical rings of radius 'a' are placed on the xy-plane so that each ring touches the other two tangentially. Each ring, which may be considered to be infinitesimally thin, has a mass M. The centres of the three rings are equidistant from the z-axis— this common distance being $2a/\sqrt{3}$. What is the moment of inertia of this system about the z-axis?

(a) $3Ma^2/4$ (b) $3Ma^2/2$ (c) $6Ma^2$ (d) $7Ma^2$

6. A solid sphere of mass 'm' and radius 'a' is rolling with a linear speed 'v' on a flat surface without slipping. The magnitude of the angular momentum of the sphere with respect to a point along the path of the sphere on the surface is:

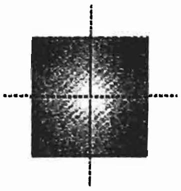
(a) $\frac{2}{m} m\,av$ (b) $\frac{7}{5} m\,av$ (c) $m\,av$ (d) $\frac{3}{2} m\,av$

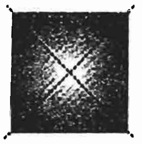
27. Moment of inertia of a solid cylinder of mass m, height 'h' and radius 'r' about an axis (shown in the figure by dashed line) passing through its centre of mass and perpendicular to its symmetry axis is:

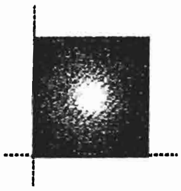


(a) $\frac{1}{4}mr^2 + \frac{1}{12}mh^2$ (b) $\frac{1}{2}mr^2 + \frac{1}{8}mh^2$ (c) $\frac{1}{2}mr^2 + \frac{1}{12}mh^2$ (c) $\frac{1}{2}mr^2 + \frac{1}{4}mh^2$

28. The figure shows a thin square sheet of metal of uniform density along with possible choices for a set of principal axes (indicated by dashed lines) of the moment of inertia, lying in the plane of the sheet. The correct choice(s) for the principal axes would be


p


q


r

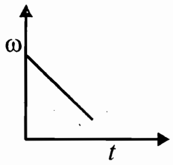
(a) p, q and r (b) p and r (c) p and q (d) p only

29. The moment of inertia of a disc about one of its diameters is I_M. The mass per unit area of the disc is proportional to the distance from its centre. If the radius of the disc is R and its mass is M, the value of I_M is

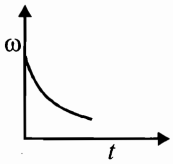
(a) $\frac{1}{2}MR^2$ (b) $\frac{2}{5}MR^2$ (c) $\frac{3}{10}MR^2$ (d) $\frac{3}{5}MR^2$

30. A spherical ball of ice has radius R_0 and is rotating with an angular speed ω about an axis passing through its centre. At time $t = 0$, it starts acquiring mass because the moisture (at rest) around it starts to freeze on it uniformly. As a result its radius increases as $R(t) = R_0 + \alpha t$, where α is a constant. The curve which best describes its angular speed with time is

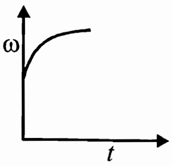
(a)



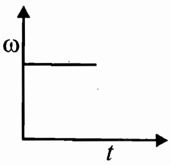
(b)



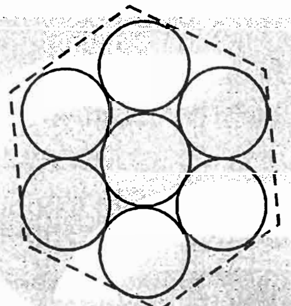
(c)



(d)

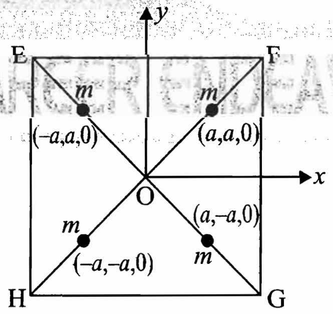


31. Seven uniform disks, each of mass m and radius r, are inscribed inside a regular hexagon, as shown. The moment of inertia of this system of seven disks, about an axis passing through the central disk and perpendicular to the plane of the disks, is



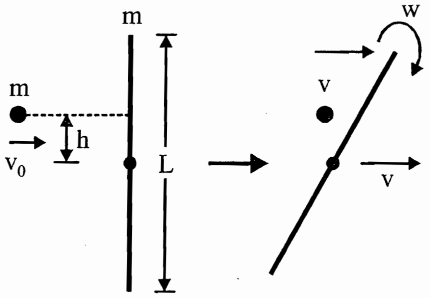
(a) $\frac{7}{2}mr^2$ (b) $7mr^2$ (c) $\frac{13}{2}mr^2$ (d) $\frac{55}{2}mr^2$

32. EFGH is a thin square plate of uniform density σ and side 4a. Four point masses, each of mass m, are placed on the plate as shown in the figure. In the moment of inertia matrix 'I' of the composite system,



(a) Only I_{xy} is zero
(b) Only I_{xz} and I_{yz} are zero.
(c) All the product of inertia terms are zero
(d) None

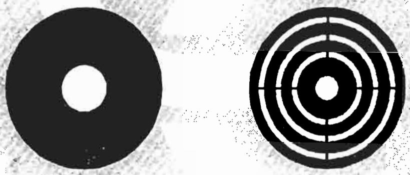
A mass m travels in a straight line with velocity v_0 perpendicular to a uniform stick of mass m and length L, which is initially at rest. The distance from the centre of the stick to the path of the travelling mass is h(see figure). Now the travelling mass m collides elastically with the stick, and the centre of the stick and the mass 'm' are observed to move with equal speed v after the collision. Assuming that the travelling mass m can be treated as a point mass, and the moment of inertia of the stick about its center is $I = \frac{mL^2}{12}$, it follows that the distance h must be **[TIFR 2010]**



(a) $\frac{L}{2}$ (b) $\frac{L}{4}$ (c) $\frac{L}{\sqrt{6}}$ (d) $\frac{L}{\sqrt{3}}$

(e) $\frac{L}{3}$ (f) zero

4. A scientist is given two heavy spheres made of the same metal, which have the same diameter and weight, and is asked to distinguish the spheres, without damaging them in any way. Though the spheres look identical, one of them is actually a hollow spherical shell, while the other is a set of concentric shells mounted on four thin rods of the same metal (see figure). **[TIFR 2011]**



To make this distinction, the scientist must perform an experiment where each sphere is

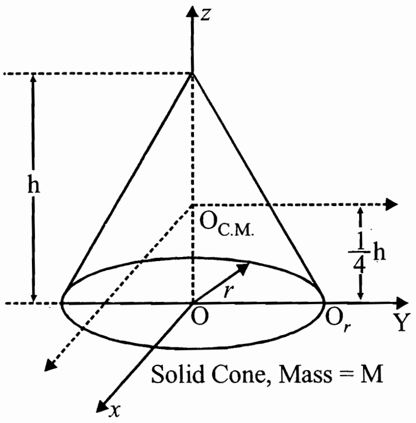
(a) set rotating under the action of a constant torque

(b) made into the bob of a long simple pendulum and set oscillating

(c) immersed fully in a non-corrosive liquid and then weighed

(d) given the same electric charge Q and the potential is measured

35. Consider the uniform solid right cone depicted in the figure on the right. This cone has mass M and a circular base of radius r. If the moment of inertia of the cone about an axis parallel to the X axis passing through the centre of mass $O_{C.M.}$ (see figure) is given by $\frac{3}{80}M(4r^2+h^2)$ then the moment of inertia about another axis parallel to the X axis, but passing through the point O_r (see figure), is **[TIFR 2013]**



(a) $\frac{3}{80}M(4r^2+h^2)$ (b) $\frac{3}{40}M(2r^2+h^2)$

(c) $\frac{1}{20}M(23r^2+2h^2)$ (d) $\frac{1}{30}M(15r^2+4h^2)$

36. A uniform cylinder of radius r and length ℓ, and a uniform sphere of radius R are released on an inclined plane when their centres of mass are at the same height. If they roll down without slipping, and if the sphere reaches the bottom of the plane with a speed V, then the speed of the cylinder when it reaches the bottom is:

(a) $V\sqrt{\frac{14r\ell}{15R^2}}$ (b) $4V\sqrt{\frac{r}{15R}}$ (c) $\frac{4V}{\sqrt{15}}$ (d) $V\sqrt{\frac{14}{15}}$

Answer Key: Classical Mechanics (Rigid Body)

1. (d)	**2. (b)**	**3. (d)**	**4. (a)**	**5. (c)**	**6. (c)**	**7. (b)**
8. (c)	**9. (a)**	**10_A(c)**	**10_B(d)**	**11. (d)**	**12. (c)**	**13. (c)**
14. (b)	**15. (c)**	**16. (c)**	**17. (c)**	**18. (d)**	**19. (c)**	**20. (b)**
21. (b)	**22. (a)**	**23. (d)**	**24. (c)**	**25. (d)**	**26. (b)**	**27. (a)**
28. (c)	**29. (c)**	**30. (b)**	**31. (d)**	**32. (c)**	**33. (c)**	**34. ( )**
35. (c)	**36. (d)**					

ELECTROMAGNETIC THEORY

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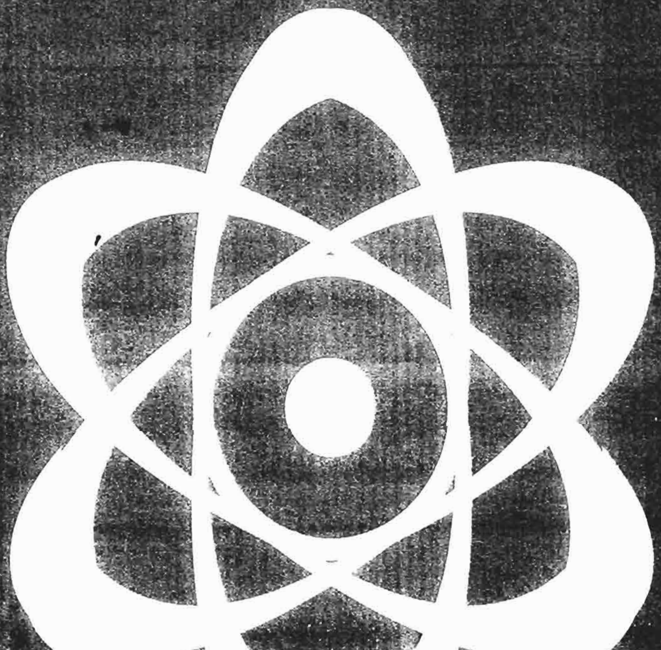

Previous Years' Questions

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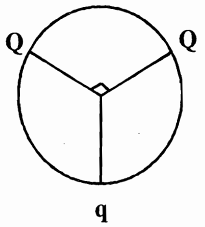
ELECTROMAGNETIC THEORY

Assignment-1 : COULOMB'S & GAUSS LAW

1. Three charges are located on the circumference of a circle of radius 'R' as shown in the figure below. The two charges Q subtend an angle 90° at the centre of the circle .The charge 'q' is symmetrically placed with respect to the charges Q. If the electric field at the centre of the circle is zero, what is the magnitude of Q?



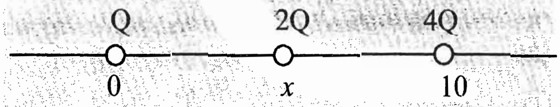
[NET Dec. 2012]

(a) $q/\sqrt{2}$ (b) $\sqrt{2}q$ (c) 2q (d) 4q

2. Two points charges $+q_1$ and $+q_2$ are fixed with a finite a distance 'd' between them. It is desired to put a third charge q_3 in between these two charges on the line joining them so that the charge q_3 is in equilibrium. That is:

(a) Possible only if q_3 is positive (b) Possible only if q_3 is negative
(c) Possible irrespective of the sign of q_3. (d) Not possible at all.

3. Three positively charged particles lie on a straight line at positions 0, x and 10 as indicated in the figure below. Their charges are Q, 2Q and 4Q cm respectively.



If the charges at x = 0 and x = 10 are fixed and the charge at x is movable, the system will be in equilibvrium when x =

(a) 8 (b) 2 (c) 20/3 (d) 10/3

4. Two point charges Q_1 = 1 nC and Q_2 = 2 nC are kept in free space such that the distance between them is 0.1 m. **[GATE 2001]**

(a) The force on Q_2 is along the direction from Q_2 to Q_1
(b) The force on Q_2 is the same in magnitude as that on Q_1
(c) The force on Q_1 is attractive
(d) A point charge Q_3 = –3 nC, placed at the midpoint between Q_1 and Q_2, experiences no net force.

5. In a region of electrostatic field which of the following statements is true for a charged particle :

(a) There exists only one stable equilibrium point
(b) Many stable equilibrium points may exist.
(c) No stable equilirium point exists.
(d) Number of stable equilibrium points depends on how the field is produced

6. Consider two point charges q and λq located at the points, $x = a$ and $x = \mu a$, respectively. Assuming that the sum of the two charges is constant, what is the value of λ for which the magnitude of the electrostatic force is maximum? **[JEST 2015]**

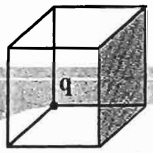
(a) μ (b) 1 (c) $\frac{1}{\mu}$ (d) $1+\mu$

7. When a particle of charge q is at rest, it produces
(a) Only electric field
(b) Only magnetic field
(c) Both electric and magnetic fields.
(d) A magnetic field depending upon the sign of the charge.

8. A charge distribution has the charge density given by $\rho = Q\{\delta(x-x_0)-\delta(x+x_0)\}$. For this charge distribution the electric field at $(2x_0, 0, 0)$ **[GATE 2013]**

(a) $\frac{2Q\hat{x}}{9\pi \in_0 x_0^2}$ (b) $\frac{Q\hat{x}}{4\pi \in_0 x_0^3}$ (c) $\frac{Q\hat{x}}{4\pi \in_0 x_0^2}$ (d) $\frac{Q\hat{x}}{16\pi \in_0 x_0^2}$

9. A point charge 'q' sits at a corner of a cube of side 'a', as shown in the figure on the right. The flux of the electric field vector through the shaded side is **[TIFR 2013]**



(a) $\frac{q}{8\varepsilon_0}$ (b) $\frac{q}{16\varepsilon_0}$ (c) $\frac{q}{24\varepsilon_0}$ (d) $\frac{q}{6\varepsilon_0}$

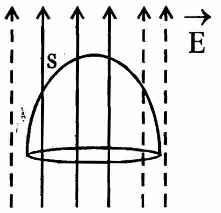
10. The magnitude of the electrostatic field of an infinite straight wire carrying a line charge density l at a distance Z from it is given by

(a) $E = \frac{1}{4\pi\varepsilon_0}.\frac{\lambda}{Z^2}$ (b) $E = \frac{1}{4\pi\varepsilon_0}.\frac{2\lambda}{Z^2}$ (c) $E = \frac{1}{4\pi\varepsilon_0}.\frac{2\lambda}{Z}$ (d) $E = \frac{1}{4\pi\varepsilon_0}.\frac{\lambda}{Z}$

11. A static charge distribution gives rise to an electric field of the form $\vec{E} = \alpha\left(1-e^{-r/R}\right)\frac{\hat{r}}{r^2}$, where α and R are positive constants. The charge contained within a sphere of radius R, centred at the origin is:

(a) $\pi\alpha\varepsilon_0\frac{e}{R^2}$ (b) $\pi\alpha\varepsilon_0\frac{e^2}{R^2}$ (c) $4\pi\alpha\varepsilon_0\frac{R^2}{e}$ (d) $\pi\alpha\varepsilon_0\frac{R^2}{e}$ **[NET Dec. 2011]**

12. A hemispherical surface of radius R is in a constant electric field $\vec{E}$ directed perpendicular to the base, as shown. The flux through the curved surface S of the hemisphere is



(a) 0 (b) $\pi R^2 E$ (c) $2\pi R^2 E$ (d) $4\pi R^2 E$

13. An electric field $\vec{E}(\vec{r}) = \frac{1}{r}\left(\alpha\hat{r} + \beta \sin\theta \cos\phi\hat{\phi}\right)$ exists in space. What will be the total charge enclosed in a sphere of unit radius centered at the origin?

(a) $4\pi\varepsilon_0 \alpha$ (b) $4\pi\varepsilon_0(\alpha+\beta)$ (c) $4\pi\varepsilon_0(\alpha-\beta)$ (d) $4\pi\varepsilon_0\beta$

14. Consider a hollow charged shell of inner radius 'a' and outer radius 'b'. The volume charge density is $\rho(r) = \frac{k}{r^2}$ (where k is a constant) in the region $a < r < b$. The magnitude of the electric field produced at distance $r > a$ is: **[NET Dec. 2012]**

(a) $\frac{k(b-a)}{\varepsilon_0 r^2}$ for $r > a$

(b) $\frac{k(b-a)}{\varepsilon_0 r^2}$ for $a < r < b$ and $\frac{kb}{\varepsilon_0 r^2}$ for $r > b$

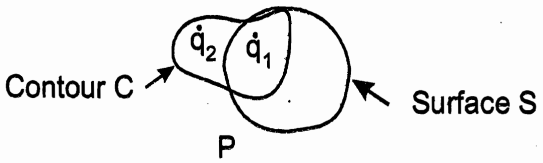
(c) $\frac{k(r-a)}{\varepsilon_0 r^2}$ for $a < r < b$ and $\frac{k(b-a)}{\varepsilon_0 r^2}$ for $r > b$

(d) $\frac{k(r-a)}{\varepsilon_0 a^2}$ for $a < r < b$ and $\frac{k(b-a)}{\varepsilon_0 a^2}$ for $r > b$

15. A solid sphere of radius R has a charge density, given by $\rho(r) = \rho_0\left(1 - \frac{ar}{R}\right)$, where r is the radial coordinate and ρ_0, a and R are positive constants. If the magnitude of the electric field at $r = R/2$ is 1.25 times that at $r = R$, then the value of a is **[NET June 2014]**

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

16. Consider a set of two stationary point charges q_1 and q_2 as shown in the figure. Which of the following statements is correct? **[GATE 2002]**



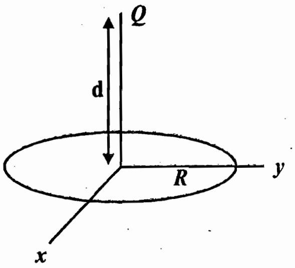
(a) The electric field at P is independent of q_2

(b) The electric flux crossing the closed surface S is independent of q_2

(c) The line integral of the electric field $\vec{E}$ over the closed contour C depends on q_1 and q_2.

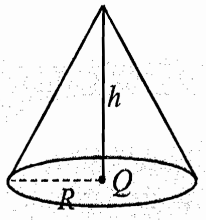
(d) $\vec{\nabla}.\vec{E} = 0$ everywhere

17. The flux of electric field through a circle of radius R placed in the x-y plane with its centre at the origin due to a point charge Q placed at (0, 0, d) is



(a) $\frac{Q}{2\varepsilon_0}\left[1-\frac{d}{\left(d^2+R^2\right)^{1/2}}\right]$ (b) $\frac{Q}{2\varepsilon_0}\frac{d^3}{\left(d^2+R^2\right)^{3/2}}$ (c) $\frac{Q}{4\pi\varepsilon_0}\frac{d}{R}$ (d) $\frac{Q}{4\varepsilon_0}\frac{R^2}{d^2}$

18. Consider a charge Q at the origin of 3-dimensional coordinate system. The flux of the electric field through the curved surface of a cone that has a height 'h' and a circular base of radius R (as shown in the figure) is



(a) $\frac{Q}{\epsilon_0}$ (b) $\frac{Q}{2\epsilon_0}$ (c) $\frac{hQ}{R\epsilon_0}$ (d) $\frac{QR}{2h\epsilon_0}$

19. A solid sphere of radius R carries a uniform volume charge density ρ. The magnitude of electric field inside the sphere at a distance r from the centre is : **[GATE 2005]**

(a) $\frac{r\rho}{3\varepsilon_0}$ (b) $\frac{R\rho}{3\varepsilon_0}$ (c) $\frac{R^2\rho}{r\varepsilon_0}$ (d) $\frac{R^3\rho}{r^2\varepsilon_0}$

20. A shpere of radius R carrries a volume charge density $\rho=\rho_0 r$, where ρ_0 is a constant and r is the radial distance from the centre of the charged sphere. The intensity of the electric field is proportional to:

(a) r^2 for $r \le R$ and $1/r^2$ for $r \ge R$ (b) r for $r \le R$ and $1/r^2$ for $r \ge R$

(c) $1/r^2$ for both $r \le R$ and $r \ge R$ (d) $1/r$ for $r \le R$ and $1/r^2$ for $r \ge R$

21. The electric field due to a charge q is given by $\vec{E}=\frac{q}{r^2}\hat{r}$. The value of the surface integral $\oint_s \vec{E}.\vec{ds}$ depends only

(a) On the area of the surface (b) On the radial distance r.
(c) On the charge (d) On the shape of the surface

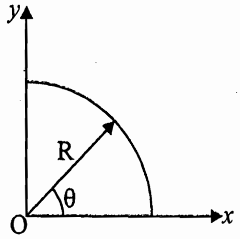
22. The potential due to an electric dipole p placed at the origin is known to be of the form $\phi(r)=p.r/r^3$. The total flux of the electric field through a spherical surface of radius R with the dipole at the centre is given by

(a) Zero (b) $-\frac{2\pi}{3}\frac{|p|}{R}$ (c) $+\frac{2\pi}{3}\frac{|p|}{R}$ (d) $+\frac{4\pi}{3}\frac{|p|}{R}$

23. If $\vec{D}=\left(2y^2+z\right)\hat{i}+4xy\hat{j}+x\hat{k}$, units of C/m^2, the total charge enclosed by the cube $0\le x<1, 0\le y<1,\ 0\le z\le 1$

(a) 2C (b) 4C (c) 0 C (d) 1C

24. A segment of a circular wire of radius R, extending from $\theta=0$ to $\pi/2$, carries a constant linear charge density, λ. The electric field at origin 'O' is:



(a) $\frac{\lambda}{4\pi\varepsilon_0 R}(-\hat{x}-\hat{y})$ (b) $\frac{\lambda}{4\pi\varepsilon_0 R}\left(-\frac{1}{\sqrt{2}}\hat{x}-\frac{1}{\sqrt{2}}\hat{y}\right)$ (c) $\frac{\lambda}{4\pi\varepsilon_0 R}\left(-\frac{1}{2}\hat{x}-\frac{1}{2}\hat{y}\right)$ (d) 0

25. The electric fields of a long straight wire varies with distance r (perpendicular to the wire) as

(a) $\frac{1}{r}$ (b) ln r (c) $1/r^2$ (d) $1/r^3$

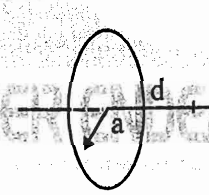
26. The concentric thin spherical shells of radii a, b and c (a < b < c) carry uniform surface electric charge of densities $\sigma, -\sigma$ and σ, respectively. The electric potential at the surface of the outermost shell is

(a) $\frac{\sigma}{\varepsilon_0}(c-b+a)$ (b) $\frac{\sigma}{\varepsilon_0}\left(\frac{c^2}{a}-\frac{b^2}{a}+a\right)$ (c) $\frac{\sigma}{\varepsilon_0}\left(\frac{c^2}{b}-b\frac{a^2}{b}\right)$ (d) $\frac{\sigma}{\varepsilon_0}\left(c-\frac{b^2}{c}+\frac{a^2}{c}\right)$

27. If $\vec{V}=\hat{i}+\hat{j}+\hat{k}$, where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors in three-dimensions, the value of $\vec{\nabla}.\vec{V}$ is :

(a) -3 (b) 2 (c) 0 (d) 3

28. A very circular wire of radius a carries electric charge of uniform liner density ρ. On its axis, the magnitude of the electric field attains its maximum value at a perpendicular distance d form the plane of the wire (see the figure below). The value of d is



(a) 0 (b) 0.32 a (c) 0.50a (d) 0.71a

29. Consider the plane defined by $(x, y, z=10\text{ m})$. If it carries a charge of 20 nC/m^2, what is the electric field intensity at the origin?

(a) $-720\pi\hat{k}\frac{V}{m}$ (b) $-360\pi\hat{k}\frac{V}{m}$ (c) $-72\pi\hat{k}\frac{V}{m}$ (d) $-18\pi\hat{k}\frac{V}{m}$

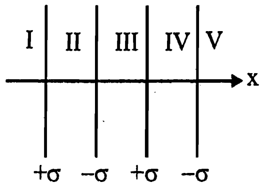
30. A circular loop of radius *R*, carries a uniform line charge density λ. The electric field, calculated at a distance *z* directly above the center of the loop, is maximum if *z* is equal to, **[JEST 2015]**

(a) $\frac{R}{\sqrt{3}}$ (b) $\frac{R}{\sqrt{2}}$ (c) $\frac{R}{2}$ (d) $2R$

31. Consider a uniformly charged sphere of radius, R. The charge per unit volume is ρ. The magnitude of the electric field at a distance of r from the centre of the sphere (r < R), is given by: (ε_0 is the permittivity of free space)

(a) $E = \frac{\rho r}{3\varepsilon_0}$ (b) $E = \frac{\rho r^2}{3\varepsilon_0}$ (c) $E = \frac{\rho R^3}{4\pi\varepsilon_0 r^2}$ (d) $E = \frac{\rho r}{4\pi\varepsilon_0}$

32. Four infinite conducting plates carry equal but opposite uniform charge densities $+\sigma$ as shown. The electrostatic fields in the regions I, II, III, IV and V are given respectively by



(a) $0, \frac{\sigma}{\varepsilon_0}\hat{x}, 0, \frac{\sigma}{\varepsilon_0}\hat{x}, 0$ (b) $\frac{\sigma}{\varepsilon_0}\hat{x}, 0, \frac{\sigma}{\varepsilon_0}\hat{x}, 0, \frac{\sigma}{\varepsilon_0}\hat{x}$

(c) $0, \frac{\sigma}{\varepsilon_0}\hat{x}, -\frac{\sigma}{\varepsilon_0}\hat{x}, \frac{\sigma}{\varepsilon_0}\hat{x}, 0$ (d) $0, +\frac{\sigma}{\varepsilon_0}\hat{x}, \frac{\sigma}{2\varepsilon_0}\hat{x}, +\frac{\sigma}{\varepsilon_0}\hat{x}, 0$

33. An electric field in a region is given by $\vec{E}(x, y, z) = ax\hat{i} + cz\hat{j} + 6by\hat{k}$. For which of a, b, c does this represent an electrostatic field ? **[JEST 2012]**

(a) 13, 1, 12 (b) 17, 6, 1 (c) 13, 1, 6 (d) 45, 6, 1

34. Consider an infinite line charge with linear charge density λ. At a distance r from the line, the electrostatic potential has the form **[NET 2008]**

(a) $\frac{\lambda a}{4\pi\varepsilon_0 r}$ (b) $\frac{\lambda}{4\pi\varepsilon_0}\exp(-r/a)$ (c) $\frac{\lambda}{4\pi\varepsilon_0}\ln(r/a)$ (d) $\frac{\lambda}{4\pi\varepsilon_0}\frac{r}{a}$

35. Consider an infinite horizontal surface with fixed surface charge density σ, where $\hat{n}$ is the upward normal to the surface. If an electric field $\vec{E}_b = E\hat{n}$ is applied from below, the electric field $\vec{E}_a$ in the region above the surface is: **[NET 2008]**

(a) $\vec{E}_a = \vec{E}_b - \frac{\sigma}{2\varepsilon_0}\hat{n}$ (b) $\vec{E}_a = \vec{E}_b + \frac{\sigma}{2\varepsilon_0}\hat{n}$

(c) $\vec{E}_a = \vec{E}_b - \frac{\sigma}{\varepsilon_0}\hat{n}$ (d) $\vec{E}_a = \vec{E}_b + \frac{\sigma}{\varepsilon_0}\hat{n}$

36. Consider an electric field $\vec{E}$ existing in the interface between a conductor and free space. Then the electric field $\vec{E}$ is: **[GATE 2001]**

(a) External to the conductor and normal to the conductor's surface
(b) Internal to the conductor and normal to the conductor's surface
(c) External to the conductor and tangential to the conductor's surface
(d) Both external and internal to the conductor and normal to the conductor's surface

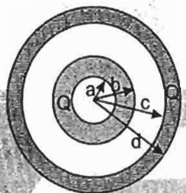
37. An electric charge, +Q is placed on the surface of a solid, conduction sphere of radius a. The distance measured from the centre of the sphere is denoted as r. Then : **[GATE 2003]**

(a) The charge gets distributed uniformly through the volume of the sphere
(b) The electrostatic potential has the same value for $r < a$
(c) An equal and opposite charge gets induced in the bottom half of the sphere
(d) The electric field is given by $1/\left(4\pi\varepsilon_0 r^2\right)$ for $r < a$

38. A shere of radius R has uniform volume charge density. The electric potential at a point r $(r < R)$ is

(a) Due to the charge inside a sphere of radius r only **[GATE 2006]**
(b) Due to the entire charge of the sphere
(c) Due to charge in the spherical shell of inner and outer radii r and R, only
(d) Independent of r

Common data for Q. 39 and Q.40.
Consider two concentric conducting spherical shells with inner and outer radii a, b and c, d as shown in the figure. Both the shells are given Q amount of positive charges.



39. The electric field in different regions are : **[GATE 2008]**

(a) $\vec{E} = 0$ for $r < a$; $\vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $a < r < b$; $\vec{E} = 0$ for $b < r < c$; $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r > d$

(b) $\vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r < a$; $\vec{E} = 0$ for $a < r < b$; $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $b < r < c$; $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r < d$

(c) $\vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r < a$; $\vec{E} = 0$ for $a < r < b$; $\vec{E} = 0$ for $b < r < c$; $\vec{E} = \frac{2Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r > d$

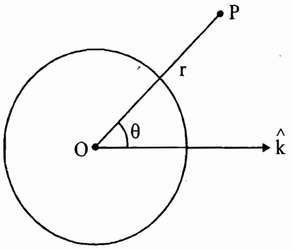
(d) $\vec{E} = 0$ for $r < a$; $\vec{E} = 0$ for $a < r < b$; $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $b < r < c$; $\vec{E} = \frac{2Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r > d$

40. In order to have equal surface charge densities on the outer surfaces of both the shells, the following conditions should be satisfied **[GATE 2008]**

(a) $d = 4b$ and $c = 2a$
(b) $d = 2b$ and $c = \sqrt{2}a$
(c) $d = \sqrt{2}b$ and $c > a$
(d) $d > b$ and $c = \sqrt{2}a$

41. A spherical conductor of radius a is placed in a uniform electric field $\vec{E} = E_0\hat{k}$. The potential at a point $P(r,\theta)$ for $r > a$, is given by, $\varphi(r,\theta) = \text{constant} - E_0 r\cos\theta + \frac{E_0 a^3}{r^2}\cos\theta$ **[GATE 2011]**

Where 'r' is the distance of P from the centre O of the sphere and θ is the angle OP makes with the z-axis.

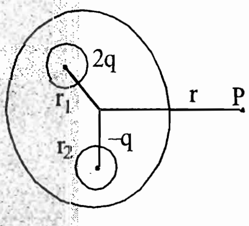


The charge density on the sphere at $\theta = 30^\circ$ *is* :

(a) $3\sqrt{3}\,\varepsilon_0 E_0/2$ (b) $3\varepsilon_0 E_0/2$ (c) $\sqrt{3}\varepsilon_0 E_0/2$ (d) $\varepsilon_0 E_0/2$

42. A spherical conductor of radius R has two cavities centered at r_1 and r_2 respectively from the centre of the conductor. The cavities contain point charges +2q and –q at their respective centers as shown in the figure. The magnitude of the electric field at a point P is :

(a) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ (b) $\frac{1}{4\pi\varepsilon_0}\left[\left(\frac{2q}{(r-r_1)^2}\right)^2+\left(\frac{-q}{(r-r_2)^2}\right)^2\right]^{1/2}$



(c) $\frac{1}{4\pi\varepsilon_0}\frac{q}{(r-r_1-r_2)^2}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{q}{(r-r_1)^2+(r-r_2)^2}$

43. A uniformly charged thin spherical shell of radius R carries a total charge q. At a point inside ($r < R$) the spherical shell, if E and V are intensity of the electric field and electric potential respectively then :

(a) Both E and V are zero.
(b) E is zero but V is a nonzero constant.
(c) E is proportional to $1/r^2$ and V is proportional to 1/r.
(d) Both E and V are nonzero constants.

44. A hollow, closed conductor is placed in an external electric field $\vec{E}$. Then, at equilibrium, the electric field within the cavity is:

(a) $\vec{E}$ (b) 0
(c) Can be less than $\vec{E}$ but non-zero (d) Greater than $\vec{E}$

45. If $\vec{E}_1 = xy\hat{i} + 2yz\hat{j} + 3xz\hat{k}$ and $\vec{E}_2 = y^2\hat{i} + (2xy + z^2)\hat{j} + 2yz\hat{k}$ then **[JEST 2013]**

(a) Both are impossible electrostatic fields. (b) Both are possible electrostatic fields.
(c) Only $\vec{E}_1$ is a possible electrostatic field. (d) Only $\vec{E}_2$ is a possible electrostatic field.

46. Four equal charges of $+Q$ each are kept at the vertices of a square of side R. A particle of mass m and charge $+Q$ is placed in the plane of the square at a short distance $a\,(\ll R)$ from the center. If the motion of the particle is confined to the plane, it will undergo small oscillations with an angular frequency

[NET June 2016]

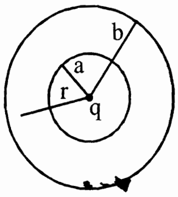
(a) $\sqrt{\dfrac{Q^2}{2\pi\varepsilon_0 R^3 m}}$ (b) $\sqrt{\dfrac{Q^2}{\pi\varepsilon_0 R^3 m}}$ (c) $\sqrt{\dfrac{\sqrt{2}\,Q^2}{\pi\varepsilon_0 R^3 m}}$ (d) $\sqrt{\dfrac{Q^2}{4\pi\varepsilon_0 R^3 m}}$

CSIR-UGC-NET/JRF|GATE-PHYSICS

ELECTROMAGNETIC THEORY

Assignment-2 : Electric Potential and Conductors

1. A conducting spherical shell of inner radius a and outer radius b has a point charge q located at the centre of the shell. The potential at a distance r from the centre (a < r < b) is:



(a) 0 (b) $\frac{1}{4\pi\varepsilon_0}\frac{q}{a}$ (c) $\frac{1}{4\pi\varepsilon_0}\frac{q}{b}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

2. A conducting sphere of radius R is placed in a uniform electric field $\vec{E}_0$ directed along + z axis. The electric potential for outside points is given as :

$V_{out} = -E_0\left(1-\frac{R^3}{r^3}\right)r\cos\theta,$ where r is the distance from the centrer and q is the polar angle. The charge density on the surfae of the sphere is :

(a) $3\varepsilon_0 E_0\cos\theta$ (b) $\varepsilon_0 E_0\cos\theta$ (c) $3\varepsilon_0 E_0\cos\theta$ (d) $\frac{\varepsilon_0}{3}E_0\cos\theta$

3. The field $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ may represent
(a) Only magnetostatic field (b) Only an electrostatic field
(c) Both electrostatic and magnetostatic field (d) Neither electrostatic nor magnetostatic field.

4. A particle of mass m and charge q is constrained to move along a straight line joining two other equal charges q fixed at $x = \pm a$. The time period of small oscillation is:

(a) $T = \frac{2\pi a}{q}\sqrt{\varepsilon_0 am}$ (b) $T = \frac{4\pi a}{q}\sqrt{\pi\varepsilon_0 am}$ (c) $T = \frac{a}{q}\sqrt{\pi\varepsilon_0 am}$ (d) $T = \frac{2\pi a}{q}\sqrt{\pi\varepsilon_0 am}$

5. The electrostatic potential due to a charge distribution is given by $V(r) = A\frac{e^{-\lambda r}}{r}$, where A and λ are constants. The total charge enclosed within a sphere of radius $1/\lambda$, with its origin at $r = 0$ is given by,

(a) $\frac{8\pi\varepsilon_0 A}{e}$ (b) $\frac{4\pi\varepsilon_0 A}{e}$ (c) $\frac{\pi\varepsilon_0 A}{e}$ (d) 0 **[JEST 2015]**

6. A static, spherically symmetric charge distribution is given by $\rho(r) = \frac{A}{r}e^{-Kr}$ where A and K are positive constants. The electrostatic potential corresponding to this charge distribution varies with r as

[NET June 2011]

(a) $r\,e^{-Kr}$ (b) $\frac{1}{r}e^{-Kr}$ (c) $\frac{1}{r^2}e^{-Kr}$ (d) $\frac{1}{r}\left(1-e^{-Kr}\right)$

7. Three equal chrages Q are successively brought from infinity and each is placed at one of the three vertices of an equilateral tringle. Assuming the rest of the Universe as a whole to be neutral, the energy E_0 of the electrostatic field will increase, successively, to **[TIFR 2012]**

$E_0+\Delta_1,\ E_0+\Delta_1+\Delta_2,\ E_0+\Delta_1+\Delta_2+\Delta_3$ Where $\Delta_1:\Delta_2:\Delta_3=$

(a) 1 : 2 : 3 : (b) 1 : 1 : 1 (c) 0 : 1 : 1 (d) 0 : 1 : 2

8. The electric fields outside ($r>R$) and inside ($r<R$) a solid sphere with a uniform volume charge density are given by $\vec{E}_{r>R}=\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$ and $\vec{E}_{r<R}=\frac{1}{4\pi\epsilon_0}\frac{q}{R^3}r\,\hat{r}$ respectively, while the electric field outside a spherical shell with a uniform surface charge density is given by $\vec{E}_{r>R}=\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$, q being the total charge. The correct ratio of the electrostatic energies for the second case to the first case is **[JEST 2013]**

(a) 1:3 (b) 9:16 (c) 3:8 (d) 5:6

9. One of electrostatic boundary conditions for the potential V for a surface carrying a surface charge density s can be written as

(a) $V_{\text{above}}-V_{\text{below}}=-\frac{\sigma}{\epsilon_0}$ (b) $V_{\text{above}}-V_{\text{below}}=\frac{\sigma}{\epsilon_0}$

(c) $\frac{\partial V_{\text{above}}}{\partial n}-\frac{\partial V_{\text{below}}}{\partial n}=\frac{\sigma}{\epsilon_0}$ (d) $\frac{\partial V_{\text{above}}}{\partial n}-\frac{\partial V_{\text{below}}}{\partial n}=-\frac{\sigma}{\epsilon_0}$

10. In an electrostatic boundary value problem, the surface charge density σ is specified at x = 0. The corresponding boundary condition for potential V can be written as

(a) $\left(\frac{\partial^2 V}{\partial x^2}\right)_{x=0}=\frac{1}{\varepsilon_0}\sigma(y.z)$ (b) $\left(\frac{\partial^2 V}{\partial x^2}\right)_{x=0}=-\frac{1}{\varepsilon_0}\sigma(y.z)$

(c) $\left(\frac{\partial V}{\partial x}\right)_{x=0}=+\frac{1}{\varepsilon_0}\sigma(y.z)$ (d) $\left(\frac{\partial V}{\partial x}\right)_{x=0}=-\frac{1}{\varepsilon_0}\sigma(y.z)$

11. Two point charges q and 4q are located at r_1 and r_2, respectively, on the xy-plane. The magnitude Q and location R of a third charge to be placed on the plane such that total force on the each of the three charges vanishes, are

(a) $Q=-4q/9$ and $R=(2r_1+r_2)/3$ (b) $Q=-4q/5$ and $R=(r_1+2r_2)/3$

(c) $Q=-4q/25$ and $R=(16r_1+r_2)/\sqrt{17}$ (d) $Q=-9q/4$ and $R=(r_1+2r_2)/3$

12. The amount of energy required to charge a metallic sphere of radius 2.0 m to a potential of 3000 V in air is:

(a) 1×10^{-3} Joule (b) 2×10^{-3} Joule (c) 3×10^{-3} Joule (d) 4×10^{-3} Joule

13. A small charged spherical shell of radius 0.01 m is at a potential of 30V. The electrostatic energy of the shell is

(a) 10^{-10} J (b) 5×10^{-10} J (c) 5×10^{-9} J (d) 10^{-9} J

14. Suppose that the electric field on the xy-plane is given by

$$\vec{E} = P\left[xy\hat{i} + \left(\frac{1}{2}x^2 + y^2\right)\hat{j}\right]$$

The magnitude of the difference in potential between the origin (0, 0) and the point (1, 2) is:

(a) $\frac{5}{4}P$ (b) $\frac{11}{3}P$ (c) $\frac{7}{2}P$

(d) Not unique, it depends on the path joining the two points.

15. The electrostatic potential V(x,y) in free space in a region where the charge density ρ is zero is given by $V(x,y) = 4e^{2x} + f(x) - 3y^2$. Given that the x-component of the electric field, E_x, and V are zero at the origin, f(x) is: **[NET June 2011]**

(a) $3x^2 - 4e^{2x} + 8x$ (b) $3x^2 - 4e^{2x} + 16x$ (c) $4e^{2x} - 8$ (d) $3x^2 - 4e^{2x}$

16. If the electrostatic potential $V(r,\theta,\phi)$ in *a* charge free region has the form **[NET Dec. 2013]**

$V(r,\theta,\phi) = f(r)\cos\theta$, then the functional form of $f(r)$ (in the following a and b are constants) is

(a) $ar^2 + \frac{b}{r}$ (b) $ar + \frac{b}{r^2}$ (c) $ar + \frac{b}{r}$ (d) $a\ln\left(\frac{r}{b}\right)$

17. If the electrostatic potential in spherical polar coordinates is **[NET June 2014]**

$$\varphi(r) = \varphi_0 e^{-r/r_0}$$

where φ_0 and r_0 are constants, then the charge density at a distance $r = r_0$ will be

(a) $\frac{\varepsilon_0\varphi_0}{er_0^2}$ (b) $\frac{e\varepsilon_0\varphi_0}{2r_0^2}$ (c) $-\frac{\varepsilon_0\varphi_0}{er_0^2}$ (d) $-\frac{2e\varepsilon_0\varphi_0}{r_0^2}$

18. A conducting sphere of radius R has charge + Q on its surface. If the charge on the sphere is doubled and its radius is halved, the energy associated with the electric field will: **[GATE 2004]**

(a) Increase four times (b) Increase eight times

(c) Remain the same (d) Decrease four times

19. If the electrostatic potential were given by $\phi = \phi_0(x^2 + y^2 + z^2)$, where ϕ_0 is constant, then the charge density giving rise to the above potential would be: **[GATE 2005]**

(a) 0 (b) $-6\phi_0\varepsilon_0$ (c) $-2\phi_0\varepsilon_0$ (d) $-\frac{6\phi_0}{\varepsilon_0}$

20. Equipotential surfaces corresponding to a particular charge distribution are given by $4x^2 + (y-2)^2 + z^2 = V_i$, where the values of V_i are constants. The electric field $\vec{E}$ at the origin is:

(a) $\vec{E} = 0$ (b) $\vec{E} = 2\hat{x}$ (c) $\vec{E} = 4\hat{y}$ (d) $\vec{E} = -4\hat{y}$

21. Solving Poisson's equation $\nabla^2\varphi = -\rho_0/\varepsilon_0$ for the electrostatic potential $\varphi(\vec{x})$ in a region with a constant charge density ρ_0, two students find different answers, viz, **[TIFR 2014]**

$$\varphi_1(\vec{x}) = -\frac{1}{2}\frac{\rho_0 x^2}{\varepsilon_0} \text{ and } \varphi_2(\vec{x}) = -\frac{1}{2}\frac{\rho_0 y^2}{\varepsilon_0}$$

The reason why these different solutions are both correct is because

(a) space is isotropic and hence x and y are physically equivalent.

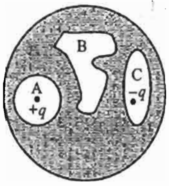
(b) We can add solutions of Laplace's equation to both $\varphi_1(\vec{x})$ and $\varphi_2(\vec{x})$

(c) The electrostatic energy is infinite for a constant charge density.

(d) The boundary conditions are different in the two cases.

22. A solid spherical conductor encloses 3 cavities, a cross-section of which are as shown in the figure. A net charge $+q$ resides on the outer surface of the conductor. Cavities A and C contain point charges $+q$ and $-q$, respectively. **[TIFR 2015]**

The net charges on the surfaces of these cavities are



(a) $A=-q, B=q, C=0$ (b) $A=-q, B=0, C=-q$

(c) $A=+q, B=0, C=-q$ (d) $A=-q, B=0, C=+q$

23. In a region of space that does not contain any electric charge, the electrostatic potential satisfies Laplace's equation. Suppose that the potential in such a situation is given by the formula

$$V(x,y,z)=A\left(bx^2+\frac{1}{2}y^2-z^2\right)$$

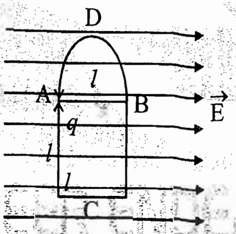
Where A and b are constants. Then the value of b

(a) Can be arbitrary (b) Must be zero (c) Must be 1/2 (d) Must be 1

24. The one dimensional potential $V(x)$ for an electrostatic fields in a region of space which does not contain any charge has the form (in the following a and b are constants)

(a) $a\sin bx$ (b) ae^{-x^2} (c) $\frac{1}{2}ax^2$ (d) $ax+b$

25. A charge q is taken from point A to point B along two different paths ACB and ADB in an uniform electric field.



The difference in the work done between these two paths is:

(a) $3qlE-(\pi/2)lqE$ (b) 0 (c) $\frac{1}{2}\pi lqE$ (d) 3qlE

26. The electrostatic potential is given by, $\Phi=0$ at $x=0$ and $x=a$; $\Phi=V$ $0\le x\le a$. Then Φ is given by

$\Phi=\sum_{n=1}^{\infty}A_n\sin(\alpha x)$ where α is:

(a) $\frac{n\pi}{2}$ (b) $\frac{n\pi}{a}$ (c) $\frac{na}{\pi}$ (d) $\frac{na}{2\pi}$

27. The electrostatic potential in a given region of space is $\phi(\bar{r})=\frac{1}{\epsilon_0}\left[x^2+y^2-2z^2\right]$. The charge density $\rho(\bar{r})$ in the region is

(a) $\frac{-x-y+4z}{r^3}$ (b) $\frac{x+y-2z}{r}$ (c) zero (d) $\frac{4}{r^2}$

28. Which one of the following is not correct?

(a) $\nabla\cdot(\nabla\phi)=\nabla^2\phi$ (b) $\nabla\times(\nabla\times A)=\nabla(\nabla\cdot A)-\nabla^2 A$

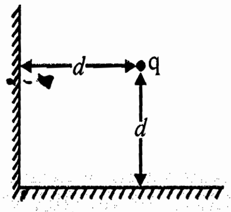
(c) $\nabla\cdot(\nabla\times A)=0$ (d) $\nabla\times(\nabla\phi)=\phi$

29. A point charge 'q' of mass 'm' is kept at a distance 'd' below a grounded infinite conducting sheet which lies in the xy-plane. What is the value of 'd' for which the charge remains stationary?

(a) $q/4\sqrt{mg\pi\varepsilon_0}$ (b) $q/\sqrt{mg\pi\varepsilon_0}$ **[NET Dec. 2012]**

(c) There is no finite value of 'd' (d) $\sqrt{mg\pi\varepsilon_0}/q$

30. A point charge q is placed symmetrically at a distance d from two perpendicularly placed grounded conducting infinite plates as shown in the figure. The net force on the charge (in units of $1/4\pi\in_0$) is



[NET Dec. 2013]

(a) $\frac{q^2}{8d^2}(2\sqrt{2}-1)$ away from the corner (b) $\frac{q^2}{8d^2}(2\sqrt{2}-1)$ towards the corner

(c) $\frac{q^2}{2\sqrt{2}d^2}$ towards the corner (d) $\frac{3q^2}{8d^2}$ away from the corner

31. A grounded conducting sphere of radius 'a' is placed with its centre at the origin. A point dipole of dipole moment, $\vec{p}=p\hat{k}$ is placed at a distance 'd' along the x-axis, where $\hat{i},\hat{k}$ are the units vector along the x and z-axes respectively. This leads to the formation of an image dipole of strength $\vec{p}\,'$ at a distance $d\,'$ from the centre along the x-axis. If $d\,'=\frac{a^2}{d}$, then $\vec{p}\,'$

(a) $-\frac{a^4p}{d^4}\hat{k}$ (b) $-\frac{a^3p}{d^3}\hat{k}$ (c) $-\frac{a^2p}{d^2}\hat{k}$ (d) $-\frac{ap}{d}\hat{k}$

32. A charged particle is at a distance d from an infinite conducting plane maintained at zero potential. When released from rest, the particle reaches a speed u at a distance d/2 from the plane. At what distance from the plane will the particle reach the speed 2u? **[NET June 2014]**

(a) d/6 (b) d/3 (c) d/4 (d) d/5

33. A charge (–e) is placed in vacuum at the point (d, 0, 0), where d > 0. The region $x\le 0$ is filled uniformly with a metal. The electric field at the point $\left(\frac{d}{2},0,0\right)$ is

(a) $-\frac{10e}{9\pi\varepsilon_0 d^2}(1,0,0)$ (b) $\frac{10e}{9\pi\varepsilon_0 d^2}(1,0,0)$ (c) $\frac{e}{\pi\varepsilon_0 d^2}(1,0,0)$ (d) $-\frac{e}{\pi\varepsilon_0 d^2}(1,0,0)$

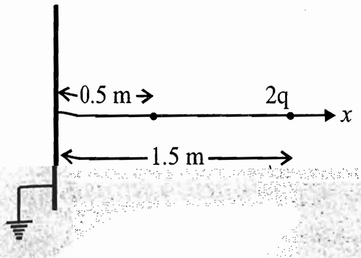
34. A charge + q is kept at a distance of 2R from the center of a grounded conducting sphere of radius R. The image charge and its distance from the centre are, respectively : **[GATE 2004]**

(a) $-\frac{q}{2}$ and $\frac{R}{2}$ (b) $-\frac{q}{2}$ and $\frac{R}{4}$ (c) $-q$ and $\frac{R}{2}$ (d) $+\frac{q}{2}$ and $\frac{R}{2}$

35. The work done in bringing a charge +q from infinity in free space, to a position at a distance d in front of a semi-infinite grounded metal surface is : **[GATE 2005]**

(a) $-\frac{q^2}{4\pi\varepsilon_0(d)}$ (b) $-\frac{q^2}{4\pi\varepsilon_0(2d)}$ (c) $-\frac{q^2}{4\pi\varepsilon_0(4d)}$ (d) $-\frac{q^2}{4\pi\varepsilon_0(6d)}$

36. Two charges q and 2q are placed along the x-axis in front of a grounded, infinite conducting plane, as shown in the figure. They are located respectively at a distance of 0.5 m and 1.5 m from the plane. The force acting on the charge q is: **[GATE 2010]**



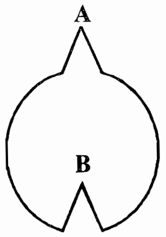
(a) $\frac{1}{4\pi\varepsilon_0}\frac{7q^2}{2}$ (b) $\frac{1}{4\pi\varepsilon_0}2q^2$ (c) $\frac{1}{4\pi\varepsilon_0}q^2$ (d) $-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2}$

37. For a scalar function φ satisfying the Laplace equation, $\nabla\varphi$ has **[GATE 2013]**

(a) zero curl and non-zero divergence (b) non-zero curl and zero divergence
(c) zero curl and zero divergence (d) non-zero curl and non-zero divergence

38. A solid spherical conductor has a conical hole made at one end, ending in a point B, and a small conical projection of the same shape and size at the opposite side, ending in a point A. A cross-section through the centre of the conductor is shown in the figure on the right. **[TIFR 2014]**

If now, a positive charge Q is transferred to the sphere, then
(a) The charge density at both A and B will be undefined.
(b) The charge density at A will be the same as the charge density at B.
(c) The charge density at A will be more than the charge density at B.
(d) The charge density at B will be more than the charge density at A.



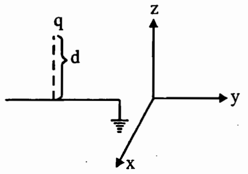
39. A point charge +q placed at (0, 0, d) above a grounded infinite conducting plane defined by z = 0. There are no charges present anywhere else. What is the magnitude of electric field at (0, 0, –d) ?

(a) $q/(8\pi\varepsilon_0 d^2)$ (b) $-\infty$ (c) 0 (d) $q/(16\pi\varepsilon_0 d^2)$ **[JEST 2012]**

40. A point charge is at a distance d from a metal plate. The force acting on the charge is:

(a) Proportional to $1/d^2$ and repulsive. (b) zero
(c) Proportional to $1/d^2$ and attractive. (d) Proportional to 1/d and attractive.

41. Let a point charge +q be at a distance d from an infinite conducting plate which is grounded.

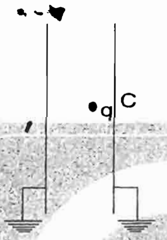


The force experienced by the charge is given by

(a) $\vec{F} = +\frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}\hat{k}$ (b) $\vec{F} = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}\hat{k}$

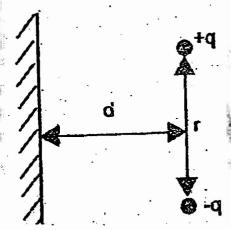
(c) $\vec{F} = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2d)^2}\hat{k}$ (d) $\vec{F} = +\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2d)^2}\hat{k}$

42. A point charge q is kept at the mid-point between two large parallel grounded conducting plates. Assume no gravity. The charge is displaced a little towards the right plate. The charge will now,



(a) Stay where it is (b) Move towards the right plate
(c) Move towards the left plate (d) Oscillate between the plates

43. Two equal and opposite charges are placed at a distance r from each other. A large metallic sheet is placed at a distance d from them as shown in the figure. Due to the presence of the sheet, the attractive force between the charges along the direction joining them



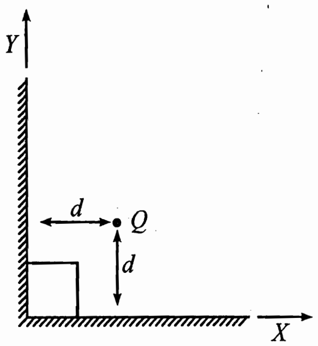
(a) decreases (b) increases (c) remains unchanged
(d) decreases or increases depending on the conductivity of the sheet.

44. A point charge is placed between two semi-infinite conducting plates which are inclined at an angle of 30° with respect to each other. The number of image charges is_______ **[GATE 2015]**

45. A point charge +q is placed outside a grounded conducting sphere of radius *a*. Which of the following statements is not true?

(a) there is an attractive force between the sphere and the point charge
(b) the induced surface charge density on the sphere is not the same everywhere
(c) the electric field inside the sphere is zero
(d) the total induced charge on the sphere is -q

46. A positive point charge Q is located at a distance d from two grounded conducting half planes perpendicular to each other, as shown in the figure. The force on Q caused by the charges induced on the planes is



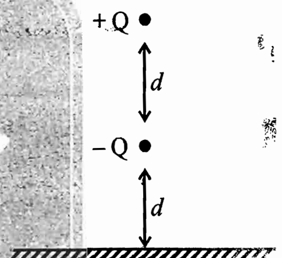
(a) 0

(b) $\frac{Q^2}{4\pi\in_0 d^2}\left(\frac{-1}{4}\hat{i}-\frac{1}{4}\hat{j}\right)$

(c) $\frac{Q^2}{4\pi\in_0 d^2}\left(\frac{-1}{2\sqrt{2}}\hat{i}-\frac{1}{2\sqrt{2}}\hat{j}\right)$

(d) $\frac{Q^2}{4\pi\in_0 d^2}\left(\frac{1-2\sqrt{2}}{8\sqrt{2}}\hat{i}+\frac{1-2\sqrt{2}}{8\sqrt{2}}\hat{j}\right)$

47. Two points charges +Q and –Q are placed away from a grounded conducting plane as shown in the figure. The net force on the charge –Q is



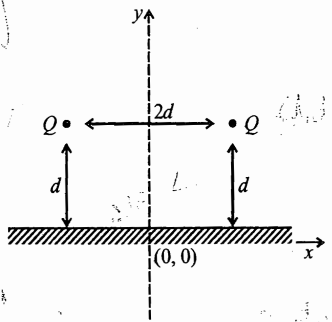
(a) $\frac{Q^2}{4\pi\in_0 d^2}$ away from the conducting plane

(b) $\frac{Q^2}{4\pi\in_0 d^2}\left(\frac{137}{144}\right)$ towards the conducting plane

(c) $\frac{Q^2}{4\pi\in_0 d^2}\left(\frac{31}{36}\right)$ away from the conducting plane

(d) $\frac{Q^2}{4\pi\in_0 d^2}\left(\frac{137}{144}\right)$ away from the conducting plane

48. Two positive point charges Q are placed as shown in the figure a distance d away from a grounded conducting plane. The distance between the point charges is $2d$. The net force on the point charges located at the point (d, d) is



(a) $\frac{Q^2}{4\pi\in_0(4d^2)}i$

(b) $\frac{Q^2}{4\pi\in_0(4d^2)}\left(\frac{2\sqrt{2}-1}{2\sqrt{2}}i-\frac{2\sqrt{2}+1}{2\sqrt{2}}j\right)$

(c) $\frac{Q^2}{4\pi\in_0(4d^2)}\left(\frac{1}{2}i-\frac{3}{2}j\right)$

(d) $\frac{Q^2}{4\pi\in_0(4d^2)}\left(\frac{\sqrt{2}+1}{\sqrt{2}}i-\frac{\sqrt{2}-1}{\sqrt{2}}j\right)$

CSIR-UGC-NET/JRF|GATE-PHYSICS

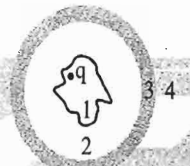
ELECTROMAGNETIC THEORY

Assignment-3 : Dielectric Polarization

1. A point charge q of mass m is released from rest at a distance d from an infinite grounded conducting plane (ignore gravity). How long does it takes for the charge to hit the plane ?

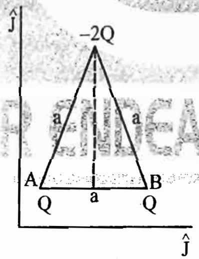
(a) $\frac{\sqrt{2\pi^3\varepsilon_0 md^3}}{q}$ (b) $\frac{\sqrt{2\pi^3\varepsilon_0 md}}{q}$ (c) $\frac{\sqrt{\pi^3\varepsilon_0 md^3}}{q}$ (d) $\frac{\sqrt{\pi^3\varepsilon_0 md}}{q}$

2. An arbitrarily shaped conductor encloses a charge q and is surrounded by a conducting hollow sphere as shown in the figure. Four different regions of space, 1, 2, 3 and 4, are indicated in the figure. Which one of the following statements is correct?



(a) The electric field lines in region 2 are not affected by the position of the charge q
(b) The surface charge density on the inner wall of the hollow sphere is uniform
(c) The surface charge density on the outer surface of the sphere is always uniform irrespective of the position of charge q in region 1
(d) The electric field in region 2 has a radial symmetry

3. Charges Q, Q and -2Q are placed on the vertices of an equilateral triangle ABC of sides of length a, as shown in the figure **[NET June 2012]**



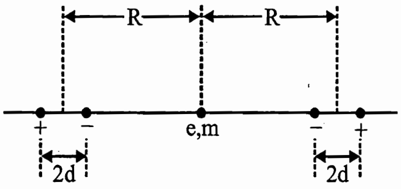
The dipole moment of this configuration of charges, irrespective of the choice of origin, is

(a) $+2aQ\hat{i}$ (b) $+\sqrt{3}aQ\hat{j}$ (c) $-\sqrt{3}aQ\hat{j}$ (d) 0

4. Consider an axially symmetric static charge distribution of the form, $\rho = \rho_0\left(\frac{r_0}{r}\right)^2 e^{-r/r_0}\cos^2\varphi$. The radial component of the dipole moment due to this charge distribution is **[NET June 2013]**

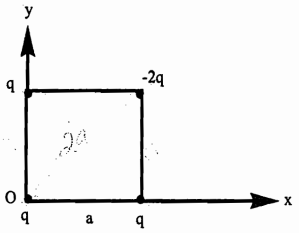
(a) $2\pi\rho_0 r_0^4$ (b) $\pi\rho_0 r_0^4$ (c) $\rho_0 r_0^4$ (d) $\pi\rho_0 r_0^4/2$

5. A particle of charge e and mass m is located at the midpoint of the line joining two mixed collinear dipoles with unit charges as shown in the figure. (The particle is constrained to move only along the line joining the dipoles). Assuming that the length of the dipoles is much shorter than their separation, the natural frequency of oscillation of the particle is **[NET June 2013]**



(a) $\sqrt{\dfrac{6e^2R^2}{\pi\varepsilon_0 md^5}}$ (b) $\sqrt{\dfrac{6e^2R}{\pi\varepsilon_0 md^4}}$ (c) $\sqrt{\dfrac{6e^2d^2}{\pi\varepsilon_0 mR^5}}$ (d) $\sqrt{\dfrac{6e^2d}{\pi\varepsilon_0 mR^4}}$

6. Four charges are placed at the four corners of a square of side a as shown in the figure. The electric dipole moment of this configuration is : **[GATE 2002]**



(a) $\vec{p} = qa\hat{i} + qa\hat{j}$ (b) $\vec{p} = -qa\hat{i} + qa\hat{j}$ (c) $\vec{p} = -qa\hat{i} - qa\hat{j}$ (d) $\vec{p} = qa\hat{i} - qa\hat{j}$

7. Two electric dipoles P_1 and P_2 are placed at (0, 0, 0) and (1, 0, 0) respectively, with both of them pointing in the +z direction. Without changing the orientations of the dipoles, P_2 is moved to (0, 2, 0). The ratio of the electrostatic potential energy of the dipoles after moving to that before moving is:

(a) $\dfrac{1}{16}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{4}$ (d) $\dfrac{1}{8}$

8. Three point charges q, q and $-2q$ are located at $(0, -a, a)$, $(0, a, a)$ and $(0, 0, -a)$, respectively. The net dipole moment of this charge distribution is : **[GATE 2006]**

(a) $4qa\hat{k}$ (b) $2qa\hat{k}$ (c) $-4qa\hat{i}$ (d) $-2qa\hat{j}$

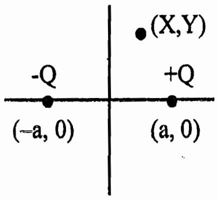
9. Four electric charges of strength Q, Q, –Q and –Q are placed respectively at the points (–b, 0), (b, 0), (0, –a) and (0, a) on the xy plane. At a distance d very far away from these charges (d >> a, b), the electric field will appear to be approximately that of **[GATE 2007]**

(a) A point charge (b) A dipole
(c) A quadrupole (d) A mixture of a dipole and a quadrupole

10. An insulating sphere of radius a carries a charge density $\rho(\vec{r}) = \rho_0\left(a^2 - r^2\right)\cos\theta;\ r < a$. The leading order term for the electric field at a distance d, far away from the charge distribution, is proportional to **[GATE 2010]**

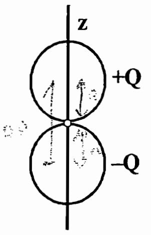
(a) d^{-1} (b) d^{-2} (c) d^{-3} (d) d^{-4}

11. Consider two charges +Q and –Q placed at the points (a, 0) and (–a, 0) in a plane, as shown in the figure on the right. If the origin is moved to the point (X, Y), the magnitude of the dipole moment of the given charge distribution with respect to this origin will be **[TIFR 2013]**



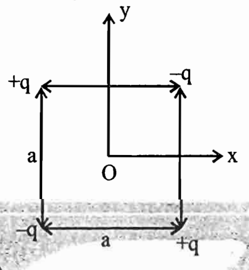
(a) $Q\sqrt{(a-X)^2 + y^2} - Q\sqrt{(a+X)^2 + y^2}$ (b) $Q(a-X) - Q(-a+X)$
(c) 2Qa (d) $2Q\sqrt{X^2 + Y^2}$

12. An electric dipole is constructed by fixing two circular charged rings, each of radius 'a', with an insulating contact (see figure). One of these rings has total charge +Q and the other has total charge −Q. If the charge is distributed uniformly along each ring, the dipole moment about the point of contact will be **[TIFR 2014]**



(a) $\frac{Qa}{\pi}\hat{z}$ (b) $4\pi Qa\hat{z}$ (c) $2Qa\,\hat{z}$ (d) Zero

13. The dipole moment of the following configuration about the centre:

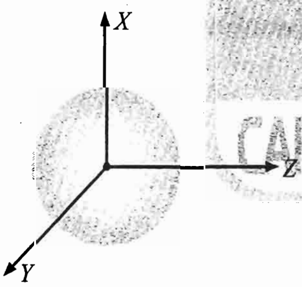


(a) $4\,\text{qa}\left(\hat{i}+\hat{j}\right)$ (b) $2\,\text{qa}\left(\hat{i}-\hat{j}\right)$ (c) 0 (d) $8\,\text{qa}^2\,\hat{k}$

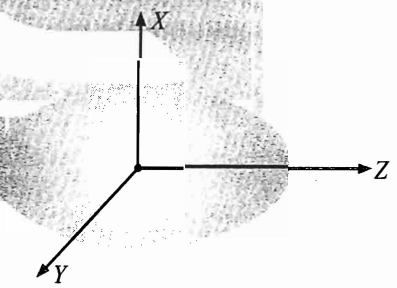
14. A certain charge distribution consists of two equivalent dipoles which are oppositely oriented. The leading term in the expansion of the potential is that due to a

(a) Monopole (b) Dipole (c) Quadrupole (d) Octupole

15. A charge $-q$ is distributed uniformly over a sphere, with a positive charge q at its center in (i). Also in (ii), a charge $-q$ is distributed uniformly over an ellipsoid with a positive charge q at its center. With respect to the origin of the coordinate system, which one of the following statements is correct ?



(i)



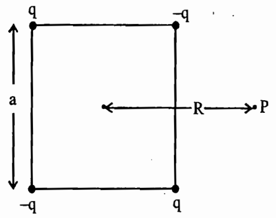
(ii)

(a) The dipole moment is zero in both (i) and (ii)
(b) The dipole moment is non-zero in (i) but zero in (ii)
(c) The dipole moment is zero in (i) but non-zero in (ii)
(d) The dipole moment is non-zero in both (i) and (ii)

16. An electric point charge +q is placed at the point (1, 1) of the xy-plane in which two grounded semi-infinite conducting plates along the positive x and y-axes meet see figure). The electric potential in the positive quadrant at a large distance r goes as **[NET 2008]**

(a) $V(r)\sim r^{-1}$ (b) $V(r)\sim r^{-2}$ (c) $V(r)\sim r^{-3}$ (d) $V(r)\sim r^{-4}$

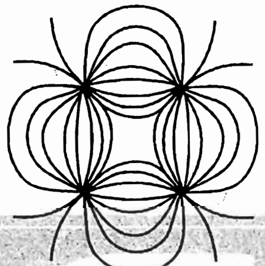
17. Four charges (two +q and two –q) are kept fixed at the four vertices of a square of side ‘a’ as shown



At the point P which is at a distance R from the centre (R > > a), the potential is proportional to

(a) $\frac{1}{R}$ (b) $\frac{1}{R^2}$ (c) $\frac{1}{R^3}$ (d) $\frac{1}{R^4}$ **[NET Dec. 2012]**

18. The electrostatic lines of force due to a system of four point charges is sketched below.



At a large distance *r*, the leading asymptotic behaviour of the electrostatic potential is proportional to

(a) r (b) r^{-1} (c) r^{-2} (d) r^{-3} **[NET Dec. 2014]**

19. Consider a spherical dielectric material of radius ‘a’, centered at origin. If the polarization vector $\vec{P} = P_0\hat{e}_x$, where P_0 is a constant of appropriate dimensions, then

($\hat{e}_x$, $\hat{e}_y$, and $\hat{e}_z$ are unit vectors in Cartesian - coordinate system)

(A) the bound volume charge density is zero

(B) the bound surface charge density is zero at (0, 0, a)

(C) the electric field is zero inside the dielectric

(D) the sign of the surface charge density changes over the surface.

20. For an electric dipole with moment $\vec{p} = p_0\hat{e}_z$ placed at the origin, (p_0 is a constant of appropriate dimensions and $\hat{e}_x$, $\hat{e}_y$, and $\hat{e}_z$ are unit vectors in Cartesisn coordinate system)

(A) potential falls as $\frac{1}{r^2}$, where r is the distance from origin.

(B) a spherical surface centered at origin is an equipotential surface.

(C) electric flux through a spherical surface enclosing the origin is zero

(D) radial component of $\vec{E}$ is zero on the *xy*-plane.

21. A dielectric sphere of radius R carries a polarization $\vec{P} = k\vec{r}$ throughout its volume. The volume charge density Pb of the bound charges is **[NET June 2009]**

(a) –3k (b) 3k (c) –k (d) k

22. Consider an electromagnetic wave at the interface between two homogeneouos dielectric media of the dielectric constants ε_1 and ε_2. Assuming $\varepsilon_2 > \varepsilon_1$ and non charges on the surface, the electric field vector E and the displacement vector $\vec{D}$ in the two media satisfy the following inequalities **[NET June 2014]**

(a) $|\vec{E}_2| > |\vec{E}_1|$ and $|\vec{D}_2| > |\vec{D}_1|$ (b) $|\vec{E}_2| < |\vec{E}_1|$ and $|\vec{D}_2| < |\vec{D}_1|$

(c) $|\vec{E}_2| < |\vec{E}_1|$ and $|\vec{D}_2| > |\vec{D}_1|$ (d) $|\vec{E}_2| > |\vec{E}_1|$ and $|\vec{D}_2| < |\vec{D}_1|$

23. Suppose *yz*-plane forms the boundary between two linear dielectric media *I* and *II* with dielectric constant $\varepsilon_I = 3$ and $\varepsilon_{II} = 4$, respectively. If the electric field in region. *I* at the interface is given by $\vec{E}_1 = 4\hat{x} + 3\hat{y} + 5\hat{z}$, then the electric field $\vec{E}_{II}$ at the interface in region *II* is:

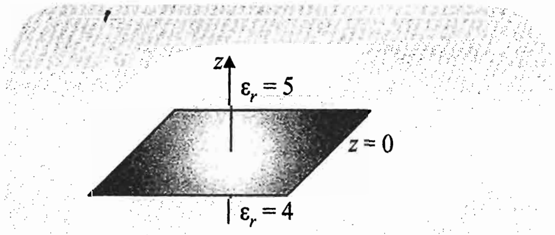
$\vec{E}_1^{\parallel} = 3\hat{y} + 5\hat{z}$

(a) $4\hat{x} + 3\hat{y} + 5\hat{z}$ (b) $4\hat{x} + 0.75\hat{y} - 1.25\hat{z}$

(c) $-3\hat{x} + 3\hat{y} + 5\hat{z}$ (d) $3\hat{x} + 3\hat{y} + 5\hat{z}$

$E_2^{\perp} = 3\hat{x}$

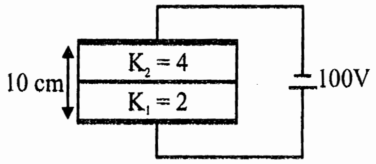
24. Assume that z = 0 plane is th interface between two linear and homogeneous dielectrics (see figure). The relative permitivities are $\varepsilon_r = 5$ for $z > 0$ and $\varepsilon_r = 4$ for $z < 0$. The electric field in the region $z > 0$ is $\vec{E}_1 = (3\hat{i} - 5\hat{j} + 4\hat{k})\ kV/m$. If there are no free charges on the interface, the electric field in the region $z < 0$ is given by



(a) $\vec{E}_2 = \left(\frac{3}{4}\hat{i} - \frac{5}{4}\hat{j} + \hat{k}\right) kV/m$ (b) $\vec{E}_2 = (3\hat{i} - 5\hat{j} + \hat{k})\, kV/m$

(c) $\vec{E}_2 = (3\hat{i} - 5\hat{j} - 5\hat{k})\, kV/m$ (d) $\vec{E}_2 = (3\hat{i} - 5\hat{j} + 5\hat{k})\, kV/m$

25. In a parallel plate capacitor the distance between the plates is 10 cm. Two dielectric slabs of thickness 5cm each and dielectric constants $K_1 = 2$ and $K_2 = 4$ respectively, are inserted between the plates. A potential of 100V is applied across the capacitor as shown in the figure. The value of the net bound surface charge density at the interface of the two dielectrics is



(a) $-\frac{2000}{3}\varepsilon_0$ (b) $-\frac{1000}{3}\varepsilon_0$ (c) $-250\varepsilon_0$ (d) $\frac{2000}{3}\varepsilon_0$

26. A unit cube made of dielectric material has a polarization $\vec{P} = 3\hat{i} + 4\hat{j}$ units. The edges of the cube are parallel to the Cartesian axes. Which of the following statements are true?

(a) the cube carries a volume bound charge of magnitude 5 units.

(b) there is a charge of magnitude 3 units on both the surfaces parallel to the y-z plane.

(c) there is a charge of magnitude 4 units on both the surfaces parallel to the x-z plane.

(d) there is a net non-zero induced charge on the cube

27. An infinite, conducting slab kept in a horizontal plane carries a uniform charge density σ. Another infinite slab of thickness t, made of a linear dielectric material of dielectric constant k, is kept above the conducting slab. The bound charge density on the upper surface of the dielectric slab is

(a) $\frac{\sigma}{2k}$ (b) $\frac{\sigma}{k}$ (c) $\frac{\sigma(k-2)}{2k}$ (d) $\frac{\sigma(k-1)}{k}$

28. Electric charges Q and –Q are distributed uniformly on the surfaces of two concentric shells of radii a and b, respectively ($a < b$). The energy required to assemble this charge distribution is

(a) $\frac{Q^2}{4\pi\varepsilon_0}\frac{(b-a)}{(b+a)^2}$ (b) $\frac{Q^2}{8\pi\varepsilon_0}\frac{(b-a)}{ab}$ (c) $\frac{Q^2}{4\pi\varepsilon_0}\frac{(b-a)}{b^2}$ (d) $\frac{Q^2}{8\pi\varepsilon_0}\frac{(b-a)}{b^2}$

Statement for Linked Answer Q. 29 and Q. 30 :

A dielectric sphere of radius R carries polarization $\vec{P} = kr^2\,\hat{r}$, wherer r is the distance from the centre and k is a constant. In the spherical polar coordinate system, $\hat{r}$, $\hat{\theta}$ and $\hat{\varphi}$ are the unit vectors.

29. The bound volume charge density inside the sphere at a distance r from the centre is :

(a) –4kR (b) –4kr (c) $-4kr^2$ (d) $-4kr^3$ **[GATE 2006]**

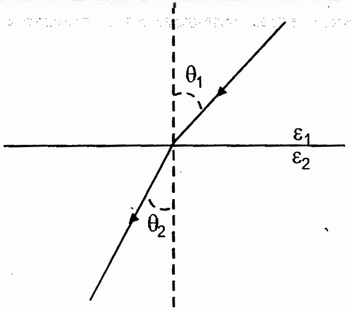
30. The electric field inside the sphere at a distance d from the centre is : **[GATE 2006]**

(a) $\frac{-kd^2}{\varepsilon_0}\hat{r}$ (b) $\frac{-kR^2}{\varepsilon_0}\hat{r}$ (c) $\frac{-kd^2}{\varepsilon_0}\hat{\theta}$ (d) $\frac{-kR^2}{\varepsilon_0}\hat{\theta}$

31. A long, solid dielectric cylinder of radius 'a' is permanently polarized so that the polarization is everywhere radially outward, with a magnitude proportional to the distance from the axis of the cylinder, i.e., $\vec{p} = \frac{1}{2}P_0 r\hat{r}$. The bound charge density in the cylinder is given by

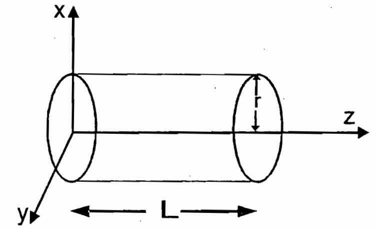
(a) $-P_0$ (b) P_0 (c) $-P_0/2$ (d) $P_0/2$

32. At the interface between two linear dielectrics (with dielectric constants (ε_1 and ε_2), the electric field lines bend, as shown in the figure. Assume that there are no free charges at the interface. The ratio ε_1 and ε_2 is :



(a) $\frac{\tan\theta_1}{\tan\theta_2}$ (b) $\frac{\cos\theta_1}{\cos\theta_2}$ (c) $\frac{\sin\theta_1}{\sin\theta_2}$ (d) $\frac{\cot\theta_1}{\cot\theta_2}$ **[GATE 2006]**

33. A cylindrical rod of length L and radius r, made of an inhomogeneous dielectric, is placed with its axis along the z direction with one end at the origin as shown below : **[GATE 2009]**

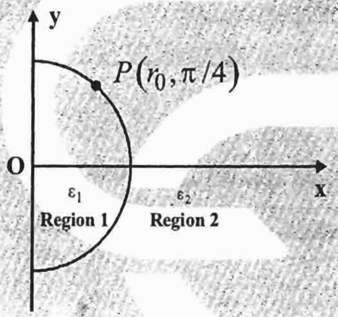


If the rod carries a polarization, $\vec{P} = \left(5z^2 + 7\right)\hat{k}$, the volume bound charge inside the dielectric is :

(a) Zero (b) $10\pi r^2 L$ (c) $-5\pi r^2 L$ (d) $-5\pi r^2 L^2$

34. A ray of light insid Region 1 in the xy-plane is incident at the semicircle boundary that carries no free charges. The electric field at the point $P\left(r_0, \pi/4\right)$ in plane polar coordinates is $\vec{E}_1 = 7\hat{e}_r - 3\hat{e}_\varphi$, where $\hat{e}_r$ and $\hat{e}_\varphi$ are the unit vectors. The emerging ray in Region 2 has the electric field $\vec{E}_2$ parallel to x-axis. If ε_1 and ε_2 are the dielectric constants of Region 1 and Region 2 respectively, then $\dfrac{\varepsilon_2}{\varepsilon_1}$ is ________

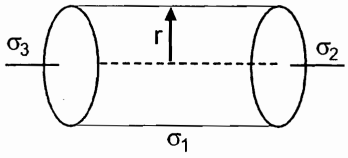
[GATE 2014]



35. A charge q is placed at the centre of an otherwise neutral dielectric sphere of radius a and relative permittivity ϵ_r. We denote the expression $q/4\pi\epsilon_0 r^2$ by $E(r)$. Which of the following statements is false?

(a) The electric field inside the sphere, $r < a$, is given by $E(r)/\epsilon_r$. **[JEST 2013]**
(b) The field outside the sphere, $r > a$, is given by $E(r)$.
(c) The total charge inside a sphere of radius $r > a$ is given by q.
(d) The total charge inside a sphere of radius $r < a$ is given by q.

36. A cylinder of radius R and length L carries a polarization $P = P_0\vec{r}$, where P_0 is a constant and $\vec{r}$ is a radial vector from the axis. The surface bound charges on the cylindrical surface $\left(\sigma_1\right)$ and at the circular surfaces $\left(\sigma_2 \text{ and } \sigma_3\right)$ as shown in the firgure are :



(a) $\sigma_1 = 0, \sigma_2 = \sigma_3 = P_0 R$ (b) (b) $\sigma_1 = P_0 R, \sigma_2 = \sigma_3 = 0$

(c) $\sigma_1 = P_0 R^2/L, \sigma_2 = \sigma_3 = P_0 R$ (d) $\sigma_1 = P_0 R, \sigma_2 = \sigma_3 = P_0 R^2/L$

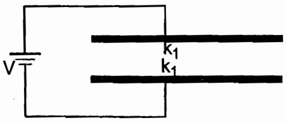
37. The space between the plates of a parallel plate capacitor is filled with two dielectric slabs of dielectric constants k_1 and k_2 as shown in the figure. If the capacitor is charged to a potential V, then at the interface between the two dielectrics.

(a) $\vec{E}$ is discontinuous and $\vec{D}$ is continuous

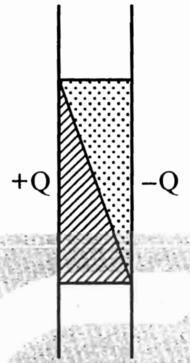
(b) $\vec{E}$ is discontinuous and $\vec{D}$ is discontinuous

(c) $\vec{E}$ is continuous and $\vec{D}$ is continuous

(d) $\vec{E}$ is continuous and $\vec{D}$ is discontinous



38. The space between two plates of a capacitor carrying charges +Q and –Q is filled with two different dielectric materials, as shown in the figure. Across the interface of the two dielectric materials, which one of the following statements is correct ? **[GATE 2015]**



(a) $\vec{E}$ and $\vec{D}$ are continuous

(b) $\vec{E}$ is continuous and $\vec{D}$ is discontinuous

(c) $\vec{D}$ is continuous and $\vec{E}$ is discontinuous

(d) $\vec{E}$ and $\vec{D}$ are discontinuous

39. A spherical shell of inner and outer radii a and b, respectively, is made of a dielectric material with frozen polarization $\vec{P}(r) = \frac{k}{r}\hat{r}$, where k is a constant and r is the distance from the its centre. The electric field in the region $a < r < b$ is, **[JEST 2015]**

(a) $\vec{E} = \frac{k}{\varepsilon_0 r}\hat{r}$

(b) $\vec{E} = -\frac{k}{\varepsilon_0 r}\hat{r}$

(c) $\vec{E} = 0$

(d) $\vec{E} = \frac{k}{\varepsilon_0 r^2}\hat{r}$

40. The relation $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$ where, $\vec{D} \rightarrow$ dielectric constant; $\vec{E} \rightarrow$ electric field intensity; $\vec{P} \rightarrow$ polarization vector is applicable

(a) For all medium

(b) For dielectric medium

(c) Free space

(d) Magnetic medium

41. Two parallel plate capacitors, separated by distances x and $1.1x$ respectively, have a dielectric material of dielectric constant 3.0 inserted between the plates, and are connected to a battery of voltage V. The difference in charge on the second capacitor compared to the first is **[NET June 2016]**

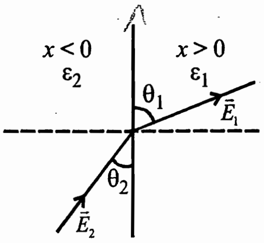
(a) $+66\%$

(b) $+20\%$

(c) -3.3%

(d) -10%

42. The half space regions $x > 0$ and $x < 0$ are filled with dielectric media of dielectric constants ε_1 and ε_2 respectively. There is a uniform electric field in each part. In the right half, the electric field makes an angle θ_1 to the interface. The corresponding angle θ_2 in the left half satisfies **[NET June 2016]**



tan θ = 90°

$\frac{\tan(90°-\theta_1)}{\tan(90-\theta_2)} = \frac{\epsilon_1}{\epsilon_2}$

$\frac{\cot(\theta_1)}{\cot(\theta_2)} = \frac{\epsilon_1}{\epsilon_2}$

$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_1}{\epsilon_2}$

(a) $\varepsilon_1 \sin\theta_2 = \varepsilon_2 \sin\theta_1$ (b) $\varepsilon_1 \tan\theta_2 = \varepsilon_2 \tan\theta_1$

(c) $\varepsilon_1 \tan\theta_1 = \varepsilon_2 \tan\theta_2$ (d) $\varepsilon_1 \sin\theta_1 = \varepsilon_2 \sin\theta_2$

43. A large charged copper sheet is surrounded by a dielectric medium of dielectric constant $\in$. At any point on the flat surface of the sheet.

(a) The normal component of $\vec{E}$ is zero. (b) The normal component of $\vec{E}$ is not zero.

(c) The tangential component of $\vec{E}$ is zero. (d) The tangential component of $\vec{E}$ is not zero.

44. An electric dipole $\vec{p}$ located in a uniform electric field with its direction along the electric field $\vec{E}$. The work required to rotate the dipole through 180° is:

(a) 2pE (b) pE (c) $\vec{p}\times\vec{E}$ (d) $2(\vec{p}\times\vec{E})$

45. Suppose the *yz*-plane forms a chargeless boundary between two media of permittivities e_{left} and e_{right} where $e_{\text{left}} : e_{\text{right}} = 1:2$. If the uniform electric field on the left is $\vec{E}_{\text{left}} = c(\hat{i}+\hat{j}+\hat{k})$ (where c is a constant), then the electric field on the right $\vec{E}_{\text{right}}$ is **[NET June 2015]**

(a) $c(2\hat{i}+\hat{j}+\hat{k})$ (b) $c(\hat{i}+2\hat{j}+2\hat{k})$ (c) $c\left(\frac{1}{2}\hat{i}+\hat{j}+\hat{k}\right)$ (d) $c\left(\hat{i}+\frac{1}{2}\hat{j}+\frac{1}{2}\hat{k}\right)$

46. A thin dielectric slab is slowly introduced between the plates of a charged parallel plate capacitor, as shown in the figure. **[NET 2008]**

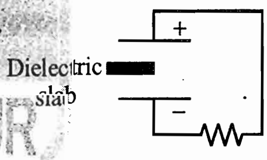
Which of the following is true?

(a) The slab is pushed out of the region between the capacitor plates.

(b) The slab is sucked into the region between the capacitor plates.

(c) The slab moves towards the positively charged plate.

(d) The slab moves towards the negatively charged plate.



47. Consider a parallel plate capacitor connected to an AC voltage source (as shown in fig.1). A conducting slab is introduced in the space between the plates from above (as shown in fig.2) **[NET 2008]**

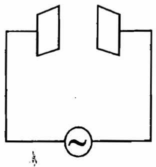


Fig. 1

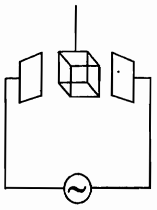


Fig. 2

The capacitance of the parallel plate capacitor.

(a) Goes to zero. (b) Increases to a finite value.

(c) Decreases to a non-zero value. (d) Becomes infinite.

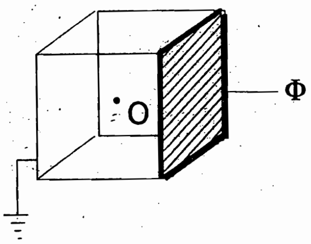
48. A coaxial cable of uniform cross-section contains an insulating material of dielectric constant 3.5. The radius of the central wire is 0.01 m and that of the sheath is 0.02 m. The capacitance per kilometer of a cable is :

(a) 280.5 nF (b) 28.05 nF (c) 56.10nF (d) 2.805 nF **[GATE 2001]**

49. An electric field applied along the length of a long cylinder produces a polarization P. The depolarization field produced in this configuration is : **[GATE 2003]**

(a) $4\pi P/3$ (b) $-4\pi P/3$ (c) $2\pi P$ (d) 0

50. Five sides of a hollow metallic cube are grounded and the sixth side is insulated from the rest and is held at a potential Φ (see figure). **[TIFR 2012]**



The potential at the centre O of the cube is

(a) 0 (b) $\Phi/6$ (c) $\Phi/5$ (d) $2\Phi/3$

51. A parallel plate capacitor of circular cross section with radius r >>d, where 'd' is the spacing between the plates, is charged to a potential V and then disconnected from the charging circuit. If, now, the plates are slowly pulled apart (keeping then parallel) so that their separation is increased from d to d¢, the work done will be

(a) $\frac{\pi\varepsilon_0 r^2V^2}{2d}\left(1-\frac{d}{d'}\right)$ (b) $\frac{\pi\varepsilon_0 r^2V^2}{2d}\frac{d'}{d}$ (c) $\frac{\pi\varepsilon_0 r^2V^2}{2d}\left(\frac{d'}{d}-1\right)$ (d) $\frac{\pi\varepsilon_0 r^2V^2}{2d}\frac{d}{d'}$ **[TIFR 2013]**

52. A cube has a constant electric potential V on its surface. If there are no charges inside the cube. the potential at the center of the cube is **[JEST 2012]**

(a) V (b) V/8 (c) 0 (d) V/6

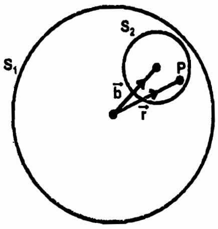
53. Consider a particle of electric charge e and mass moving under the influence of constant horizontal electric field E and constant vertical gravitational field described by acceleration due to gravity 'g'. If the particle starts from rest, what will be its trajectory ? **[JEST 2012]**

(a) Parabolic (b) elliptic (c) straight line (d) circular

54. The conductors in a 0.75 km long two-wire transmission line are separated by a centre-to-centre distance of 0.2m. If each conductor has a diameter of 4cm, then the capacitance of the line is **[JEST 2013]**

(a) 8.85 μF (b) 88.5 nF (c) 8.85 pF (d) 8.85 nF

55. Consider a sphere S_1 of radius R which carries a uniform charge of density ρ. A smaller sphere S_2 of radius $a<\frac{R}{2}$ is cut out and removed from it. The centers of the two spheres are separated by the vector $\vec{b}=\hat{n}\frac{R}{2}$, as shown in the figure. **[NET June 2016]**



The electric field at a point P inside S_2 is

(a) $\frac{\rho R}{3\varepsilon_0}\hat{n}$ (b) $\frac{\rho R}{3\varepsilon_0 a}(\vec{r}-\hat{n}a)$ (c) $\frac{\rho R}{6\varepsilon_0}\hat{n}$ (d) $\frac{\rho a}{3\varepsilon_0 R}\vec{r}$

56. A charged capacitor is made of two concentric cylinders of radii r_1 and r_2. The region between the cylindrical plates is filled with a medium of dielectric constant, k. The displacement vector $(\vec{D})$ at a distance $r(r_1 < r < r_2)$.

(a) Is independent of r.
(b) Varies as ln r.
(c) Is proportional to r.
(d) Varies as 1/r.

(31)
(42) (c)
(43)
(50)
51 (c)
(55) (c)
(56) (d)

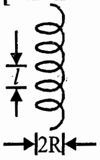
(a)
(c)
(b)
(a)

CSIR-UGC-NET/JRF|GATE-PHYSICS

ELECTROMAGNETIC THEORY

Assignment-4 : Magnetostatics

1. A charged particle moves in a helical path under the influence of a constant magnetic field. The initial velocity is such that the component along the magnetic field is twice the component in the plane normal to the magnetic field. The ratio l/R of the pitch l to the radius R of the helical path is **[NET Dec. 2014]**



(a) $\pi/2$ (b) 4π

(c) 2π (d) π

2. The value of the magnetic field required to maintain non-relativistic protons of energy 1 MeV in a circular orbit of radius 100 mm is ________ Tesla. **[GATE 2014]**

(Given: $m_p = 1.67\times10^{-27}\,kg, e = 1.6\times10^{-19}\,C$

3. An electron with kinetic energy of 10 keV is moving in a circular in a plane perpendicular to a uniform magnetic field of strength 1 tesla $\left(=1\times10^4 \text{ gauss}\right)$. The period of rotation is nearest to

(a) 2.8×10^{-12}s (b) 5.6×10^{-12}s (c) 1.8×10^{-11}s (d) 1.1×10^{-10}s

4. Consider a region of space in which there are uniform electric and magnetic fields which are mutually parallel. A charged particle released from rest in this region will move in a

(a) straight line (b) circle (c) helix (d) cycloid

5. A charged particle in a uniform magnetic field $\vec{B} = B_0\hat{e}_z$ starts moving from the origin with velocity $\vec{v} = (3\hat{e}_x + 2\hat{e}_z)$ m/s. The trajectory of the particle and the time t at which it reaches 2 meters above the xy-plane are ($\hat{e}_x, \hat{e}_y$, and $\hat{e}_z$ are unit vectors in Cartesian - coordinate system.)

(a) Helical path; $t = 1s$ (b) Helical path; $t = 2/3\,s$

(c) Circular path; $t = 1s$ (d) Circular path; $t = 2/3\,s$

6. A magnetic field $\vec{B} = B_0\left(\hat{i} + 2\hat{j} - 4\hat{k}\right)$ exists at a point. If a test charge moving with a velocity $\vec{\upsilon} = \upsilon_0\left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ experiences no force at a certain point. The electric field at that point in SI units is

(a) $\vec{E} = -\upsilon_0 B_0\left(3\hat{i} - 2\hat{j} - 4\hat{k}\right)$ (b) $\vec{E} = -\upsilon_0 B_0\left(\hat{i} + \hat{j} + 7\hat{k}\right)$ **[JEST 2012]**

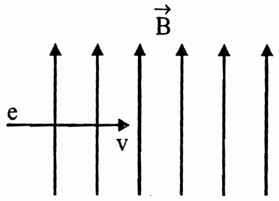
(c) $\vec{E} = \upsilon_0 B_0\left(14\hat{j} + 7\hat{k}\right)$ (d) $\vec{E} = -\upsilon_0 B_0\left(14\hat{j} + 7\hat{k}\right)$

7. The x- and z-components of a static magnetic field in a region are $B_x = B_0(x^2 - y^2)$ and $B_z = 0$, respectively. Which of the following solutions for its y-component is consistent with the Maxwell equations ?

(a) $B_y = B_0 xy$ (b) $B_y = -2B_0 xy$ **[NET June 2016]**

(c) $B_y = -B_0(x^2 - y^2)$ (d) $B_y = B_0\left(\frac{1}{3}x^3 - xy^2\right)$

8. A stream of electrons eneters, from left to right, a uniform magnetic field (B) which is pointing upwards as shown in the figure. The field deflects the stream of electrons.



(a) Downward (in the plane of the paper) (b) Upward (in the plane of the paper)
(c) Into the paper (d) Out of the Paper.

9. A charged particle moving with a constant velocity enters a magnetic field perpendicular to its velocity. In which direction y an electric field should be applied to compensate the magnetic force?
(a) Along the initial velocity
(b) Perpendicular to the initial velocity and parallel to the magnetic field.
(c) Perpendicular to both, the initial velocity and the magnetic field.
(d) Along the magnetic field.

10. A charge q moves in a combined electric field $\vec{E}$ and magnetic induction $\vec{B}$. Then, as the charge moves
(a) Both the electric field and magnetic induction do work on it.
(b) The electric field does work but magnetic induction does no work on it.
(c) Magnetic induction does work but electric field does no work on it.
(d) Both electric field and magnetic induction do no work on it.

11. When a charged particle with kinetic energy E enters a region of uniform magnetic field $\vec{B}$ directed perpendicular to its velocity, it is found to move in circle of radius R. If the particle had kinetic energy 2E, the radius of the path would have been

(a) $R/\sqrt{2}$ (b) $\sqrt{2}R$ (c) $R/2$ (d) $2R$

12. A proton moves with a speed of 300 m/s in a circular orbit in the *xy*-plane in a magnetic field 1 tesla along the positive *z*-direction. When an electric field of 1 V/m is applied along the positive *y*-direction, the center of the circular orbit **[NET June 2015]**

(a) remains stationary
(b) moves at 1 m/s along the negative *x*-direction
(c) moves at 1 m/s along the positive *z*-direction
(d) moves at 1 m/s along the positive *x*-direction

13. An alpha particle is accelerated in a cyclotron. It leaves the cyclotron with a kinetic energy of 16 MeV. The potential difference between the D electrodes is 50 kilovolts. The number of revolutions the alpha particle makes in its spiral path before it leaves the cyclotron is _______ **[GATE 2016]**

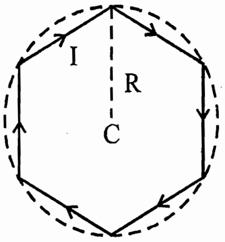
14. The force between two long and parallel wires carrying currents I_1 and I_2 and separated by a distance D is proportional to **[NET Dec. 2013]**

(a) $I_1 I_2 / D$ (b) $(I_1 + I_2) / D$ (c) $(I_1 I_2 / D)^2$ (d) $I_1 I_2 / D^2$

15. An infinite conducting sheet in the x-y plane carries a surface current density K along the y-axis. The magnetic field B for z > 0 is : **[GATE 2003]**

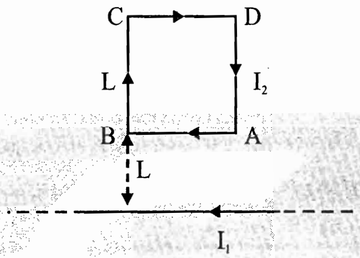
(a) $B = 0$ (b) $B = \mu_0 K \hat{k} / z$ (c) $B = \mu_0 K \hat{i} / 2$ (d) $B = \mu_0 K \hat{j} / \left(x^2 + z^2\right)^{0.5}$

16. A conducting wire is in the shape of a regular hexagon, which is inscribed inside an imaginary circle of radius R, as shown. A current I flows through the wire. The magnitude of the magnetic field at the centre of the circle is



(a) $\frac{\sqrt{3}\mu_0 I}{2\pi R}$ (b) $\frac{\mu_0 I}{2\sqrt{3}\pi R}$ (c) $\frac{\sqrt{3}\mu_0 I}{\pi R}$ (d) $\frac{3\mu_0 I}{2\pi R}$

17. A rigid current-carrying square loop ABCD of side L is situated at a distance L from an infinite straight wire, with the side AB parallel to the latter, as shown in the figure below. The infinite wire, which is in the plane of the loop, carries a current I_1 from right to left, while the loop carries a current I_2 flowing clockwise.



The net force between thest two current-carrying elements is

(a) repulsive of magnitude $\frac{3\mu_0}{4\pi} I_1 I_2$ (b) repulsive of magnitude $\frac{\mu_0}{2\pi} I_1 I_2$

(c) attractive of magnitude $\frac{3\mu_0}{2\pi} I_1 I_2$ (d) zero (no net force)

18. Consider an infinitely long straight cylindrical conductor of radius R with its axis along the z-direction, which carries a current of 1 A uniformly distributed over its cross section. Which of the following statements is correct? **[GATE 2002]**

(a) $\vec{\nabla} \times \vec{B} = 0$ everywhere (b) $\vec{\nabla} \times \vec{B} = \frac{\mu_0}{\pi R^2} \hat{z}$ everywhere

(c) $\vec{\nabla} \times \vec{B} = 0$ for $r > R$ (d) $\vec{\nabla} \times \vec{B} = \frac{\mu_0}{\pi R^2} \hat{z}$ for $r > R$

where r is the radial distance from the axis of the cylinder.

19. If a charged particle of charge q and mass m moving with speed v enters a magnetic field $\boldsymbol{B}$ perpendicular to its direction of motion, it will move in a circular orbit of radius R. Then,

(a) R is directly proportional to q/m and $|\boldsymbol{B}|$

(b) R is inversely proportional to q/m and $|\boldsymbol{B}|$

(c) R is directly proportional to q/m and inversely proportional to $|\boldsymbol{B}|$

(d) R is inversely proportional to q/m and directly proportional to $|\boldsymbol{B}|$

20. The x and y plane carries a uniform surface current of density $\vec{K} = 50\, \hat{e}_z$ A / m. The magnetic field at the point $z = -0.5$ m is: **[GATE 2001]**

(a) 10×10^{-6} Wb (b) 1×10^{-6} Wb (c) $\pi \times 10^{-6}$ Wb (d) $10\pi \times 10^{-6}$ Wb

21. The magnetic field at a distance R from a long straight wire carrying a steady current I is proportional to

(a) IR (b) I/R^2 (c) I^2/R^2 (d) I/R **[NET June 2012]**

22. A uniform surface current is flowing in the positive y-direction over an infinite sheet lying in x-y plane. The direction of the magnetic field is: **[GATE 2011]**

(a) Along $\hat{i}$ for $z > 0$ and along $-\hat{i}$ for $z < 0$. (b) Along $\hat{k}$ for $z > 0$ and along $-\hat{k}$ for $z < 0$

(c) Along $-\hat{i}$ for $z > 0$ and along $\hat{i}$ for $z < 0$ (d) Along $-\hat{k}$ for $z > 0$ and along $\hat{k}$ for $z < 0$

23. A long cylindrical kept along z-axis carries a current density $\hat{J} = J_0 r\hat{k}$, where J_0 is a constant and r is the radial distance form the axis of the cylinder. The magnetic induction $\hat{B}$ inside the conductor at a distance d from the axis of the cylinder is : **[GATE 2006]**

(a) $\mu_0 J_0 \hat{\phi}$ (b) $\frac{\mu_0 J_0 d}{2}\hat{\phi}$ (c) $\frac{\mu_0 J_0 d^2}{3}\hat{\phi}$ (d) $\frac{\mu_0 J_0 d^3}{4}\hat{\phi}$

24. The magnetic field (in A m^{-1}) inside a long solid cylindrical conductor of radius a = 0.1 *m* is,
$H = \frac{10^4}{r}\left[\frac{1}{\alpha^2}\sin(\alpha r) - \frac{r}{\alpha}\cos(\alpha r)\right]\hat{\phi}$, where $\alpha = \frac{\pi}{2a}$. What is the total current (in A) in the conductor?

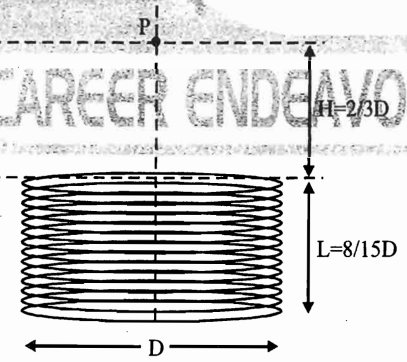
(a) $\frac{\pi}{2a}$ (b) $\frac{800}{\pi}$ (c) $\frac{400}{\pi}$ (d) $\frac{300}{\pi}$ **[GATE 2009]**

25. A ring of radius R carries a linear charge density λ. It is rotating with angular speed ω. The magnetic field at its centre is

(a) $\frac{3\mu_0\lambda\omega}{2}$ (b) $\frac{\mu_0\lambda\omega}{2}$ (c) $\frac{\mu_0\lambda\omega}{\pi}$ (d) $\mu_0\lambda\omega$

26. A short solenoid with n turns per unit length has diameter D and length L = 8D/15, as shown in the figure, and it carries a constant current I. The magnetic field B at a point P on the axis of the solenoid at a distance H = 2D/3 from its near end (see figure) is **[TIFR 2014]**

[use $\int dx\left(1+x^2\right)^{-3/2} = x\left(1+x^2\right)^{-1/2}$]



(a) $\frac{4}{65}\mu_0 nI$ (b) $\frac{4}{13}\mu_0 nI$ (c) $\frac{24}{15}\mu_0 nI$ (d) $\frac{112}{65}\mu_0 nI$

27. Three infinitely long wires are placed equally apart on the circumference of a circle of radius a, perpendicular to its plane. Two of the wires carry current I each, in the same direction, while the third carries current 2I along the direction opposite to the other two. The magnitude of the magnetic induction $\vec{B}$ at a distance r from the centre of the circle, for r > a, is :

(a) 0 (b) $\frac{2\mu_0}{\pi}\frac{I}{r}$ (c) $-\frac{2\mu_0}{\pi}\frac{I}{r}$ (d) $\frac{2\mu_0}{\pi}\frac{Ia}{r^2}$

28. A very long solenoid with n turns per unit length carries a current I. The magnetic field at a point, which is on its axis and its end face, is :

(a) $\mu_0 nI$ (b) $(2/3)\mu_0 nI$ (c) $(1/3)\mu_0 nI$ (d) $(1/2)\mu_0 nI$

29. **STATEMENT I :**

The work done by the magnetic force on a moving charged particle is zero.

STATEMENT II :

The magnetic force tends to change the direction of a moving charged particle in a direction perpendicular to both the magnetic field and the instantaneous velocity of the particle.

(a) Statements I and II both are ture. Statement II is the correct explanation for statement I.
(b) Statements I and II both are true. Statement II is not the correct explanation for statement I.
(c) Statement I is true but statement II is false.
(d) Statement I is false but statement II is true

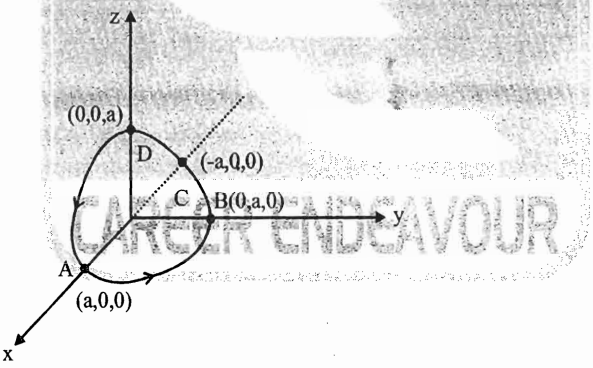
30. A particle of mass m carrying charge q is moving in a circle in a magnetic field B. According to Bohr's model, the energy of the particle in the n^{th} level is

(a) $\frac{1}{n^2}\left(\frac{hqB}{\pi m}\right)$ (b) $n\left(\frac{hqB}{\pi m}\right)$ (c) $n\left(\frac{hqB}{2\pi m}\right)$ (d) $n\left(\frac{hqB}{4\pi m}\right)$

31. Electric current of uniform current density is flowing through an infinitely long wire with a circular cross-section of constant radius R. Which of the following relations gives the magnitude B of the magnetic field measured inside the wire at a distance r from the central axis of the wire (so that r < R)?

(a) $B \propto r$ (b) $B \propto \ln r$ (c) $B \propto 1/r$ (d) $B \propto 1/r^2$

32. In the following figure, ABC and CDA are two semicircular conducting wires of radius a with the origin as their common centre. The arc ABC lies on the xy-plane whereas CDA lies on the xz-plane. They are joined as shown. This combined loop carries a current in the direction indicated in the figure below. Which of the following unit vectors denotes the direction of the resulting magnetic field vector at the origin?



(a) $(1,0,0)$ (b) $\left(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (c) $\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$ (d) $(-1,0,0)$

33. A wire of circular cross section of radius R carries a volume current density $\overline{J} = kr^2\hat{z}$ along the length of the wire. The magnetic field at a position r < R is :

(a) $\frac{\mu_0}{2}kr^3\hat{z}$ (b) $\frac{\mu_0}{4}k\,r^3\hat{\phi}$ (c) $\frac{\mu_0}{4\pi}k\,r^3\hat{r}$ (d) $\frac{\mu_0}{4\pi}k\,r^2\hat{\phi}$

34. Three infinitely long parallel wires separated by a distance d from each other, carrying current I each are kept in the same plane. The magnetic field is found to be zero at a distance x on both sides of the central wire. The values of x are :

(a) $\pm d/2$ (b) $\pm d/\sqrt{2}$ (c) $\pm d/\sqrt{3}$ (d) $\pm d/3$

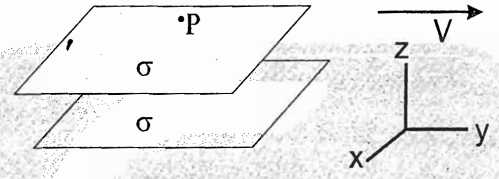
35. Magnetic induction $|B|$ due to current I in a coil of radius R placed in y-z plane at a distance X along the axis of the coil is given by $\frac{\mu_0 I}{2}\frac{R^2}{\left(R^2+X^2\right)^{3/2}}$. Another similar coil of radius R, carrying equal current I is placed in the y-z plane at a distance 2R from the first coil along the same axis. The value of $|B|$ at a point midway between the two coils and on the x-axis is:

(a) $\frac{\mu_0 I}{2\sqrt{2}\,R}$ (b) $\frac{\mu_0 I\,R}{2\sqrt{2}}$ (c) 0 (d) $\frac{\mu_0 I}{4\sqrt{2}\,R}$

36. An electron moves with velocity $\hat{v}=\hat{j}.v_0$ under the combined influence of $\vec{E}=\hat{i}E_0$ and $\vec{B}=-\hat{k}B_0$ fields. The force on the electron is in the

(a) Y direction (b) Z direction (c) YZ plane (d) X direction

37. Two large parallel plates move with a constant speed v in the positive y-direction as shown in the figure. If both the plates have a surface charge density $\sigma>0$, the magnetic field at the point P just above the top plate will have :



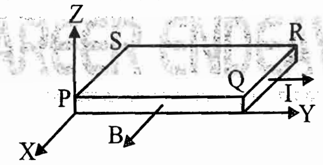
(a) Larger magnitude than the field at the mid-point between the plates and point towards $-\hat{x}$

(b) Smaller magnitude than the field at the mid-point between the plates and point towards $+\hat{x}$.

(c) Larger magnitude than the field at the mid-point between the plates and point towards $+\hat{x}$

(d) Smaller magnitude than the field at the mid-point between the plates and point towards $-\hat{x}$

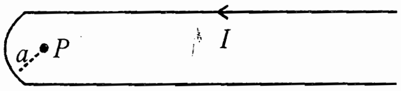
38. A conducting slab of copper PQRS is kept on the *xy* plane in a uniform magnetic field along *x*-axis as indicated in the figure. A steady current *I* flows through the cross section of the slab along the *y*-axis. The direction of the electric field inside the slab, arising due to the applied magnetic field is along the



(a) negative Y direction (b) positive Y direction

(c) negative Z direction (d) positive Z direction

39. An infinite conducting wire, which is bent in the shape of a hairpin (the curved part being a semicircle of radius *a*) as shown in the figure, carries a current *I*.



(a) 1.3 gauss (b) 7.1 gauss (c) 31.4 gauss (d) 0.5 gauss

40. The strength of magnetic field at the center of a regular hexagon with sides of length *a* carrying a steady current *I* is : **[JEST 2016]**

(a) $\frac{\mu_0 I}{\sqrt{3}\pi a}$ (b) $\frac{\sqrt{6}\mu_0 I}{\pi a}$ (c) $\frac{3\mu_0 I}{\pi a}$ (d) $\frac{\sqrt{3}\mu_0 I}{\pi a}$

CSIR-UGC-NET/JRF | GATE-PHYSICS

ELECTROMAGNETIC THEORY

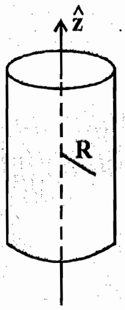
Assignment-5 : Vector Potential and Magnetisation

1. For constant uniform electric and magnetic fields $\vec{E}=\vec{E}_0$ and $\vec{B}=\vec{B}_0$, it is possible to choose a gauge such that the scalar potential ϕ and vector potential $\vec{A}$ are given by **[NET June 2011]**

(a) $\phi=0$ and $\vec{A}=\frac{1}{2}\left(\vec{B}_0\times\vec{r}\right)$ (b) $\phi=-\vec{E}_0.\vec{r}$ and $\vec{A}=\frac{1}{2}\left(\vec{B}_0\times\vec{r}\right)$

(c) $\phi=-\vec{E}_0.\vec{r}$ and $\vec{A}=0$ (d) $\phi=0$ and $\vec{A}=-\vec{E}_0 t$

2. An infinite solenoid with its axis of symmetry along the z-direction carries a steady current I.



The vector potential $\vec{A}$ at a distance R from the axis. **[NET Dec. 2012]**
(a) Is constant inside and varies as R outside the solenoid.
(b) Varies as R inside and is constant outside the solenoid.
(c) Varies as 1/R inside and as R outside the solenoid.
(d) Varies as R inside and as 1/R outside the solenoid.

3. The vector potential $\vec{A}$ due to a magnetic moment 'm' at a point 'r' is given by $\vec{A}=\frac{\vec{m}\times\vec{r}}{r^3}$. If $\vec{m}$ is directed along the positive z-axis, the x-component of the magnetic field, at the point r, is **[NET June 2012]**

(a) $\frac{3myz}{r^5}$ (b) $-\frac{3mxy}{r^5}$ (c) $\frac{3mxz}{r^5}$ (d) $\frac{3m\left(z^2-xy\right)}{r^5}$

4. The magnetic field corresponding to the vector potential, $\overline{A}=\frac{1}{2}\overline{F}\times\overline{r}+\frac{10}{r^3}\overline{r}$ where $\overline{F}$ is a constant vector, is

(a) $\overline{F}$ (b) $-\overline{F}$ (c) $\overline{F}+\frac{30}{r^4}\overline{r}$ (d) $\overline{F}-\frac{30}{r^4}\overline{r}$ **[NET June 2012]**

5. A thin infinitely long solenoid placed along the z-axis contains a magnetic flux ϕ. Which of the following vector potentials corresponds to the magnetic field at an arbitrary point (x,y,z)? **[NET June 2014]**

(a) $\left(A_x,A_y,A_z\right)=\left(-\frac{\phi}{2\pi}\frac{y}{x^2+y^2},\frac{\phi}{2\pi},\frac{x}{x^2+y^2},0\right)$

(b) Only $\vec{A}_2$ produces the magnetic field $\vec{B}$ but not $\vec{A}_1$

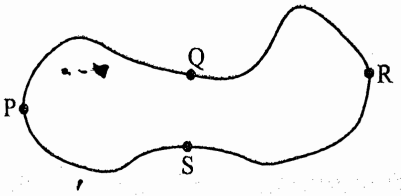
(c) Neither $\vec{A}_1$ nor $\vec{A}_2$ produces the magnetic field $\vec{B}$

(d) Both $\vec{A}_1$ and $\vec{A}_2$ produce the magnetic field $\vec{B}$.

20. There is a uniform magnetic field B in the z-direction in a certain region of space. The corresponding vector potential is given by.

(a) $\vec{A} = Bx\,\vec{j}$ (b) $\vec{A} = \frac{B\vec{r}}{r^2}$ (c) $\vec{A} = B\vec{r}$ (d) $\vec{A} = B\left(x\vec{i} + y\vec{k}\right)$

21. Given that magnetic flux through the closed loop PQRSP is ϕ. If $\int_P^R \vec{A}\cdot\vec{dl} = \phi_1$ along PQR, the value of $\int_P^R \vec{A}\cdot\vec{dl}$ along PSR is **[GATE 2015]**



(a) $\phi - \phi_1$ (b) $\phi_1 - \phi$ (c) $-\phi_1$ (d) ϕ_1

22. Given a uniform magnetic field $B = B_0\hat{k}$ (where B_0 is a constant), a possible choice for the magnetic vector potential A is **[NET Dec. 2015]**

(a) $B_0 y\hat{i}$ (b) $-B_0 y\hat{i}$ (c) $B_0(x\hat{j} + y\hat{i})$ (d) $B_0(x\hat{i} - y\hat{j})$

23. A dipole of moment $\vec{p}$, oscillating at frequency ω, radiates spherical waves. The vector potential at large distance is $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} i\omega \frac{e^{ikr}}{r}\vec{p}$. To order $(1/r)$ the magnetic field $\vec{B}$ at a point $\vec{r} = r\hat{n}$ is

(a) $-\frac{\mu_0}{4\pi}\frac{\omega^2}{c}(\hat{n}\cdot\vec{p})\hat{n}\frac{e^{ikr}}{r}$ (b) $-\frac{\mu_0}{4\pi}\frac{\omega^2}{c}(\hat{n}\times\vec{p})\frac{e^{ikr}}{r}$ **[NET Dec. 2015]**

(c) $-\frac{\mu_0}{4\pi}\omega^2 k(\hat{n}\cdot\vec{p})\vec{p}\frac{e^{ikr}}{r}$ (d) $-\frac{\pi_0}{4\pi}\frac{\omega}{c}p\frac{e^{ikr}}{r}$

24. Which of the following magnetic vector potentials gives rise to a uniform magnetic field $B_0\hat{k}$?

(a) $B_0 z\hat{k}$ (b) $-B_0 x\hat{j}$ **[GATE 2016]**

(c) $\frac{B_0}{2}(-y\hat{i} + x\hat{j})$ (d) $\frac{B_0}{2}(y\hat{i} + x\hat{j})$

25. A current carrying loop lying in the plane of the paper is in the shape of an equilateral triangle of side a. It carries a current I in the clockwise sense. If $\hat{k}$ denotes the outward normal to the plane of the paper, the magnetic moment $\vec{m}$ due to the current loop is: **[NET 2008]**

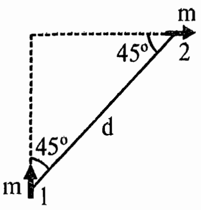
(a) $\vec{m} = a^2 I\,\hat{k}$ (b) $\vec{m} = -\frac{1}{2}a^2 I\,\hat{k}$ (c) $\vec{m} = \frac{\sqrt{3}}{2}a^2 I\,\hat{k}$ (d) $\vec{m} = -\frac{\sqrt{3}}{2}a^2 I\,\hat{k}$

26. A magnetic dipole of dipole moment $\vec{m}$ is placed in a non-uniform field $\vec{B}$. If the position vector of the dipole is $\vec{r}$, the torque acting on the dipole about the origin is: **[GATE 2011]**

(a) $\vec{r}\times\left(\vec{m}\times\vec{B}\right)$ (b) $\vec{r}\times\vec{\nabla}\left(\vec{m}.\vec{B}\right)$ (c) $\vec{m}\times\vec{B}$ (d) $\vec{m}\times\vec{B}+\vec{r}\times\vec{\nabla}\left(\vec{m}.\vec{B}\right)$

27. The magnitude of the magnetic dipole moment associated with a square shaped loop carrying a steady current I is m. If this loop is changed to a circular shape with the same current I passing through it, the magnetic dipole moment becomes $\frac{pm}{\pi}$. The value of p is ____________

28. Two magnetic dipoles of magnitude m each are placed in a plane as shown



The energy of interaction is given by **[GATE 2010]**

(a) Zero (b) $\frac{\mu_0}{4\pi}\frac{m^2}{d^3}$ (c) $\frac{3\mu_0}{2\pi}\frac{m^2}{d^3}$ (d) $\frac{3\mu_0}{8\pi}\frac{m^2}{d^3}$

29. At a surface current, which one of the magnetostatic boundary condition is NOT CORRECT? **[GATE 2013]**
(a) Normal component of the magnetic field is continuous.
(b) Normal component of the magnetic vector potential is continuous.
(c) Tangential component of the magnetic vector potential is continuous.
(d) Tangential component of the magnetic vector potential is not continuous.

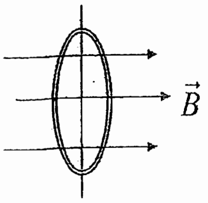
30. A dipole of magnetic moment $\vec{m}$ is placed in an inhormogeneous magnetic field $\vec{B}$. The force experienced by the dipole is :

(a) $\vec{F}=\vec{m}\times\left(\vec{\nabla}\times\vec{B}\right)$ (b) $\vec{F}=\left(\vec{m}.\vec{\nabla}\right)\vec{B}$ (c) $\vec{F}=\left(\vec{\nabla}.\vec{m}\right)\vec{B}$ (d) $\vec{F}=\vec{\nabla}\times\left(\vec{m}\times\vec{B}\right)$

31. A circular coil of radius 120 mm with 10 turns is placed with its plane parallel to the earth's magnetic field. When a current of 0.45 A flows through the coil, a compass needle placed at the center of the coil is seen to be deflected by an angle of 45° to the plane of the coil. The intensity of the earth's magnetic field is calculated to be

(a) 0.3 μT (b) 24 μT (c) 42 μT (d) 100 μT

32. A loop of wire of radius a and carrying a current I is suspended vertically in a uniform horizontal magnetic field B as shown. Calculate the work done in rotating the loop around the vertical axis through an angle π



(a) $\frac{\pi^2 I^2}{2a}$ (b) $2\pi a^2 I$ (c) $\frac{\pi a^2 I}{2}$ (d) 0

33. A small magnetic needle is kept at (0, 0) with its moment along the x-axis. Another small magnetic needle is at the point (1, 1) and is free to rotate in the xy-plane. In equilibrium the angle θ between their magnetic moments is such that **[NET Dec. 2015]**

(a) $\tan\theta = 1/3$ (b) $\tan\theta = 0$ (c) $\tan\theta = 3$ (d) $\tan\theta = 1$

34. A spherical shell of radius R carries a constant surface charge density σ and is rotating about one of its diameters with an angular velocity ω. The magnitude of the magnetic moment of the shell is: **[JEST 2016]**

(a) $4\pi\sigma\omega R^4$ (b) $4\pi\sigma\omega R^4/3$ (c) $4\pi\sigma\omega R^4/15$ (d) $4\pi\sigma\omega R^4/9$

35. A circular loop of fine of radius R carrying a current I is placed in a uniform magnetic field B perpendicular to the plane of the loop. If the breaking tension of the wire is T_b, the wire break when the magnetic field exceeds

(a) $\dfrac{T_b}{IR}$ (b) $\dfrac{T_b}{2\pi IR}$ (c) $\dfrac{\mu_0 T_b}{2\pi IR}$ (d) $\dfrac{\mu_0 T_b}{4\pi IR}$ **[TIFR 2016]**

36. The relation between $\vec{H}.\vec{B}$ and $\vec{M}$ representing the magnetic intensity, magnetic field and magnetic dipole moment vectors, respectively, is

(a) $\vec{H} = -(\mu_0\vec{M} + \vec{B})$ (b) $\vec{H} = \mu_0\vec{M} - \vec{B}$ (c) $\vec{H} = \vec{B}/\mu_0 - \vec{M}$ (d) $\vec{H} = \vec{B} + \mu_0\vec{M}$

37. According to Kirchoff's laws for the circuits, the sum of the currents at any junction is equal to zero. Which of the following equations for the current density $\vec{j}$ describes this situation?

(a) $\vec{\nabla}\times\vec{j} = 0$ (b) $\vec{\nabla}.\vec{j} = 0$ (c) $\nabla^2\vec{j} = 0$ (d) $\partial\vec{j}/\partial t = 0$

38. Two small circular loops, each of area 1 cm^2 carry currents 1 A and 2 A respectively. They are placed in a plane at a distance of 5 meters from each other. If one of the loops is lifted 5 meters in the vertical direction while maintaining it flat, the electrical work done in the process is (μ_0 is the magnetic permeability of vacuum)

(a) $2.75\times10^{-9}\,\mu_0^2\,J$ (b) $1.46\times10^{-10}\,\mu_0^2\,J$

(c) $1.88\times10^{-10}\,\mu_0^2\,J$ (d) $4.74\times10^{-9}\,\mu_0^2\,J$

39. The magnetic induction $\vec{B}$ and the field $\vec{H}$ inside a bar magnet are

(a) parallel to the bar, and opposite to each other with $\vec{B}$ directed from south to north pole

(b) parallel to each other along the bar, but their magnitude differ by a factor of permeability

(c) parallel to the bar, and opposite to each other with $\vec{B}$ directed from north to south pole

(d) parallel to each other along the bar with equal magnitude

40. In a conducting medium, the magnetic field is given by $H = y^2z\hat{i} + 2(x+1)yz\hat{j} + (x+1)yz^2\hat{k}$ A/m. The current enclosed by the square loop $y = 1, 0 \le x \le 1, 0 \le z \le 1$ is

(a) $\dfrac{2}{3}A$ (b) $\dfrac{4}{3}A$ (c) $\dfrac{11}{6}A$ (d) $\dfrac{5}{2}A$

41. Given a uniform magnetic field $\boldsymbol{B} = B_0\hat{k}$ (where B_0 is a constant), a possible choice for the magnetic vector potential $\boldsymbol{A}$ is **[NET Dec. 2015]**

(a) $B_0 y\hat{i}$ (b) $-B_0 y\hat{i}$ (c) $B_0(x\hat{j} + y\hat{i})$ (d) $B_0(x\hat{i} - y\hat{j})$

42. Consider the operations $P:\vec{r}\rightarrow-\vec{r}$ (parity) and $T:t\rightarrow-t$ (time-reversal). For the electric and magnetic field $\vec{E}$ and $\vec{B}$, which of the following set of transformations is correct? **[GATE 2010]**

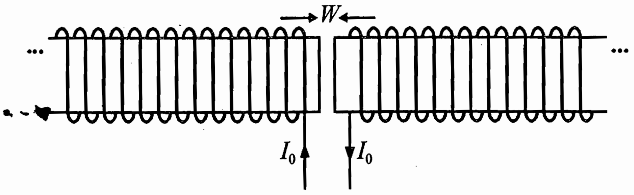
(a) $P:\vec{E}\rightarrow-\vec{E},\vec{B}\rightarrow\vec{B};$ $T:\vec{E}\rightarrow\vec{E},\vec{B}\rightarrow-\vec{B}$

(b) $P:\vec{E}\rightarrow\vec{E},\vec{B}\rightarrow\vec{B};$ $T:\vec{E}\rightarrow\vec{E},\vec{B}\rightarrow\vec{B}$

(c) $P:\vec{E}\rightarrow\vec{E},\vec{B}\rightarrow\vec{B};$ $T:\vec{E}\rightarrow-\vec{E},\vec{B}\rightarrow-\vec{B}$

(d) $P:\vec{E}\rightarrow\vec{E},\vec{B}\rightarrow-\vec{B};$ $T:\vec{E}\rightarrow-\vec{E},\vec{B}\rightarrow\vec{B}$

43. Two semi-infinite solenoids placed next to each other are separated by a small gap of width W as shown in the figure. **[TIFR 2015]**



The current I_0 in the solenoids flows in the direction as shown. If the solenoids have a circular cross-section of radius R and are filled with a magnetic material of permeability $\mu(\mu>\mu_0)$, then the magnetic energy densities u_i inside the solenoid and u_g in the gap are best related by

(a) $u_g > u_i$ (b) $u_g < u_i$ (c) $u_g = cu_i$ (d) $u_g > cu_i$

44. Choose the correct statement from the following:

(a) The magnetic field $\vec{B}$ is a vector and not a pseudovector.

(b) $\vec{E}.\vec{B}$ is a scalar and not a pseudoscalar.

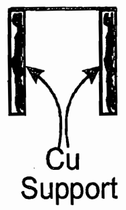
(c) $\vec{E}\times\vec{B}$ is a pseudovector.

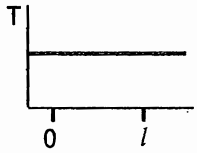
(d) The magnetic vector potential $\vec{A}$ is a pseudovector.

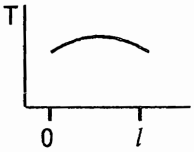
45. For the magnetic field measurement, Gauss-meters employ the principle of

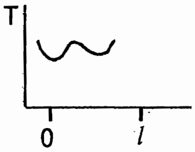
(a) double exchange (b) super exchange (c) Jahn-Teller effect (d) Hall effect

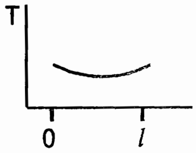
46. A tungsten wire of uniform cross section and high resistance is supported by two copper supports of low resistance in vacuum. The length of the wire is l. A constant current I is sent through the wire. The temperature profile (T vs. x) of the wire will look like.



(a)


(b)


(c)


(d)


47. The x-y plane is the boundary between free space and a magnetic material with relative permeability μ_r. The magnetic field in the free space is $B_x\hat{i} + B_z\hat{k}$. The magnetic field in the magnetic material is

(a) $B_x\hat{i} + B_z\hat{k}$ (b) $B_x\hat{i} + \mu_r B_z\hat{k}$ **[GATE 2016]**

(c) $\frac{1}{\mu_r}B_x\hat{i} + B_z\hat{k}$ (d) $\mu_r B_x\hat{i} + B_z\hat{k}$

48. A small magnetic dipole is kept at the origin in the x-y plane. One wire L_1 is located at $z = -a$ in the x-z plane with a current I flowing in the positive x direction. Another wire L_2 is at $z = +a$ in y-z plane with the same current I as in L_1, flowing in the positve y direction. The angle φ made by the magnetic dipole with respect to the positive x-axis is:

(a) 225° (b) 120° (c) 45° (d) 270°

49. A vector field is given by $\vec{A} = y\hat{i} + z\hat{j} + x\hat{k}$. The curl of the vector is:

(a) $\hat{i} + \hat{j} - \hat{k}$ (b) $-\hat{i} - \hat{j} - \hat{k}$ (c) $-\hat{i} - \hat{j} + \hat{k}$ (d) $\hat{i} + \hat{j} + \hat{k}$

50. A loop of radius a, carrying a current I, is placed in a uniform magnetic field $\boldsymbol{B}$. If the normal to the loop is denoted by $\hat{n}$, the force $\boldsymbol{F}$ and the torque $\boldsymbol{T}$ on the loop are

(a) $\boldsymbol{F} = 0$ and $\boldsymbol{T} = \pi a^2 I\,\hat{n} \times \boldsymbol{B}$ (b) $\boldsymbol{F} = \frac{\mu_0}{4\pi} I \times \boldsymbol{B}$ and $\boldsymbol{T} = 0$

(c) $\boldsymbol{F} = \frac{\mu_0}{4\pi} I \times \boldsymbol{B}$ and $\boldsymbol{T} = I\,\hat{n} \times \boldsymbol{B}$ (d) $\boldsymbol{F} = 0$ and $\boldsymbol{T} = \frac{1}{\mu_0\varepsilon_0} I\boldsymbol{B}$

CSIR-UGC-NET/JRF|GATE-PHYSICS

ELECTROMAGNETIC THEORY

Assignment-6 : Electromagntic Theory

1. The magnetic flux density in a region is given by $B=\left(2\hat{i}+3\hat{j}-4\hat{k}\right)e^{-2t}\text{Wb/m}^2$. A square loop of 2m side is placed in the region, with its center at the origin and its sider along the co-ordinate axes. The emf induced in the loop at $t=1$ sec is
(a) 1.35 volts (b) 3.24 volts (c) 4.33 volts (d) 5.41 volts

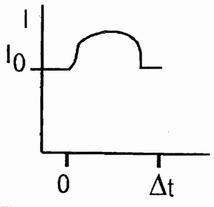
2. A uniform magnetic field in the positive z-direction passes through a circular wire loop of radius 1 cm and resistance 1 Ω lying in the xy-plane. The field strength is reduced from 10 tesla to 9 tesla in 1 s. The charge transferred across any point in the wire is approximately **[NET June 2015]**
(a) 3.1×10^{-4} coulomb (b) 3.4×10^{-4} coulomb
(c) 4.2×10^{-4} coulomb (d) 5.2×10^{-4} coulomb

3. A long solenoid is embedded in a conducting medium and is insulated from the medium. If the current through the solenoid is increased at a constant rate, the induced current in the medium as a function of the radial distance r from the axis of the solenoid is proportional to **[GATE 2015]**
(a) r^2 inside the solenoid and $\frac{1}{r}$ outside (b) r inside the solenoid and $\frac{1}{r^2}$ outside
(c) r^2 inside the solenoid and $\frac{1}{r^2}$ outside (d) r inside the solenoid and $\frac{1}{r}$ outside

4. A small loop of wire of area $A=0.01\text{ m}^2$, $N=40$ turns and resistance $R=20\Omega$ is initially kept in a uniform magnetic field B in such a way that the field is normal to the plane of the loop. When it is pulled out of the magnetic field, a total charge of $Q=2\times10^{-5}C$ flows through the coil. The magnitude of the field B is:
(a) $1\times10^{-3}T$ (b) $4\times10^{-3}T$
(c) Zero (d) Unobtainable, as the data is insufficient

5. A metal rod AB rotates clockwise in the plane of the paper about the end B with an angular velocity ω in a uniform magnetic field emerging out of the plane of the paper normally. The end B about which the rod rotates will be
(a) Positive with respect to the end A
(b) Negative with respect to the end A.
(c) Having same potential as the remaining part of the rod.
(d) Having same potential as the end B, but the centre of the rod will be at higher potential.

6. A solenoid with an iron core is connected in series with a battery of emf V and it is found that a constant current I_0 passes through the solenoid. If at $t=0$, the iron core is pulled out from the solenoid quickly in a time Δt, which one of the following could be a correct description of the current passing through the solenoid? **[GATE 2000]**

(a)


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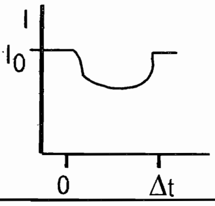

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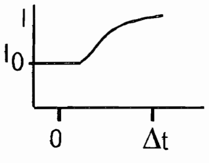

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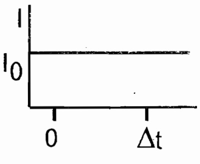
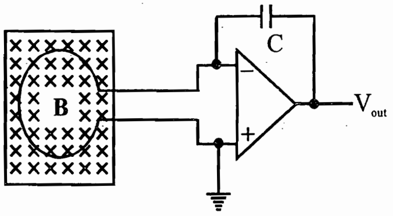


7. Consider a conducting loop of radius a and total loop resistance R placed in a region with a magnetic field B thereby enclosing a flux ϕ_0. The loop is connected to an electronic circuit as shown, the capacitor being initially uncharged. If the loop is pulled out of the region of the magnetic field at a constant speed 'u', the final output voltage V_{out} is independent of **[GATE 2010]**

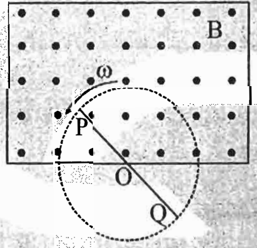


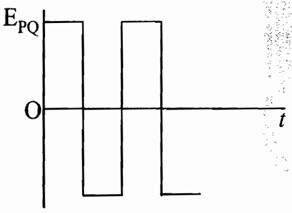
(a) ϕ_0 (b) u (c) R (d) C

8. A circular conducting ring of radius 'R' rotates with constant angular velocity ω about its diameter placed along the x-axis. A uniform magnetic field B is applied along the y-axis. If at time t = 0 the ring is entirely in the xy- plane, the emf induced in the ring at time t > 0 is **[JEST 2012]**

(a) $B\omega^2\pi R^2 t$ (b) $B\omega\pi R^2\tan(\omega t)$ (c) $B\omega\pi R^2\sin(\omega t)$ (d) $B\omega\pi R^2\cos(\omega t)$

9. A uniform and costant magnetic field B coming out of the plane of the paper exists in a rectangular region as shown in the figure. A conducting rod PQ is rotated about O with a uniform angular speed ω in the plane of the paper. The emf E_{PQ} induced between P and Q is best represented by the graph.



(a)


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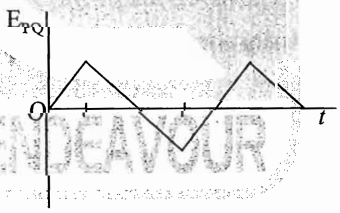

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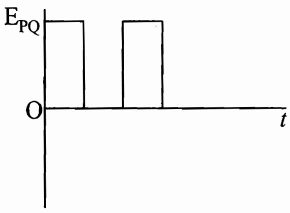

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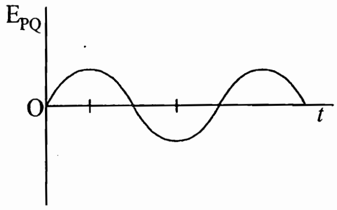


10. Which of the following vector fields can represent an electrostatic field?

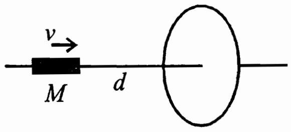
(a) $(2xz-y^2)\hat{i}+(2xy-z^2)\hat{j}+(2yz-x^2)\hat{k}$

(b) $xy\hat{i}+2yz\hat{j}+3xz\hat{k}$ (c) $yz\hat{i}-xz\hat{j}+xy\hat{k}$

(d) $y^2\hat{i}+(2xy+z^2)\hat{j}+(2yz+x^2)\hat{k}$

(e) $y^2\hat{i}+(2xy+z^2)\hat{j}+2yz\hat{k}$

11. A bar magnet, of magnetic moment M, is moving with a constant velocity v along the axis of a circular coil of radius R. The direction of the magnetic moment of the bar magnet is parallel to its velocity as shown in the figure below



What is the magnitude of the e.m.f. induced in the coil at an instant when the magnet is at a distance d (where $d >> R$) from it?

(a) $\frac{\mu_0 R^2 Mv}{2d^3}$ (b) $\frac{3\mu_0 R^2 Mv}{2d^4}$ (c) $\frac{\mu_0 R^2 Mv}{4d^2}$ (d) $\frac{\mu_0 R^2 Mv}{4d^4}$ (e) $\frac{3\mu_0 R^2 Mv}{4d^3}$

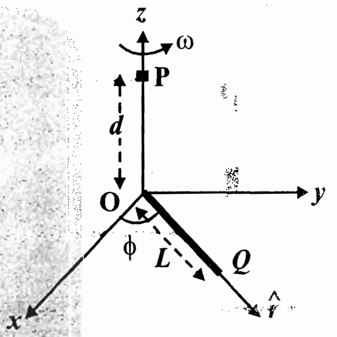
12. A rod of length L with uniform charge density λ per unit length is in the xy-plane and rotating about z-axis passing through one of its edge with an angular velocity $\vec{\omega}$ as shown in the figure below. $(\hat{r}, \hat{\phi}, \hat{z})$ refer to the unit vectors at Q, $\vec{A}$ is the vector potential at a distance d from the origin O along z-axis for $d << L$ and $\vec{J}$ is the current density due to the motion of the rod. **[GATE 2008]**
Which one of the following statements is correct?

(a) $\vec{J}$ along $\hat{r}$; $\vec{A}$ along $\hat{z}$; $|\vec{A}| \propto \frac{1}{d}$

(b) $\vec{J}$ along $\hat{\phi}$; $\vec{A}$ along $\hat{\phi}$; $|\vec{A}| \propto \frac{1}{d^2}$

(c) $\vec{J}$ along $\hat{r}$; $\vec{A}$ along $\hat{z}$; $|\vec{A}| \propto \frac{1}{d^2}$

(d) $\vec{J}$ along $\hat{\phi}$; $\vec{A}$ along $\hat{\phi}$; $|\vec{A}| \propto \frac{1}{d}$



13. A magnetic field $\boldsymbol{B}$ is $B\hat{z}$ in the region $x > 0$ and zero elsewhere. A rectangular loop, in the xy-plane, of sides l (along the x-direction) and h (along the y-direction) is inserted into the $x > 0$ region from the $x < 0$ region at a constant velocity $\mathbf{v} = v\hat{x}$. Which of the following values of l and h will generate the largest EMF ?

(a) $l = 8,\ h = 3$ (b) $l = 4,\ h = 6$ (c) $l = 6,\ h = 4$ (d) $l = 12,\ h = 2$

14. A thin and straight metallic rod of length L is rotating about its middle point with angular velocity ω in a uniform magnetic field of strength B. The axis of rotation is perpendicular to the length of the rod and parallel to the direction of the magnetic field. The strength of the induced electric field is
(a) constant throughout the rod
(b) maximumat the two points halfway between the centre and the two ends
(c) minimum at the two points halfway between the centre and the two ends
(d) maximum at the ends and minimum at the centre.

15. A small magnet is dropped down a long vertical copper tube in a uniform gravitational field. After a long time, the magnet **[JEST 2012]**
(a) Attains a constant velocity (b) moves with a constant acceleration
(c) moves with a constant deceleration (d) executes simple harmonic motion.

Statement for linked Answer Q.16 and Q.17:

An infinitely long hollow cylinder of radius R carrying a surface charge density s is rotated about its cylinderical axis with a constant angular speed w

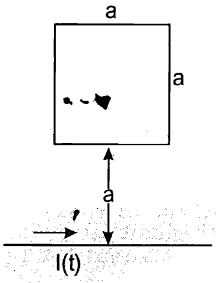
16. The magnitude of the surface current is : **[GATE 2005**

(a) $\sigma R\omega$ (b) $2\sigma R\omega$ (c) $\pi\sigma R\omega$ (d) $2\pi\sigma R\omega$

17. The magnitude of vector potential inside the cylinder at a distance from its axis is : **[GATE 2005]**

(a) $2\mu_0\sigma R\omega r$ (b) $\mu_0\sigma R\omega r$ (c) $\frac{1}{2}\mu_0\sigma R\omega r$ (d) $\frac{1}{4}\mu_0\sigma R\omega r$

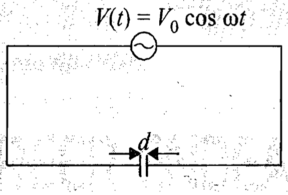
18. An infinitely long wire carrying a current $I(t) = I_0 \cos(wt)$ is placed at a distance a from a square loop of side a as shown in the figure. If the resistance of the loop is R, then the amplitude of the induced current in the loop is : **[GATE 2005]**



(a) $\frac{\mu_0}{2\pi}\frac{aI_0\omega}{R}\ln 2$ (b) $\frac{\mu_0}{\pi}\frac{aI_0\omega}{R}\ln 2$

(c) $\frac{2\mu_0}{\pi}\frac{aI_0\omega}{R}\ln 2$ (d) $\frac{\mu_0}{2\pi}\frac{aI_0\omega}{R}$

19. An oscillating voltage $V(t) = V_0 \cos\omega t$ is applied across a parallel plate capacitor having a plate separation d. The displacement current density through the capacitor is:



(a) $\frac{\varepsilon_0\omega V_0\cos\omega t}{d}$ (b) $\frac{\varepsilon_0\mu_0\omega V_0\cos\omega t}{d}$ (c) $-\frac{\varepsilon_0\mu_0\omega V_0\sin\omega t}{d}$ (d) $-\frac{\varepsilon_0\omega V_0\sin\omega t}{d}$

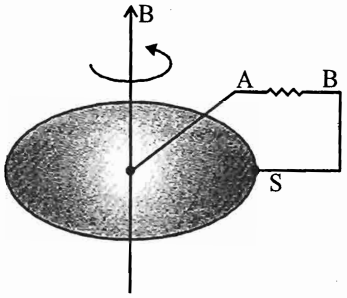
20. A thin conducting wire is bent into a circular loop of radius r and placed in a time dependent magnetic field of magnetic induction. **[GATE 2001]**

$$\vec{B}(r) = B_0 e^{-\alpha t}\hat{e}_z, (B_0 > 0 \text{ and } \alpha > 0),$$

such that, the plane of the loop is perpendicular to $\vec{B}(t)$. Then the induced emf in the loop is :

(a) $\pi r^2\alpha B_0 e^{-\alpha t}$ (b) $\pi r^2 B_0 e^{-\alpha t}$ (c) $-\pi r^2\alpha B_0 e^{-\alpha t}$ (d) $-\pi r^2 B_0 e^{-\alpha t}$

21. A horizontal metal disc rotates about the vertical axis in a uniform magnetic field pointing up as shown in the figure. A circuit is made by connecting one end A of a resistor to the centre of the disc and the other end B to its edge through a sliding contact S. The current that flows through the resistor is



(a) zero
(b) DC from A to B **[NET Dec. 2013]**
(c) DC from B to A
(d) AC

22. Which of the following expressions represents an electric field due to a time varying magnetic field ?
(a) $K(x\hat{x}+y\hat{y}+z\hat{z})$
(b) $K(x\hat{x}+y\hat{y}-z\hat{z})$ **[JEST 2015]**
(c) $K(x\hat{x}-y\hat{y})$
(d) $K(y\hat{y}-x\hat{y}+2z\hat{z})$

23. A charged particle is released at time $t=0$, from the origin in the presence of uniform static electric and magnetic fields given by $\mathbf{E}=E_0\hat{y}$ and $\mathbf{B}=B_0\hat{z}$, respectively. Which of the following statements is true for $t>0$? **[JEST 2015]**
(a) The particle moves along the x-axis
(b) The particle moves in a circular orbit
(c) The particle moves in the (x, y) plane
(d) The particle moves in the (y, z) plane

24. The condition that the magnetic field $\vec{B}$ is a curl of a vector potential $\vec{A}$ should be
(a) $\vec{\nabla}.B=0$
(b) $\vec{\nabla}\times\vec{A}=0$
(c) $\vec{\nabla}\vec{A}=0$
(d) $\vec{\nabla}\times\vec{A}-\nabla^2\vec{A}=0$

25. The magnetic field $\vec{B}$ can be expressed in terms of the vector potential as $\vec{B}=\vec{\nabla}\times\vec{A}$. If $\vec{B}=B\hat{k}$ then $\vec{A}$ can be expressed as
(a) $\left(-\frac{1}{2}By, \frac{1}{2}Bx, 0\right)$
(b) $\left(\frac{1}{2}By, -\frac{1}{2}Bx, 0\right)$
(c) $(0,0,B)$
(d) $\left(\frac{Bx}{3}, \frac{By}{3}, \frac{Bz}{3}\right)$

26. An infinitely long closely would solenoid carries a sinusoidally varying current. The induced electric field is :
(a) Zero everywhere
(b) Non-zero inside and zero outside the solenoid
(c) Non-zero inside as well as outside the solenoid
(d) Zero inside and non-zero outside the solenoid

27. A time-dependent magnetic field $\vec{B}(t)$ is produced in a circular region of space, infinitely long and of radius R. The magnetic field is given as $\vec{B}=B_0t\hat{z}$ for $0\le r<R$ and is zero for $r>R$. Where B_0 is a positive constant. The electric field for $r>R$ is **[JEST 2012]**
(a) $\frac{B_0R^2}{r}\hat{r}$
(b) $\frac{B_0R^2}{2r}\hat{\theta}$
(c) $-\frac{B_0R^2}{r}\hat{r}$
(d) $-\frac{B_0R^2}{2r}\hat{\theta}$

28. A conducting loop L of surface area S is moving with a velocity $\vec{\upsilon}$ in a magnetic field $\vec{B}(\vec{r},t)=\vec{B}_0 t^2$, B_0 is a positive constant of suitable dimensions. The emf induced, V_{emf}, in the loop is given by: **[GATE 2009]**

(a) $-\int_s \frac{\partial\vec{B}}{\partial t}.d\vec{S}$ (b) $\oint_L (\vec{\upsilon}\times\vec{B}).d\vec{L}$

(c) $-\int_s \frac{\partial\vec{B}}{dt}.d\vec{S}-\oint_L (\vec{\upsilon}\times\vec{B}).d\vec{L}$ (d) $-\int_s \frac{\partial\vec{B}}{dt}.d\vec{S}+\oint_L (\vec{\upsilon}\times\vec{B}).d\vec{L}$

29. Which one of the following Maxwell's equations implies the absence of magnetic monopoles?

(a) $\nabla.E=\pi/\varepsilon_0$ (b) $\nabla.B=0$ **[GATE 2003]**

(c) $\nabla\times E=-\partial B/\partial t$ (d) $\nabla\times B=(1/c^2)\partial B/\partial t+\mu_0 J$

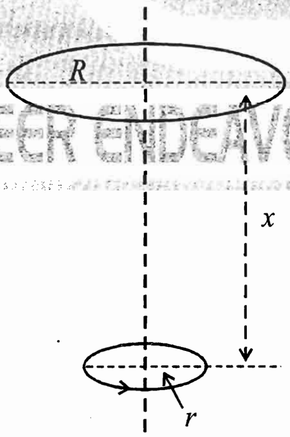
30. A large circular coil of N turns and radius R carries a time varying current $i=i_0\sin(wt)$. A small circular coil of n turns and radius r (r <<R) is placed at the center of the large coil such that the coils are concentric and coplanar. The induced emf in the small coil

(a) Leads the current in the large coil by $\pi/2$ **[GATE 2002]**
(b) Lags the current in the large coil by π
(c) Is in phase with the current in the large coil
(d) lags the current in the large coil by $\pi/2$

31. Self inductance per unit length of a long solenoid of radius R with n turns per unit length is:

(a) $\mu_0\pi R^2 n^2$ (b) $2\mu_0\pi R^2 n$ (c) $2\mu_0\pi R^2 n^2$ (d) $\mu_0\pi R^2 n$

32. Consider the following system. Two circular loops of wire are placed horizontally, having a common axis passing vertically through the centre of each coil (see figure). The lower loop has radius 'r' and carries a current 'i' as shown in the figure. The upper loop has a radius $R(R>>r)$ and is at a distance $x(x>>R)$ above it.



If the lower loop is held fixed and the upper loop moves upwards with a uniform velocity $v=\frac{dx}{dt}$, then the induced e.m.f. and the direction of the induced current in this loop will be

(a) $3i\mu_0\pi^2 r^2 R^2 v/2x^4$; anti-clockwise (b) $2i\mu_0\pi^2 r^2 R^2 v/2x^4$; clockwise

(c) $3i\mu_0\pi^2 r^2 R^2 v/2x^3$; anti-clockwise (d) $2i\mu_0\pi^2 r^2 R^2 v/2x^3$; clockwise

33. Current flowing in a circuit containing a capacitor. To satisfy the continuity equation, $\vec{\nabla}.\vec{J}=-\frac{\partial\rho}{\partial t}$, for charge and current, the current density $\vec{J}$ must be replaced by

(a) $\frac{1}{4\pi}\frac{\partial\vec{D}}{\partial t}$ (b) $\vec{J}-\frac{1}{4\pi}\frac{\partial\vec{D}}{\partial t}$ (c) $\vec{D}+\frac{1}{4\pi}\frac{\partial\vec{J}}{\partial t}$ (d) $\vec{J}+\frac{1}{4\pi}\frac{\partial\vec{D}}{\partial t}$

34. A charge capacitor (C) is connected in series with an inductor (L). When the displacement current reduces to zero, the energy of the LC circuit is **[GATE 2007]**

(a) stored entirely in its magnetic field
(b) stored entirely in its electric field
(c) distributed equally among its electric and magnetic fields.
(d) radiated out of the circuit.

35. A plane electromagnetic wave is propagating in a lossless dielectric. The electric field is given by $E(x,y,z,t)=E_0(\hat{x}+A\hat{z})\exp\left[ik_0\left\{-ct+\left(x+\sqrt{3}z\right)\right\}\right]$, where c is the speed of light in vacuum, E_0, A and k_0 are constants and $\hat{x}$ and $\hat{z}$ are unit vectors along the x- and z-axes. The relative dielectric constant of the medium, ε_r and the constant A are **[NET June 2011]**

(a) $\varepsilon_r=4$ and $A=-\frac{1}{\sqrt{3}}$ (b) $\varepsilon_r=4$ and $A=+\frac{1}{\sqrt{3}}$

(c) $\varepsilon_r=4$ and $A=\sqrt{3}$ (d) $\varepsilon_r=4$ and $A=-\sqrt{3}$

36. A conducting circular loop is placed in a uniform magnetic field B = 0.04 T with its plane perpendicular to the field. Somehow the radius of the loop starts shrinking at a constant rate of 0.5 mm/sec. The induced emf in the loop at an instant, when the radius is 2cm, will be

(a) 5.0 μV (b) 2.5 μV (c) 10.0 μV (d) 25.0 μV

37. A linearly polarized electromagnetic wave is incident on a quarter wave plate. The emerge wave will in general be

(a) linearly polarized
(b) elliptically polarized
(c) unpolarized
(d) mixture of linearly polarized and unpolarized wave

38. The magnitude of the Poynting vector gives the

(a) energy transported per unit area per unit time
(b) energy density of the electromagnetic wave
(c) angular momentum density of the electromagnetic wave
(d) angular momentum transported per unit area per unit time

CSIR-UGC-NET/JRF|GATE-PHYSICS

ELECTROMAGNETIC THEORY

Assignment-7 : Electromagntic Theory

1. In a certain region R, Maxwell's equations for the electric and magnetic fields are given by

$$\nabla.\vec{E}=0, \nabla.\vec{B}=0, \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}, \nabla\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$ **[NET 2008]**

We may conclude that

(a) Both the scalar and the vector potential are necessarily constant in the region R.

(b) The electric field $\vec{E}$ and the magnetic field $\vec{B}$ must necessarily be uniform in R.

(c) There are no sources for electric charges and currents in R.

(d) The electric field $\vec{E}$ is necessarily perpendicular to the magnetic field $\vec{B}$ at every point in R.

2. Consider the interference of two coherent electromagnetic waves whose electric field vectors are given by $\vec{E}_1=\hat{i}E_0\cos\omega t$ and $\vec{E}_2=\hat{j}E_0\cos(\omega t+\varphi)$ where φ is the phase difference. The intensity of the resulting wave is given by $\frac{\varepsilon_0}{2}\langle E^2\rangle$, where $\langle E^2\rangle$ is the time average of E^2. The total intensity is **[NET Dec. 2012]**

(a) 0 (b) $\varepsilon_0E_0^2$ (c) $\varepsilon_0E_0^2\sin^2\varphi$ (d) $\varepsilon_0E_0^2\cos^2\varphi$

3. The electric field of an electromagnetic wave is given by $\vec{E}=E_0\cos[\pi(0.3x+0.4y-1000t)]\hat{k}$. The associated magnetic field $\vec{B}$ is **[NET Dec. 2013]**

(a) $10^{-3}E_0\cos[\pi(0.3x+0.4y-1000t)]\hat{k}$ (b) $10^{-4}E_0\cos[\pi(0.3x+0.4y-1000t)](4\hat{i}-3\hat{j})$

(c) $E_0\cos[\pi(0.3x+0.4y-1000t)](0.3\hat{i}+0.4\hat{j})$ (d) $10^{2}E_0\cos[\pi(0.3x+0.4y-1000t)](3\hat{i}+4\hat{j})$

4. An electromagnetically-shielded room is designed so that at a frequency $\omega=10^7$ rad/s the intensity of the external radiation that penerates the room is 1% of the incident radiation. If $\sigma=\frac{1}{2\pi}\times10^6(\Omega m)^{-1}$ is the conductivity of the shielding material, its minimum thickness should be (given that ln 10 = 2.3) **[NET June 2014]**

(a) 4.60 mm (b) 2.30 mm (c) 0.23 mm (d) 0.46 mm

5. The electric field E(r, t) at a point r at time t in a metal due to the passage of electrons can be described by the equation **[GATE 2001]**

$$\nabla^2\vec{E}(\vec{r},t)=\frac{1}{c^2}\left[\frac{\partial^2\vec{E}(\vec{r},t)}{\partial t^2}+\omega'^2\vec{E}(\vec{r},t)\right]$$

Where ω' is a characteristic frequency associated with the metal and c is the speed of light in vacuum. The dispersion relation corresponding to the plane wave solutions of the form $\exp\left[i\left(\vec{k}.\vec{r}-\omega t\right)\right]$ is given by:

(a) $\omega^2=c^2k^2-\omega'^2$ (b) $\omega^2=c^2k^2+\omega'^2$ (c) $\omega^2=ck-\omega'$ (d) $\omega=ck+\omega'$

6. An electromagnetic wave is propagating in free space in the z-direction. If the electric field is given by $E = \cos(\omega t - kz)i$, where $\omega t = ck$, then the magnetic field is given by : **[GATE 2003]**

(a) $B = (1/c)\cos(\omega t - kz)j$ (b) $B = (1/c)\sin(\omega t - kz)j$

(c) $B = (1/c)\cos(\omega t - kz)i$ (d) $B = (1/c)\cos(\omega t - kz)ji$

Common Data for Q.7 and Q.8

Let $\tilde{E} = \hat{x}E_0 \exp\left[i\hat{k}\cdot\hat{r} - \omega t\right]$, where $\vec{k} = \hat{z}(k\cos\phi + ik\sin\phi)$, $k = 1\vec{k}$ and $\hat{x}$, $\hat{y}$ and $\hat{z}$ are cartesian unit vectors, represent an electric field of a plane electro magnetic wave of frequency ω.

7. Which one of the following statements is TRUE? **[GATE 2004]**
(a) The magnitude of the electric field is attenuated as the wave propagates
(b) The energy of the e.m. wave flows along the x-direction
(c) The magnitude of the electric field of the wave is a constant
(d) The speed of the wave is the same as c (speed of light in free space)

8. The magnetic field $\vec{B}$ of the wave is : **[GATE 2004]**

(a) $\hat{y}\frac{k}{\omega}E_0 \exp(-zk\sin\phi)\exp\left[i(zk\cos\phi - \omega t)\right]$

(b) $\hat{y}\frac{k}{\omega}E_0 \exp(-zk\sin\phi)\exp\left[i(zk\cos\phi - \omega t + \phi)\right]$

(c) $\hat{y}\frac{k}{\omega}E_0 \exp\left[i(zk\cos\phi - \omega t + \phi)\right]$

(d) $\hat{y}\frac{k}{\omega}E_0 \exp(-zk\cos\phi)\exp\left[i(zk\sin\phi - \omega t)\right]$

9. The state of polarization of light with the electric field vector **[GATE 2004]**
$\vec{E} = \hat{i}E_0\cos(kz - \omega t) - \hat{j}E_0\cos(kz - \omega t)$ is :
(a) Linearly polarized along z-direction
(b) Linearly polarized at –45° to x-axis
(c) Circularly polarized
(d) Elliptically polarized with the major axis along x- axis

Data for Q. 10 and Q. 11.

Consider two conducting plates of infinite extent, one plate at z = 0 and the other at z = L, both parallel to the xy plane. The vector and scalar potential in the region between the plates is given by :

$A(r,t) = A_0\, i\cos(kz + \alpha)\cos(kct);\ \phi(r,t) = 0$

10. For this to represent a standing wave in the empty region between the plates : **[GATE 2003]**

(a) $k = \pi/L$ and $\alpha = 0$ (b) $k = 2\pi/L$ and $\alpha = \pi/2$

(c) $k = \pi/(2L)$ and $\alpha = \pi/2$ (d) $k = \pi/2L$ and $\alpha = 0$

11. The energy density at z = 0 and t = 0 is : **[GATE 2003]**

(a) 0 (b) $\varepsilon_0 c^2 k^2 A_0^2$

(c) $(1/2)\mu_0 A_0^2 k^2$ (d) $(1/2)\mu_0 A_0^2 k^2 + (1/2)\varepsilon_0 c^2 k^2 A_0^2$

12. Consider the given statements about E(r, t) and B(r, t), the electric and magnetic vectors respectively in a region of free space **[GATE 2003]**

P. Both E and B are conservative vector fields

Q. Both E and B are central force fields

R. E and B are mutually perpendicular in the region

S. Work done by B on a moving charge in the region is zero

Choose the right combination of correct statements from the following :

(a) P and R (b) R and S (c) S only (d) P and Q

13. In a non-conducting medium characterized by $\varepsilon = \varepsilon_0, \mu = \mu_0$ and conductivity $\sigma = 0$, the electric field $\left(\text{in Vm}^{-1}\right)$ is given by $\vec{E} = 20\sin\left[10^8 t - kz\right]\hat{j}$. The magnetic field, $\vec{H}\left(\text{in A m}^{-1}\right)$, is given by : **[GATE 2009]**

(a) $20k\cos\left[10^8 t - kz\right]\hat{i}$

(b) $\dfrac{20k}{10^8\mu_0}\sin\left[10^8 t - kz\right]\hat{j}$

(c) $-\dfrac{20k}{10^8\mu_0}\sin\left[10^8 t - kz\right]\hat{i}$

(d) $-20k\cos\left[10^8 t - kz\right]\hat{j}$

14. Which one of the following sets of Maxwell's equations for time - independent charge density ρ and current density $\hat{J}$ is correct? **[GATE 2006]**

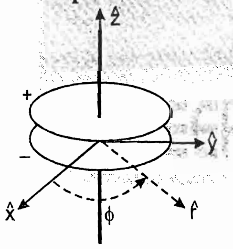
(a) $\vec{\nabla}.\vec{E} = \rho/\varepsilon_0$
$\vec{\nabla}.\vec{B} = 0$
$\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$
$\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\dfrac{\partial\vec{E}}{\partial t}$

(b) $\vec{\nabla}.\vec{E} = \rho/\varepsilon_0$
$\vec{\nabla}.\vec{B} = 0$
$\vec{\nabla}\times\vec{E} = 0$
$\vec{\nabla}\times\vec{B} = \mu_0\hat{J}$

(c) $\vec{\nabla}.\vec{E} = 0$
$\vec{\nabla}.\vec{B} = 0$
$\vec{\nabla}\times\vec{E} = 0$
$\vec{\nabla}\times\vec{B} = \mu_0\hat{J}$

(d) $\vec{\nabla}.\vec{E} = \rho/\varepsilon_0$
$\vec{\nabla}.\vec{B} = \mu_0\hat{J}$
$\vec{\nabla}\times\vec{E} = 0$
$\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\dfrac{\partial\vec{E}}{\partial t}$

15. A parallel plate capacitor is being discharged. What is the direction of the energy flow in terms of the poynting vector in the space betwen the plates? **[GATE 2008]**



(a) Along the wire in the positive z axis (b) Radially inward $(-\hat{r})$

(c) Radially outward $(\hat{r})$ (d) Circumferential (ϕ)

16. A plane electromagnetic wave is given by $E_0\left(\hat{x} + e^{i\delta}\hat{y}\right)\exp\{i(kz - \omega t)\}$. At a given location, the number of times $\vec{E}$ vanishes in one second is : **[GATE 2008]**

(a) An integer near $\dfrac{\omega}{\pi}$ when $\delta = n\pi$ and zero when $\delta \neq n\pi$, n is integer

(b) An integer near $\dfrac{\omega}{\pi}$ and is independent of δ

(c) An integer near $\dfrac{\omega}{2\pi}$ when $\delta = n\pi$ and zero when $\delta \neq n\pi$, n is integer

(d) An integer near $\dfrac{\omega}{2\pi}$ and is independent of δ

Statement for Linked Answer Q. 17 and Q. 18.

Consider the propagation of electromagnetic waves in a linear, homogenous and isotropic material medium with electric permitivity ε, and magnetic permeability μ.

17. For a plane wave of angular frequency ω and propagation vector $\vec{k}$ propagating in the medium Maxwell's equations reduce to **[GATE 2010]**

(a) $\vec{k}.\vec{E}=0;\ \vec{k}.\vec{H}=0;\ \vec{k}\times\vec{E}=\omega\varepsilon\vec{H};\ \vec{k}\times\vec{H}=-\omega\mu\vec{E}$

(b) $\vec{k}.\vec{E}=0;\ \vec{k}.\vec{H}=0;\ \vec{k}\times\vec{E}=-\omega\varepsilon\vec{H};\ \vec{k}\times\vec{H}=\omega\mu\vec{E}$

(c) $\vec{k}.\vec{E}=0;\ \vec{k}.\vec{H}=0;\ \vec{k}\times\vec{E}=-\omega\mu\vec{H};\ \vec{k}\times\vec{H}=\omega\varepsilon\vec{E}$

(d) $\vec{k}.\vec{E}=0;\ \vec{k}.\vec{H}=0;\ \vec{k}\times\vec{E}=\omega\mu\vec{H};\ \vec{k}\times\vec{H}=-\omega\varepsilon\vec{E}$

18. If ε and μ assume negative values in a certain frequency range, then the directions of the propagation vector $\vec{k}$ and the Poynting vector $\vec{S}$ in that frequency range are related as

(a) $\vec{k}$ and $\vec{S}$ are parallel **[GATE 2010]**

(b) $\vec{k}$ and $\vec{S}$ are anti-parallel.

(c) $\vec{k}$ and $\vec{S}$ are perpendicular to each other

(d) $\vec{k}$ and $\vec{S}$ make an angle that depends on the magnitude of $|\varepsilon|$ and $|\mu|$.

Statement for Linked Answer Q. 19 and Q. 20:

A plane electromagnetic wave has the magnetic field given by $\vec{B}(x,y,z,t)=B_0\sin\left[(x+y)\frac{k}{\sqrt{2}}+\omega t\right]\hat{k}$

Where k is the wave number and $\hat{i},\hat{j}$ and $\hat{k}$ are the Cartesian unit vectors is x, y and z directions, respectively.

19. The electric field $\vec{E}(x,y,z,t)$ corresponding to the above wave is given by **[GATE 2011]**

(a) $cB_0\sin\left[(x+y)\frac{k}{\sqrt{2}}+\omega t\right]\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$ (b) $cB_0\sin\left[(x+y)\frac{k}{\sqrt{2}}+\omega t\right]\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$

(c) $cB_0\sin\left[(x+y)\frac{k}{\sqrt{2}}+\omega t\right]\hat{i}$ (d) $cB_0\sin\left[(x+y)\frac{k}{\sqrt{2}}+\omega t\right]\hat{j}$

20. The average Poynting vector is given by **[GATE 2011]**

(a) $\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$ (b) $-\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$ (c) $\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$ (d) $-\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$

21. The space-time dependence of the electric field of a linearly polarized light in free space is given by $\hat{x}E_0\cos(\omega t-kz)$ where E_0, ω and k are the amplitude, the angular frequency and the wavevector, respectively. The time averaged energy density associated with the electric field is **[GATE 2012]**

(a) $\frac{1}{4}\varepsilon_0E_0^2$ (b) $\frac{1}{2}\varepsilon_0E_0^2$ (c) $\varepsilon_0E_0^2$ (d) $2\varepsilon_0E_0^2$

22. The electric field of a uniform plane wave propagating in a dielectric, non-conducting medium is given by,

$$\vec{E}=\hat{x}10\cos\left(6\pi\times10^{7}\,t-0.4\pi z\right)V/m$$

The phase velocity of the wave is _________ $\times10^{8}$ m/s. **[GATE 2014]**

23. The intensity of a laser in free space is $150\ mW/m^{2}$. The corresponding amplitude of the electric field of the laser is ______ V/m. $\left(\varepsilon_0=8.854\times10^{-12}\ C^{2}/N.m^{2}\right)$ **[GATE 2014]**

24. An electromagnetic wave with $\vec{E}(z,t)=E_0\cos(\omega t-kz)\hat{i}$ is travelling in free space and crosses a disc of radius 2m placed perpendicular to the z-axis. If $E_0=90V\ m^{-1}$, the average power, in watt, crossing the disc along the z-direction is **[GATE 2007]**

(a) 30 (b) 60 (c) 120 (d) 270

25. The magnetic field associated with the electric field vector $\vec{E}=E_0\sin(kz-\omega t)\hat{j}$ is given by

(a) $\vec{B}=-\frac{E_0}{c}\sin(kz-\omega t)\hat{i}$ (b) $\vec{B}=\frac{E_0}{c}\sin(kz-\omega t)\hat{i}$

(c) $\vec{B}=\frac{E_0}{c}\sin(kz-\omega t)\hat{j}$ (d) $\vec{B}=\frac{E_0}{c}\sin(kz-\omega t)\hat{k}$

26. A plane electromagnetic wave travelling in a vacuum is characterised by the electric and magnetic fields

$$\vec{\mathrm{E}}=\hat{\mathrm{i}}\left(30\pi\ \mathrm{Vm}^{-1}\right)\exp\mathrm{i}\left(\omega t+\mathrm{kz}\right);\ \vec{\mathrm{H}}=\hat{\mathrm{j}}\left(\mathrm{H}_0\ \mathrm{Am}^{-1}\right)\exp\mathrm{i}\left(\omega t+\mathrm{kz}\right)$$

If $\omega,k>0$, the value of H_0 must be **[TIFR 2013]**

(a) 2π (b) 0.67 (c) 0.25 (d) 0.94

27. Measurement of the magnitudes of the electric field (E) and the magnetic field (B) in a plane polarized electromagnetic wave in vaccum leads to the following results **[TIFR 2015]**

$$\frac{\partial E}{\partial y}=-\frac{\partial B}{\partial t}\qquad\qquad \frac{\partial B}{\partial y}=-\frac{1}{c^{2}}\frac{\partial E}{\partial t}$$

at all points where the measurement is made. In this case the electric vector $\vec{E}$, the magnetic vector $\vec{B}$ and the wave vector $\vec{k}$ (with magnetic k) can be written in terms of the unit vectors $(\hat{x},\hat{y},\hat{z})$ along the Cartesian axes as

(a) $\vec{E}=E\hat{x},\ \vec{B}=B\hat{y},\ \vec{k}=k\hat{z}$ (b) $\vec{E}=E\hat{x},\ \vec{B}=B\hat{z},\ \vec{k}=-k\hat{y}$

(c) $\vec{E}=E\hat{x},\ \vec{B}=-B\hat{z},\ \vec{k}=k\hat{y}$ (d) $\vec{E}=-E\hat{y},\ \vec{B}=-B\hat{z},\ \vec{k}=-k\hat{x}$

28. An electromagnetic wave of frequency ω travels in the x-direction through vacuum. It is polarized in the y-direction and the amplitude of the electric field is E_0. With $k=\omega/c$ where c is the speed of light in vacuum, the electric and the magnetic fields are then conventionally given by **[JEST 2013]**

(a) $\vec{E}=E_0\cos(ky-\omega t)\hat{x}$ and $\vec{B}=\frac{E_0}{c}\cos(ky-\omega t)\hat{z}$

(b) $\vec{E}=E_0\cos(kx-\omega t)\hat{y}$ and $\vec{B}=\frac{E_0}{c}\cos(kx-\omega t)\hat{z}$

(c) $\vec{E}=E_0\cos(kx-\omega t)\hat{z}$ and $\vec{B}=\frac{E_0}{c}\cos(kx-\omega t)\hat{y}$

(d) $\vec{E}=E_0\cos(kx-\omega t)\hat{x}$ and $\vec{B}=\frac{E_0}{c}\cos(ky-\omega t)\hat{y}$

29. For a medium of electric and magnetic field the relation between E and B with propagation vector k

(a) $\bar{E}\times\bar{k}=\omega\bar{B}$ (b) $\bar{E}\times\bar{R}=\bar{\omega}\times\bar{B}$ (c) $\bar{E}=\frac{k}{\omega}\times\bar{B}$ (d) $\vec{B}=\frac{\vec{k}\times\vec{E}}{\omega}$

30. The electric field of two linearly polarized waves are given by
$\vec{E}_1=\hat{i}E_x\cos(\omega t-\theta_1)$ and $\vec{E}_2=\hat{j}E_y\cos(\omega t-\theta_2)$. The nature of polarization of the superimposed wave is :
(a) Unpolarized (b) Linearly polarized
(c) Circularly polarized (d) Elliptically polarized

31. In a free space, which is the correct set of Maxwell's equations in absence of any charges and current densities :

(a) $\vec{\nabla}.\vec{E}=\rho/\varepsilon_0, \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$
$\vec{\nabla}.\vec{B}=0, \vec{\nabla}\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

(b) $\vec{\nabla}.\vec{E}=0, \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$
$\vec{\nabla}.\vec{B}=0, \vec{\nabla}\times\vec{B}=\mu_0\vec{J}$

(c) $\vec{\nabla}.\vec{E}=\rho/\varepsilon_0, \vec{\nabla}\times\vec{E}=0$
$\vec{\nabla}.\vec{B}=0, \vec{\nabla}\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

(d) $\vec{\nabla}.\vec{E}=0, \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$
$\vec{\nabla}.\vec{B}=0. \vec{\nabla}\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

32. Three plane waves are given as $\bar{y}_1=A_1\,\hat{y}.\cos(kx-\omega t+\delta_1), \bar{y}_2=A_2\,\hat{z}\cos(kx-\omega t+\delta_2)$, $\bar{y}_3=A_3\,\hat{y}\cos(kx-\omega t+\delta_3)$, where $\delta_1, \delta_2, \delta_3$ and A_1, A_2, A_3 are constants. If these waves are superposed pairwise, which superposition will lead to interference?

(a) $\bar{y}_1$ and $\bar{y}_2$ (b) $\bar{y}_2$ and $\bar{y}_3$ (c) $\bar{y}_1$ and $\bar{y}_3$ (d) No interference in any pair

33. A 1W point source at origin emits light uniformly in all the directions. If the units for both the axes are measured in centimetre, then the Poynting vector at the point (1, 1, 0) in W/cm^2 is

($\hat{e}_x, \hat{e}_y$, and $\hat{e}_z$ are unit vectors in Cartesian - coordinate system.)

(a) $\frac{1}{8\pi\sqrt{2}}(\hat{e}_x+\hat{e}_y)$ (b) $\frac{1}{16\pi}(\hat{e}_x+\hat{e}_y)$ (c) $\frac{1}{16\pi\sqrt{2}}(\hat{e}_x+\hat{e}_y)$ (d) $\frac{1}{4\pi\sqrt{2}}(\hat{e}_x+\hat{e}_y)$

34. For plane electromagnetic waves varying in time as e^{-iwt}, the time averaged Poynting vector is:

(a) $\frac{1}{2}R_e(\vec{E}\times\vec{H})$ (b) $R_e(\vec{E}\times\vec{H})$ (c) $\frac{1}{2}R_e(\vec{E}\times\vec{H})$ (d) $\frac{1}{2}(R_e\vec{E})\times(R_e\vec{H})$

35. An electromagnetic wave is elliptically polarized in the X-Y plane. The direction of the Poynting vector is in the

(a) Z-direction (b) X-direction (c) Y-direction (d) XY-plane

36. Which of the following proposed space-time dependent electric fields in vacuum is/are allowed by the equations of electromagnetic theory?
(I) $E_x=E_1\sin(kz-\omega t), E_y=E_2\sin(kz-\omega t), E_z=0$
(II) $E_x=E_1\sin(kz-\omega t), E_y=2E_2\cos(kz-\omega t), E_z=0$

(III) $E_x=E_1\sin(kz-\omega t), E_y=0, E_z=E_2\sin(kz-\omega t)$

[In the above E_1 and E_2 are real constants]
(a) I and II but not III (b) II and III but not I
(c) I and III but not II (d) I only (e) I, II and III

37. The poynting vector in the radial direction, at the surface of a long straight wire, of radius b and resistance per unit length R, carrying a current I is:

(a) $\frac{-RI^2}{2\pi b}\hat{r}(\text{inward})$ (b) $\frac{RI^2}{2\pi b}\hat{r}(\text{outward})$

(c) $\frac{bI}{2\pi R}\hat{r}(\text{inward})$ (d) $\frac{2\pi I}{Rb}\hat{r}(\text{inward})$

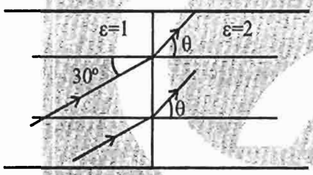
38. A medium with refractive index $n(\omega)$, where ω is the frequency, is said to posses anomalous dispersion when

(a) $\frac{\partial n}{\partial \omega} > 0$ (b) $\frac{\partial n}{\partial \omega} < 0$ (c) $\frac{\partial^2 n}{\partial \omega^2} > 0$ (d) $\frac{\partial^2 n}{\partial \omega^2} < 0$

39. In a non-magnetic dielectric medium with dielectric constant $\varepsilon_r = 4$, the electric field of a propagating plane wave with $\omega = 10^8$ rad/s is given by, $\vec{E} = \left(-\hat{i} + \sqrt{3}\hat{j}\right)\exp\left[j\left(\omega t - \vec{k}.\vec{r}\right)\right]$. The propagation vector $\vec{k}$ (in units of m^{-1}) is given by

(a) $\vec{k} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{3}\hat{j}$ (b) $\vec{k} = \frac{1}{3}\hat{z}$ (c) $\vec{k} = \frac{1}{2\sqrt{3}}\hat{i} + \frac{1}{6}\hat{j}$ (d) $\vec{k} = -\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{3}\hat{j}$

40. A large slab consists of two materials with dielectric constants 1 and 3 divided by a plane interface as shown in the figure. An electric field exists in this slab, which uniformly makes an angle of 30° with the normal to the interface on the left side of the $(\varepsilon = 1)$. The angle the field makes on the other side of the interface is:



(a) $\tan^{-1}(3)$ (b) $\sin^{-1}\left(\frac{1}{2\sqrt{3}}\right)$ (c) 60° (d) $\tan^{-1}(1/3)$

41. Two beams of light with electric field amplitudes $\hat{i}E$ and $\left(\hat{i} - 2\hat{j}\right)E$ interfere at a point with path difference $\lambda/2$. The magnitude of the electric field at the point of interference is

(a) $2\sqrt{2}$ E (b) 0 (c) 2 E (d) $\left(1+\sqrt{5}\right)$E

42. A propagating electromagnetic wave is given by $\vec{E} = E_0\left(\hat{j} + i\hat{k}\right)e^{i(kx-\omega t)}$. The wave is

(a) left circularly polarized (b) right circularly polarized
(c) linearly polarized (d) elliptically polarized

43. In a region of space it is found that $\vec{\nabla}.\vec{E} = 0$ but $\vec{\nabla}\times\vec{E} \neq 0$. Which of the following statements is not true?

(a) The charge density must be zero
(b) A displacement current must exist
(c) The divergence of magnetic field must be zero
(d) A time-varying magnetic field must exist

44. The electric field of an electromagnetic wave is given by $\vec{E} = E_0\left(3\hat{i}+4\hat{j}\right)\cos k\left(4x-3y-ct\right)$. The directions of the magnetic field $\vec{B}$ and the poynting vector $\vec{p}$ are

(a) $-\hat{k}, \frac{4\hat{i}+3\hat{j}}{5}$ (b) $\hat{k}, \frac{4\hat{i}-3\hat{j}}{5}$ (c) $\frac{4\hat{i}-3\hat{j}}{5}, \hat{k}$ (d) $\frac{4\hat{i}-3\hat{j}}{5}, \frac{4\hat{i}+3\hat{j}}{5}$

45. A microwave source of wave length $\lambda = 3$ cm is used in a double slit interference experiment. The distance D from the slits to the detector is 1m. If the distance d between the slits increases from 1mm to 5 cm, then
(a) the fringes being to overlap
(b) the central maximum becomes more intense
(c) the secondary sources at the slits lose coherence
(d) the separation between the maxima decreases.

46 The skin depth of a metal is dependent on the conductivity (σ) of the metal and the angular frequency ω of the incident field. For a metal of high conductivity, which of the following relations is correct ? (Assume that $\sigma \gg \varepsilon\omega_1$ where ε is the electrical permittivity of the medium). **[JEST 2015]**

(a) $d \propto \sqrt{\sigma/\omega}$ (b) $d \propto \sqrt{1/\sigma\omega}$ (c) $d \propto \sqrt{\sigma\omega}$ (d) $d \propto \sqrt{\omega/\sigma}$

47. The wavelength of red helium-neon laser in air is 6328 Å. What happens to its frequency in glass that has a refractive index of 1.50 ? **[JEST 2015]**

(a) Increases by a factor of 3 (b) Decreases by a factor of 1.5
(c) Remains the same (d) Decreases by a factor of 0.5

48. A monochromatic plane wave (wavelength = 600 nm) $E_0 \exp\left[i(kz-\omega t)\right]$ is incident normally on a diffraction grating giving rise to a plane wave $E_1 \exp\left[i\left(\vec{k}_1\cdot\vec{r}-\omega t\right)\right]$ in the first order of diffraction. Here $E_1 < E_0$ and $\vec{k}_1 = \left|\vec{k}_1\right|\left[\frac{1}{2}\hat{x}+\frac{\sqrt{3}}{2}\hat{z}\right]$. The period (in μm) of the diffraction grating is_______(upto one decimal place)

49. The electric and magnetic fields of an electromagnetic wave in vacuum are given by
$\mathbf{E} = \hat{i}E_0\sin(kz-\omega t)$ and $\mathbf{B} = \hat{j}B_0\sin(kz-\omega t)$ respectively. Which of the following relatins is correct ?

(a) $k^2E_0 = \omega^2B_0$ (b) $\omega E_0 = kB_0$ (c) $kE_0 = \omega B_0$ (d) $E_0B_0 = \omega k$

50. The electric field $\vec{E}(\vec{r},t)$ for a circularly polarized electromagnetic wave propagating along the positive z direction is : **[GATE 2005]**

(a) $E_0(\hat{x}+\hat{y})\exp\left[i(kz-\omega t)\right]$ (b) $E_0(\hat{x}+i\hat{y})\exp\left[i(kz-\omega t)\right]$
(c) $E_0(\hat{x}+i\hat{y})\exp\left[i(kz+\omega t)\right]$ (d) $E_0(\hat{x}+\hat{y})\exp\left[i(kz+\omega t)\right]$

51. A plane polarized electromagnetic wave in free space at time t = 0 is given by $\vec{E}(x,z)=10\hat{j}\exp\left[i(6x+8z)\right]$.

The magnetic field $\vec{B}(x,z,t)$ is given by **[GATE 2012]**

(a) $\vec{B}(x,z,t)=\frac{1}{c}\left(6\hat{k}-8\hat{i}\right)\exp\left[i(6x+8z-10ct)\right]$. (b) $\vec{B}(x,z,t)=\frac{1}{c}\left(6\hat{k}+8\hat{i}\right)\exp\left[i(6x+8z-10ct)\right]$.

(c) $\vec{B}(x,z,t)=\frac{1}{c}\left(6\hat{k}-8\hat{i}\right)\exp\left[i(6x+8z-ct)\right]$. (d) $\vec{B}(x,z,t)=\frac{1}{c}\left(6\hat{k}+8\hat{i}\right)\exp\left[i(6x+8z+ct)\right]$.

52. A plane electromagnetic wave is travelling along the positive z-direction. The maximum electric field along the x-direction is 10 V/m. The approximate maximum values of the power per unit area and the magnetic induction B, respectively, are **[NET June 2015]**

(a) 3.3×10^{-7} watts/m^2 and 10 tesla (b) 3.3×10^{-7} watts/m^2 and 3.3×10^{-8} tesla

(c) 0.265 watts/m^2 and 10 tesla (d) 0.265 watts/m^2 and 3.3×10^{-8} tesla

53. A beam of unpolarized light in a medium with dielectric constant ϵ_1 is reflected from a plane interface formed with another medium of dielectric constant $\epsilon_2 = 3\,\epsilon_1$. The two media have identical magnetic permeability. If the angle of incidence is 60°, then the reflected light

(a) is plane polarized perpendicular to the plane of incidence

(b) is plane polarized parallel to the plane of incidence

(c) is circularly polarized

(d) has the same polarization as the incident light.

54. The electric and magnetic fields in the charge free region $z>0$ are given by **[NET June 2015]**

$$\vec{E}(\vec{r},t)=E_0e^{-k_1z}\cos(k_2x-\omega t)\hat{j}$$

$$\vec{B}(\vec{r},t)=\frac{E_0}{\omega}e^{-k_1z}\left[k_1\sin(k_2x-\omega t)\hat{i}+k_2\cos(k_2x-\omega t)\hat{k}\right]$$

where ω, k_1 & k_2 are positive constants. The average energy flow in the x-direction is

(a) $\frac{E_0^2k_2}{2\mu_0\omega}e^{-2k_1z}$ (b) $\frac{E_0^2k_2}{\mu_0\omega}e^{-2k_1z}$ (c) $\frac{E_0^2k_1}{2\mu_0\omega}e^{-2k_1z}$ (d) $\frac{1}{2}c\varepsilon_0E_0^2e^{-2k_1z}$

55. At 'equilibrium' there can not be any free charge inside a metal. However, if you forcibly put charge in the interior then it takes some finite time to 'disappear', *i.e.* move to the surface. If the conductivity, σ, of a metal is $10^6(\Omega\, m)^{-1}$ and the dielectric constant $\epsilon_0 = 8.85\times10^{-12}$ Farad/m, this time will be approximately: **[JEST 2013]**

(a) 10^{-5} sec (b) 10^{-11} sec (c) 10^{-9} sec (d) 10^{-17} sec

56. The electric field of an electromagnetic wave is given by $\vec{E}=\hat{i}E_0\sin(ky-\omega t)$, where '$k$' and ω respectively denote the wave vector and angular frequency of the wave. $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively denote the unit vectors in the x, y and z directions. The magnetic field $\vec{B}$ of the wave is given by

(a) $\hat{k}E_0k\sin(ky-\omega t)$ (b) $\hat{j}E_0\cos(ky-\omega t)$

(c) $-\hat{k}\frac{E_0\omega}{k}\cos(ky-\omega t)$ (d) $-\hat{k}\frac{E_0\omega}{k}\sin(ky-\omega t)$

57. How much force does light from a 1.8 W laser exert when it is totally absorbed by an object?

(a) $6.0\times10^{-9}N$ (b) $0.6\times10^{-9}N$ (c) $6.0\times10^{-8}N$ (d) $4.8\times10^{-9}N$

58. The electric field $\vec{E} = E_0 \sin(\omega t - kz)\hat{x} + 2E_0 \sin(\omega t - kz + \pi/2)\hat{y}$ represents:

(a) a linearly polarized wave
(b) a right hand circularly polarized wave
(c) a left hand circularly polarized wave
(d) an elliptically polarized wave

59. The electric field component of a plane electromagnetic wave travelling in vacuum is given by $\vec{E}(z,t) = E_0 \cos(kz - \omega t)\hat{i}$. The Poynting vector for the wave is **[GATE 2016]**

(a) $(c\varepsilon_0/2)E_0^2 \cos^2(kz-\omega t)\hat{j}$
(b) $(c\varepsilon_0/2)E_0^2 \cos^2(kz-\omega t)\hat{k}$
(c) $c\varepsilon_0 E_0^2 \cos^2(kz-\omega t)\hat{j}$
(d) $c\varepsilon_0 E_0^2 \cos^2(kz-\omega t)\hat{k}$

60. Let a right circularly polarized light be incident normally on an ordinary mirro. The reflected light is:

(a) Elliptically polarized
(b) Right circularly polarized
(c) Left circularly polarized
(d) Linearly polarized

61. At time $t=0$, a charge distribution $\rho(\vec{r},0)$ exists within an ideal homogeneous conductor of permitivity ε and conductivity σ. At a later time $\rho(\vec{r},t)$ is given by **[GATE 2008]**

(a) $\rho(\vec{r},t) = \rho(\vec{r},0)\exp\left(-\frac{\sigma t}{\varepsilon}\right)$
(b) $\rho(\vec{r},t) = \frac{\rho(\vec{r},0)}{1+(\sigma t/\varepsilon)^2}$
(c) $\rho(\vec{r},t) = \rho(\vec{r},0)\exp\left[-\left(\frac{\sigma t}{\varepsilon}\right)^2\right]$
(d) $\rho(\vec{r},t) = \rho(\vec{r},0)\exp\frac{\varepsilon}{\sigma t}\sin\left(\frac{\sigma t}{\varepsilon}\right)$

62. The frequency dependent dielectric constant of a material is given by

$$\varepsilon(\omega) = 1 + \frac{A}{\omega_0^2 - \omega^2 - i\omega\gamma}$$

where A is a positive constant, ω_0 the resonant frequency and γ the damping coefficient. For an electromagnetic wave of angular frequency $\omega << \omega_0$, which of the following is true? (Assume that $\frac{\gamma}{\omega_0} << 1$).

(a) there is negligible absorption of the wave
(b) the wave propagation is highly dispersive
(c) there is strong absorption of the electromagnetic wave
(d) the group velocity and hte phase velocity will have opposite sign.

63. Maxwell's equations can be written in terms of two coupled equations satisfied by the vector potential A and scalar potential ϕ. The Lorentz gauge which decouples these equations is given by

(a) $\vec{\nabla}.\vec{A} = 0$
(b) $\nabla^2\phi = \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2}$
(c) $\vec{\nabla}.\vec{A} = -\frac{1}{c^2}\frac{\partial \phi}{\partial t}$
(d) $\vec{\nabla}.\vec{A} = -\frac{1}{c}\frac{\partial \phi}{\partial t}$

64. Of A and V are vector and scalar potential $V(r,t)=0$, $A(r,t)=\frac{-q}{4\pi\in_0}\frac{t\vec{r}}{r^2}$. Scalar function $\lambda=\frac{-q}{4\pi\in_0}\frac{t}{r^2}$

Due to gauge transformation transformed potential is

(a) $V'(r,t)=0 \quad A(r,t)=\frac{q}{4\pi\in_0}t\frac{1}{r^2}$ (b) $V'(r,t)=\frac{q}{4\pi\in_0 r^2} \quad A(r,t)=\frac{q}{4\pi\in_0}t\frac{1}{r^2}$

(c) $V'(r,t)=\frac{-q}{4\pi\in_0}\frac{1}{r^2} \quad A'(r,t)=\frac{-qt}{4\pi\in_0}\frac{1}{r^2}$ (d) $V'(r,t)=\frac{q}{4\pi\in_0}\frac{1}{r^2} \quad A'(r,t)=\frac{qt}{4\pi\in_0 r^3}$

65. Which of the following transformations $(V,\vec{A})\to(V',\vec{A}')$ of the electrostatic potential V and the vector potential $\vec{A}$ is a gauge transformation? **[NET June 2015]**

(a) $(V'=V+ax,\ \vec{A}'=\vec{A}+at\,\hat{k})$ (b) $(V'=V+ax,\ \vec{A}'=\vec{A}-at\,\hat{k})$

(c) $(V'=V+ax,\ \vec{A}'=\vec{A}+at\,\hat{i})$ (d) $(V'=V+ax,\ \vec{A}'=\vec{A}-at\,\hat{i})$

66. The gauge transformations of the scalar and vector potentials that leave the electric and magnetic field invariant are

(a) $\vec{A}\to\vec{A}'=A-\frac{\partial\Lambda}{\partial t}$, $\phi\to\phi'=\phi+\nabla\Lambda$ (b) $\vec{A}\to\vec{A}'=A+\nabla\Lambda$, $\phi\to\phi'=\phi-\frac{\partial\Lambda}{\partial t}$

(c) $\vec{A}\to\vec{A}'=A-\nabla\Lambda$, $\phi\to\phi'=\phi-\frac{\partial\Lambda}{\partial t}$ (d) $\vec{A}\to\vec{A}'=\vec{A}+\nabla\Lambda$, $\phi\to\phi'=\phi+\frac{\partial\Lambda}{\partial t}$

67. Let $\vec{A}$ and φ denote the vector and scalar potentials, respectively, which are Lorentz gauge invariant. $\vec{A}$ and φ satisfy.

(a) $\text{div}\vec{A}+\frac{1}{c}\frac{\partial\varphi}{\partial t}=0$ (b) $\text{div}\vec{A}+\frac{1}{c^2}\frac{\partial\varphi}{\partial t}=0$ (c) $\nabla^2\vec{A}+\frac{1}{c}\frac{\partial\varphi}{dt}=0$ (d) $\nabla^2\vec{A}+\frac{1}{c}\frac{\partial A}{dt}=0$

68. The condition for a vector potential $\vec{\Lambda}$ to satisfy the wave equation is:

(a) $\vec{\nabla}.\vec{\Lambda}=0$ (b) $\vec{\nabla}.\vec{\Lambda}+\mu_0\in_0\frac{\partial\phi}{\partial t}=0$

(c) $\frac{\partial\vec{\Lambda}}{\partial t}+\mu_0\in_0\vec{\nabla}\phi=0$ (d) $\vec{A}+\vec{\nabla}\phi=0$

69. An electro-magnetic field is defined by a scalar potential $\phi(\vec{r},t)=0$ and vector potential $\vec{A}(\vec{r},t)=\frac{1}{4\pi\varepsilon_0}\frac{qt}{r^2}\hat{r}$.

The corresponding electric field is :

(a) $\vec{E}=0$ (b) $\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$ (c) $\vec{E}=-\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$ (d) $\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\theta}$

70. For a vector potential $\vec{A}$, the divergence of $\vec{A}$ is $\vec{\nabla}.\vec{A} = -\frac{\mu_0}{4\pi}\frac{Q}{r^2}$, where Q is a constant of appropriate dimension. The corresponding scalar potential $\varphi(\vec{r},t)$ that makes $\vec{A}$ and φ Lorentz gauge invariant is : **[GATE 2005]**

(a) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{Qt}{r}$ (c) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{Qt}{r^2}$

71. The scalar and vector potentials $\varphi(\vec{x},t)$ and $\vec{A}(\vec{x},t)$ are determined up to a gauge transformation $\varphi \to \varphi' = \varphi - \frac{\partial\xi}{\partial t}$ and $\vec{A} \to \vec{A}' = \vec{A} + \vec{\nabla}\xi$ where ξ is an arbitrary continuous and differentiable function of $\vec{x}$ and t. If we further impose the Lorentz gauge condition $\vec{\nabla}\cdot\vec{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t} = 0$ then a possible choice for the gauge function $\xi(\vec{x},t)$ is (where $\omega, \vec{k}$ are non-zero constants with $\omega = c|\vec{k}|$) **[NET Dec. 2014]**

(a) $\cos\omega t \cosh\vec{k}\cdot\vec{x}$ (b) $\sinh\omega t \cos\vec{k}\cdot\vec{x}$

(c) $\cosh\omega t \cos\vec{k}\cdot\vec{x}$ (d) $\cosh\omega t \cosh\vec{k}\cdot\vec{x}$

72. If the electric and magnetic fields are unchanged when the vector potential $\vec{A}$ changes (in suitable units) according to $\vec{A} \to \vec{A} + \hat{r}$, where $\vec{r} = r(t)\hat{r}$, then the scalar potential Φ must simultaneously change to **[NET June 2013]**

(a) $\Phi - r$ (b) $\Phi + r$ (c) $\Phi - \partial r/\partial t$ (d) $\Phi + \partial r/\partial t$

73. Consider an infinite conducting sheet in the xy-plane with a time dependent current density $Kt\hat{i}$, where K is a constant. The vector potential at (x, y, z) is given by **[NET Dec/ 2012]**

$$\vec{A} = \frac{\mu_0 K}{4c}(ct - z)^2\hat{i}$$

The magnetic field $\vec{B}$ is:

(a) $\frac{\mu_0 Kt}{2}\hat{j}$ (b) $-\frac{\mu_0 Kz}{2c}\hat{j}$ (c) $-\frac{\mu_0 K}{2c}(ct - z)\hat{i}$ (d) $-\frac{\mu_0 K}{2c}(ct - z)\hat{j}$

74. A constant electric current I in an infinitely long straight wire is suddenly switched on at $t = 0$. The vector potential at a perpendicular distance r from the wire is given by $\vec{A} = \frac{\hat{k}\mu_0 I}{2\pi}\ell n\left[\frac{1}{r}\left(ct + \sqrt{c^2t^2 - r^2}\right)\right]$. The electric field at a distance $r(< ct)$ is: **[NET Dec. 2011]**

(a) 0 (b) $\frac{\mu_0 I}{2\pi t}\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$ (c) $\frac{c\mu_0 I}{2\pi\sqrt{c^2t^2 - r^2}}\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$ (d) $-\frac{c\mu_0 I}{2\pi\sqrt{c^2t^2 - r^2}}\hat{k}$

75. Consider a solenoid of radius R with n turns per unit length, in which a time dependent current $I = I_0 \sin \omega t$ (where $\omega R/c \ll 1$) flows. The magnitude of the electric field at a perpendicular distance $r < R$ from the axis of symmetry of the solenoid, is: **[NET Dec. 2011]**

(a) 0

(b) $\frac{1}{2r}\omega\mu_0 n I_0 R^2 \cos\omega t$

(c) $\frac{1}{2}\omega\mu_0 n I_0 r \sin\omega t$

(d) $\frac{1}{2}\omega\mu_0 n I_0 r \cos\omega t$

76. A time-dependent current $\vec{I}(t) = Kt\hat{z}$ (where K is a constant) is switched on at $t = 0$ in an infinite current-carrying wire. The magnetic vector potential at a perpendicular distance 'a' from the wire is given (for time $t > a/c$) by **[NET June 2014]**

(a) $\hat{z}\frac{\mu_0 K}{4\pi c}\int_{-\sqrt{c^2t^2-a^2}}^{\sqrt{c^2t^2-a^2}} dz \frac{ct-\sqrt{a^2+z^2}}{\left(a^2+z^2\right)^{1/2}}$

(b) $\hat{z}\frac{\mu_0 K}{4\pi}\int_{-ct}^{ct} dz \frac{ct-\sqrt{a^2+z^2}}{\left(a^2+z^2\right)^{1/2}}$

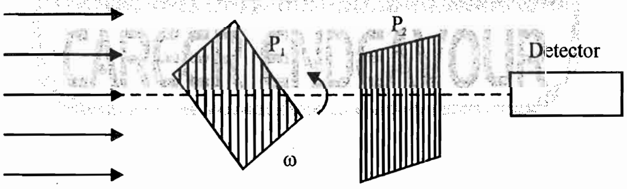
(c) $\hat{z}\frac{\mu_0 K}{4\pi c}\int_{-ct}^{ct} dz \frac{ct-\sqrt{a^2+z^2}}{\left(a^2+z^2\right)^{1/2}}$

(d) $\hat{z}\frac{\mu_0 K}{4\pi}\int_{-\sqrt{c^2t^2-a^2}}^{\sqrt{c^2t^2-a^2}} dz \frac{t}{\left(a^2+z^2\right)^{1/2}}$

77. Consider a beam of light of wavelength λ incident on a system of a polarizer and an analyzer. The analyzer is oriented at 45° to the polarizer. When an optical component is introuduced between them, the output intensity becomes zero. (Light is incident normally on all components). The optical component is:

(a) A full-wave plate
(b) A half-wave plate
(c) A quarter-wave plate
(d) An ordinary glass plate

78. A linear beam of unpolarised light passes through tow plane polarisers, the planes of which are perpendicular to the direction of propagation of the beam. The first polariser rotates around this direction with an angular velocity of 20π radians per second. If the initial intensity of the light is I_0, then the intensity when it leaves the second polariser



(a) is periodic with frequency of 20 Hz and maximum of $I_0/4$
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(c) is periodic with frequency of 10 Hz and maximum of $I_0/4$
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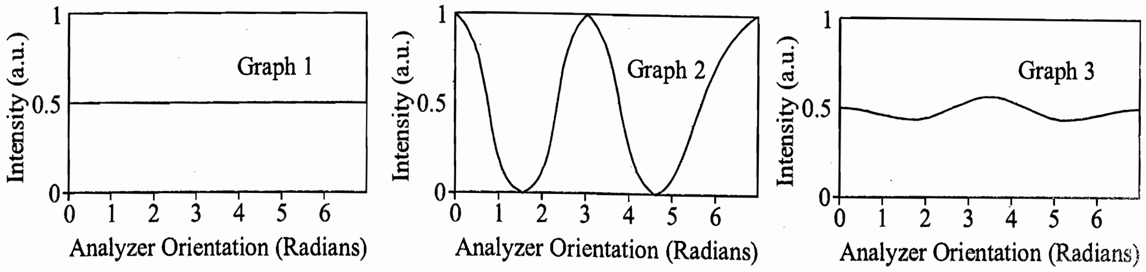
79. Intensity of three different light beams after passing through an analyzer is found to vary as shown in the following graphs. Identify the option giving the correct states of polarization of the incident beams from the graphs.



(a) Graph 1: Linear polarization, Graph 2: Circular polarization, Graph 3: Elliptic polarization.
(b) Graph 1: Circular polarization, Graph 2: Linear polarization, Graph 3: Elliptic polarization.
(c) Graph 1: Unpolarized, Graph 2: Circular polarization, Graph 3: Linear polarization.
(d) Graph 1: Unpolarized, Graph 2: Elliptic polarization, Graph 3: Circular polarization.

80. **Group-I** contains x- and y-components of the electric field and **Group-II** contains the type of polarization of light.

Group-I	**Group-II**
P. $E_x = \frac{E_0}{\sqrt{2}}\cos(\omega t + kz)$ $E_y = E_0 \sin(\omega t + kz)$	1. Linearly polarized
Q. $E_x = E_0 \sin(\omega t + kz)$ $E_y = E_0 \cos(\omega t + kz)$	2. Circularly polarized
R. $E_x = E_1 \sin(\omega t + kz)$ $E_y = E_2 \sin(\omega t + kz)$	3. Unpolarized
S. $E_x = E_0 \sin(\omega t + kz)$ $E_y = E_0 \sin\left(\omega t + kz + \frac{\pi}{4}\right)$	4. Elliptically polarized

The correct set of matches is

(a) P → 4; Q → 2; R → 4; S → 1
(b) P → 1; Q → 3; R → 1; S → 4
(c) P → 4; Q → 2; R → 1; S → 4
(d) P → 3; Q → 1; R → 3; S → 2

81. The electric field of a travelling wave is given by $100\cos(10^9 t - 4x)$, where t is in seconds and x in metres. The speed of the wave is:
(a) 3.0×10^8 m/s
(b) 2.5×10^8 m/s
(c) 3.0×10^7 m/s
(d) 5.0×10^7 m/s

82. The Poynting vector has the dimension of:
(a) Energy / Area
(b) Power / Area
(c) Energy / (Charge × Volume)
(d) Power / (Charge × Volume)

75. Consider a solenoid of radius R with n turns per unit length, in which a time dependent current $I = I_0 \sin \omega t$ (where $\omega R/c \ll 1$) flows. The magnitude of the electric field at a perpendicular distance $r < R$ from the axis of symmetry of the solenoid, is: **[NET Dec. 2011]**

(a) 0
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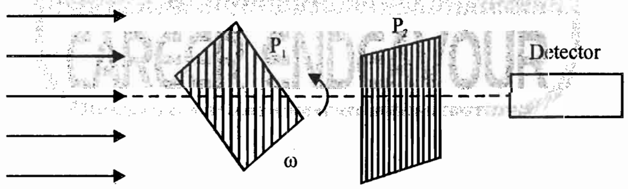

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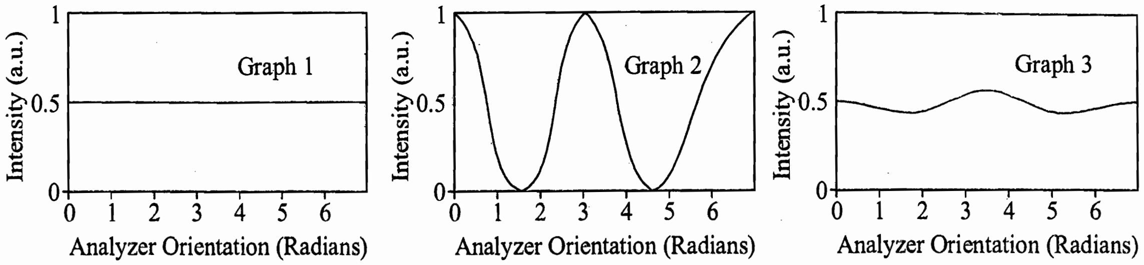

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(d) Power / (Charge × Volume)

83. A current I flows in a cylindrical wire of length L and radius a, placed in a uniform electric field, E = VL, parallel to the wire (V is the potential difference between the two ends of the wire). Which of the following statements is correct?

(a) The Poynting vector is zero for this system

(b) The magnitude of Poynting vector is $\frac{VI}{2\pi a L}$ and it is directed parallel to the wire

(c) The magnitude of Poynting vector is $\frac{VI}{2\pi a L}$ and it points radially inward

(d) The magnitude of Poynting vector is $\frac{VI}{2\pi a L}$ and it points radially outward

84. The skin depth of the electromagnetic waves in a poor conductor is:

(a) $\omega\sqrt{\frac{\varepsilon\mu}{2}}[\sqrt{1+(\frac{\sigma}{\varepsilon\omega})^2}-1]^{1/2}$

(b) $\omega\sqrt{\frac{\varepsilon\mu}{2}}$

(c) $\frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$

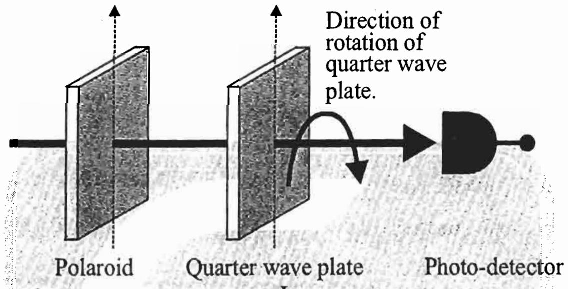
(d) $\frac{\lambda}{2\pi}$

CSIR-UGC-NET/JRF|GATE-PHYSICS

ELECTROMAGNETIC THEORY

Assignment-8 : Electromagntic Theory

1. A quarter-wave plate is placed in between a polarizer and a photo-detector. When the optic axis of the quarter-wave plate is kept initially parallel to the pass axis of the polarizer and perpendicular to the direction of light propagation. The intensity of light passing through the quarter-wave plate is measured to be I_0 (see figure). If the quarter wave plate is now rotated by 45° about an axis parallel to the light propagation, what would be the intensity of the emergent light measured by the photo-detector?



(a) $\frac{I_0}{\sqrt{2}}$ (b) $\frac{I_0}{2}$ (c) $\frac{I_0}{2\sqrt{2}}$ (d) I_0

2. A plane wave $(\hat{x}+i\hat{y})E_0\exp[i(kz-\omega t)]$ after passing through an optical element emerges as $(\hat{x}-i\hat{y})E_0\exp[i(kz-\omega t)]$, where k and ω are the wavevector and the angular frequency, respectively. The optical element is **[GATE 2015]**
(a) quarter wave plate (b) half wave plate
(c) polarizer (d) Faraday rotator

3. For a transparent material the ratio of reflected intensity to the incident intensity depends on
(a) its absorption coefficient
(b) its refractive index only
(c) its absorption coefficient and transmission coefficients
(d) its high transmission coefficient

4. The electric fields of two light sources with nearby frequencies ω_1 and ω_2, and wave vectors k_1 and k_2, are expressed as $\vec{E}_1 = E_{10}\hat{i}e^{-i(k_1z-\omega_1t)}$ and $\vec{E}_2 = E_{20}\hat{i}e^{-i(k_2z-\omega_2t)}$, respectively. The interference pattern on the screen is photographed at $t=t_0$; denote $(k_1-k_2)z-(\omega_1-\omega_2)t_0$ by θ. For this pattern

(a) a bright fringe will be obtained for $\cos\theta=-1$

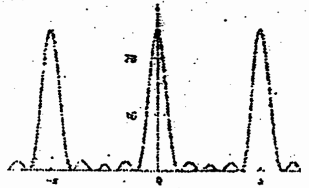
(b) a bright fringe intensity is given by $(E_{10})^2+(E_{20})^2$

(c) a dark fringe will be obtained for $\cos\theta=1$

(d) a dark fringe intensity is given by $(E_{10}-E_{20})^2$

5. A right circularly polarized beam is incident on a calcite half wave plate. The emergent wave will be
(a) linearly polarized (b) left circularly polarized
(c) elliptically polarized (d) unpolarized

6. The diffraction pattern from a grating with N slits is shown in the figure. The value of N is



(a) 3 (b) 5 (c) 4 (d) 2

7. In charge free and current free space, the following is true:

(a) $\vec{\nabla}\times\vec{H}=\frac{\partial\vec{B}}{\partial t}$ (b) $\vec{\nabla}\times\vec{H}=0$ (c) $\vec{\nabla}\times\vec{H}=\frac{\partial\vec{D}}{\partial t}$ (d) $\vec{\nabla}\times\vec{H}=-\frac{\partial\vec{D}}{\partial t}$

8. An electromagnetic wave is normally incident upon a perfectly conducting surface. In the reflected wave,

(a) $\vec{H}$ is shifted 180° in phase. (b) $\vec{E}$ is shifted 180° in phase

(c) $\vec{H}$ is shifted 90° in phase (d) $\vec{E}$ is shifted 90° in phase.

9. In which region of the electromagnetic spectrum do metals have the highest reflectivity?

(a) Infrared (b) Visible

(c) Ultraviolet (d) In the entire region of the spectrum

10. The neutral pion π^0 at rest decays into two photons. One of the photons is right circularly polarized. The other photon is:

(a) Also right circularly polarized. (b) Left circularly polarized.

(c) Unpolarized. (d) Plane polarized.

11. The instantaneous electric and magnetic fields created at a distance 'r' by a point source at the origin are given by

$$\vec{E}=\frac{A\cos\omega t}{2\pi\varepsilon_0 r}\hat{\theta} \qquad \hat{H}=\frac{B\cos\omega t}{\mu_0 r}\hat{\varphi}$$ **[TIFR 2014]**

where ω, A, B are constants, and the unit vectors $(\hat{r},\hat{\theta},\hat{\varphi})$ form an orthonormal set. The time-averaged power radiated by the source is

(a) $\frac{\omega\varepsilon_0}{\mu_0}AB$ (b) $\frac{c^3}{2\pi}AB$ (c) c^2AB (d) $\frac{2\pi\omega}{c}AB$

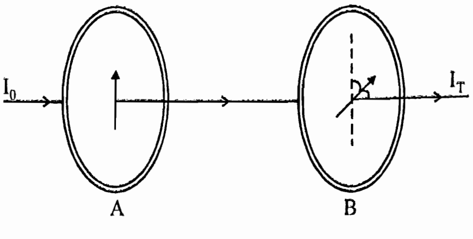
12. The magnetic vector potential $\vec{A}(\vec{r})$ corresponding to a uniform magnetic field $\vec{B}$ is taken in the form

$$\vec{A}=\frac{1}{2}\vec{B}\times\vec{r}$$

where '$\vec{r}$' is the position vector. If the electric field has the time dependent form $\vec{E}=\vec{E}_0(\vec{r})e^{i\omega t}$, where '$\omega$' is a constant, the gauge choice corresponding to this potential is a **[TIFR 2013]**

(a) Lorenz gauge (b) Coulomb gauge (c) Non-linear gauge (d) Time-varying gauge.

13. Unpolarised light of intensity I_0 Passes successively through two identical linear polarisers A and B, placed such that their polarisation axes are at an angle of 45° (see figure) with respect to one another. **[TIFR 2012]**



Assuming A and B to be perfect polarisers (i.e. no absorption losses), the intensity of the transmitted light will be I_T =

(a) $I_0/4$ (b) $I_0/2\sqrt{2}$ (c) $I_0/2$ (d) $I_0/\sqrt{2}$

Common Data for Q.14 and Q.15 **[GATE 2007]**

The Fresnel relations between the amplitudes of incident and reflected electromagnetic waves at an interface between air and a dielectric of refractive index μ, are

$$E_{\parallel}^{reflected} = \frac{\cos r - \mu\cos i}{\cos r + \mu\cos i} E_{\parallel}^{incident} \text{ and } E_{\perp}^{reflected} = \frac{\cos r - \mu\cos i}{\cos r + \mu\cos i} E_{\perp}^{incident}$$

The subscripts || and ⊥ refer to polarization, parallel to the plane of incidence respectively. Here, i and *r* are the angles of incidence and refraction respectively.

14. The condition for the reflected ray to be completely polarized is

(a) $\mu\cos i = \cos r$ (b) $\cos i = \mu\cos r$ (c) $\mu\cos i = -\cos r$ (d) $\cos i = -\mu\cos r$

15. For normal incident at an air-glass interface with μ = 1.5 the fraction of energy reflected is given by

(a) 0.40 (b) 0.20 (c) 0.16 (d) 0.04

16. Can the following scalar and vector potentials describe an electromagnetic field? **[GATE 2007]**

$$\phi(\vec{x},t) = 3xyz - 4t$$
$$\vec{A} = (\vec{x},t) = (2x - \omega t)\hat{i} + (y - 2z)\hat{j} + (z - 2e^{i\omega t})\hat{k}$$

where, ω is a constant.

(a) Yes, in the Coulomb gauge (b) Yes, in the Lorentz gauge

(c) Yes, provided $\omega = 0$ (d) No.

17. A monochromatic plane wave at oblique incidence undergoes reflection at a dielectric interface. If $\hat{k}_i$, $\hat{k}_r$ and $\hat{n}$ are the unit vectors in the directions of incident wave, reflected wave and the normal to the surface respectively, which one of the following expression is correct? **[GATE 2013]**

(a) $(\hat{k}_i - \hat{k}_r)\times\hat{n} \neq 0$ (b) $(\hat{k}_i - \hat{k}_r)\cdot\hat{n} = 0$ (c) $(\hat{k}_i \times \hat{n})\cdot\hat{k}_r = 0$ (d) $(\hat{k}_i \times \hat{n})\cdot\hat{k}_r \neq 0$

18. A circularly polarized monochromatic plane wave is incident on a dielectric interface at Brewster angle. Which one of the following statements is **CORRECT**? **[GATE 2013]**

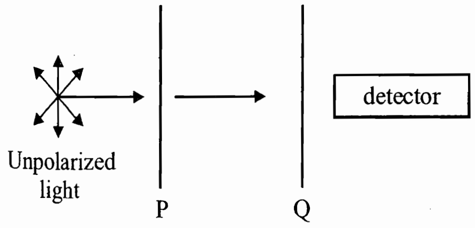
(a) The reflected light is plane polarized in the plane of incidence and the transmitted light is circularly polarized.

(b) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is plane polarized in the plane of incidence.

(c) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is elliptically polarized.

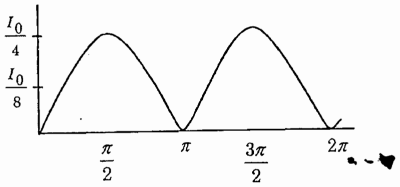
(d) There will be no reflected light and the transmitted light is circularly polarized.

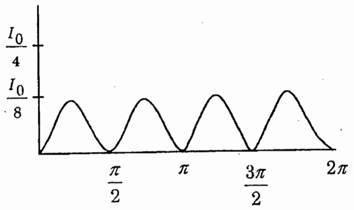
19. In the optical arrangement as shown below, the axes of two polarizing sheets P and Q are oriented such that no light is detected. Now when a third polarizing sheet (R) is placed in between P and Q, then light is detected. Which of the following statement(s) is (are) true?

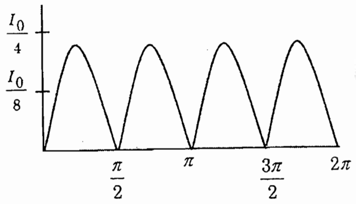


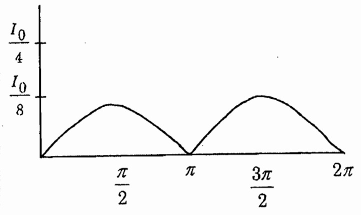
(A) polarization axes of P and Q are perpendicular to each other
(B) polarization axis of R is not parallel to P
(C) polarization axis of R is not parallel to Q
(D) polarization axes of P and Q are parallel to each other.

20. Three polarizers P, Q and R are placed parallel to each other with their planes perpendicular to the z-axis. Q is placed between P and R. Initially the polarizing directions of P and Q are parallel, but that of R is perpendicular to them. In this arrangement when unpolarized light of intensity I_0 is incident on P, the intensity coming out of R is zero. The polarizer Q is now rotated about the z-axis. As a function of angle of rotation, the intensity of light coming out of R is best represented by

(a)


(b)


(c)


(d)


21. Interference fringes are seen at an observation plane $z = 0$, by the superposition of two plane waves $A_1 \exp[i(\vec{k}_1 \cdot \vec{r} - \omega t)]$ and $A_2 \exp[i(\vec{k}_2 \cdot \vec{r} - \omega t)]$, where A_1 and A_2 are real amplitudes. The condition for interference maximum is **[GATE 2013]**

(a) $(\vec{k}_1 - \vec{k}_2) \cdot \vec{r} = (2m+1)\pi$ (b) $(\vec{k}_1 - \vec{k}_2) \cdot \vec{r} = 2m\pi$

(c) $(\vec{k}_1 + \vec{k}_2) \cdot \vec{r} = (2m+1)\pi$ (d) $(\vec{k}_1 + \vec{k}_2) \cdot \vec{r} = 2m\pi$

22. A plane electromagnetic wave traveling in free space is incident normally on a glass plate of refractive index 3/2. If there is no absorption by the glass, Its reflectivity is **[GATE 2012]**

(a) 4% (b) 16 % (c) 20% (d) 50%

23. Unpolarized light falls from air to a planar air-glass interface (refractive index of glass is 1.5) and the reflected light is observed to be plane polarized. The polarization vector and the angle of incidence q_i are : **[GATE 2008]**

(a) Perpendicular to the plane of incidence and $\theta_i = 42^\circ$

(b) Parallel to the plane of incidence and $\theta_i = 56^\circ$

(c) Perpendicular to the plane of incidence and $\theta_i = 56^\circ$

(d) Parallel to the plane of incidence and $\theta_i = 42^\circ$

24. A plane electromagnetic wave travelling in vaccum is incident normally on a non magnetic, non-absorbing medium of refractive index n. The incident (E_i), reflected (E_r) and transmitted (E_t) electric fields are given as, **[GATE 2005]**

$$E_i = E\exp\left[i(kz-\omega t)\right],\ E_r = E_{or}\exp\left[i(k_r z-\omega t)\right],\ E_t = E_{0t}\exp\left[i(k_t z-\omega t)\right]$$

If E = 2 V/m and n = 1.5 then the application of appropriate boundary conditions leads to :

(a) $E_{0r} = -\frac{3}{5}V/m,\ E_{0t} = \frac{7}{5}V/m$ (b) $E_{0r} = -\frac{1}{5}V/m,\ E_{0t} = \frac{8}{5}V/m$

(c) $E_{0r} = -\frac{2}{5}V/m,\ E_{0t} = \frac{8}{5}V/m$ (d) $E_{0r} = \frac{4}{5}V/m,\ E_{0t} = \frac{6}{5}V/m$

25. A Plane electromagnetic wave of frequency w is incident normally on an air-dielectric interface. He dielectric is linear, isotropic, non-magnetic and its refractive index is n. The reflectance (R) and transmittance (T) from the interface are : **[GATE 2004]**

(a) $R = \left(\frac{n-1}{n+1}\right)^2,\ T = \frac{4n}{(n+1)^2}$ (b) $R = -\left(\frac{n-1}{n+1}\right),\ T = \frac{2}{(n+1)^2}$

(c) $R = \left(\frac{n-1}{n+1}\right)^3,\ T = \frac{4n^3}{(n+1)^3}$ (d) $R = \left(\frac{(n-1)^2}{n+1}\right),\ T = \frac{4n^2}{(n+1)^2}$

26. A left circularly polarized light beam of wavelength 600 nm is incident on a crystal of thickness d and propagated perpendicular to its optic axis. The ordinary and extraordinary refractive indices of the crystal are $n_0 = 1.54$ and $n_e = 1.55$ respectively. The emergent light will be right circularly polarized if d is : **[GATE 2002]**

(a) 120 μm (b) 60 μm (c) 30μm (d) 15μm

27. In a two beam interference pattern, the maximum and minimum intensity values are found to be 25 I_0 and 9 I_0 respectively, where I_0 is a constant. The intensities of the two interfereing beams are : **[GATE 2002]**

(a) 16 I_0 and I_0 (b) 5 I_0 and 3 I_0 (c) 17 I_0 and 8 I_0 (d) 8 I_0 and 2 I_0

28. A plane electromagnetic wave incident normally on the surface of a material is partially reflected. Measurements on the standing wave in the region in front of the interface show that the ratio of the electric field amplitude at the maxima and the minima is 5. The ratio of the reflected intensity to the incident intensity is

(a) 4/9 (b) 2/3 (c) 2/5 (d) 1/5 **[NET Dec. 2014]**

29. A beam of light of frequency ω is reflected from a dielectric-metal interface at normal incience. the refractive index of the dielectric medium is n and that of the metal is $n_2 = n(1+i\rho)$. If the beam is polarised parallel to the interface, then the phase change experienced by the light upon reflection is

(a) $\tan\left(\frac{2}{\rho}\right)$ (b) $\tan^{-1}\left(\frac{1}{\rho}\right)$ (c) $\tan^{-1}\left(\frac{2}{\rho}\right)$ (d) $\tan^{-1}(2\rho)$

30. Let (V, A) and (V', A') denote two sets of scalar and vector potentials, and ψ a scalar function. Which of the following transformations leave the electric and magnetic fields (and hence Maxwell's equations) unchanged? **[NET Dec. 2013]**

(a) $A' = A + \nabla\psi$ and $V = V - \dfrac{\partial\psi}{\partial t}$
(b) $A' = A - \nabla\psi$ and $V' = V + 2\dfrac{\partial\psi}{\partial t}$

(c) $A' = A + \nabla\psi$ and $V' = V + \dfrac{\partial\psi}{\partial t}$
(d) $A' = A - 2\nabla\psi$ and $V' = V - \dfrac{\partial\psi}{\partial t}$

31. A current I is created by a narrow beam of protons moving in vacuum with constant velocity $\vec{u}$. The direction and magnitude, respectively, of the Poynting vector $\vec{S}$ outside the beam at a radial distance r (much larger than the width of the beam) from the axis, are **[NET June 2013]**

(a) $\vec{S} \perp \vec{u}$ and $|\vec{S}| = \dfrac{I^2}{4\pi^2 \epsilon_0 |\vec{u}| r^2}$
(b) $\vec{S} \parallel (-\vec{u})$ and $|\vec{S}| = \dfrac{I^2}{4\pi^2 \epsilon_0 |\vec{u}| r^4}$

(c) $\vec{S} \parallel \vec{u}$ and $|\vec{S}| = \dfrac{I^2}{4\pi^2 \epsilon_0 |\vec{u}| r^2}$
(d) $\vec{S} \parallel \vec{u}$ and $|\vec{S}| = \dfrac{I^2}{4\pi^2 \epsilon_0 |\vec{u}| r^4}$

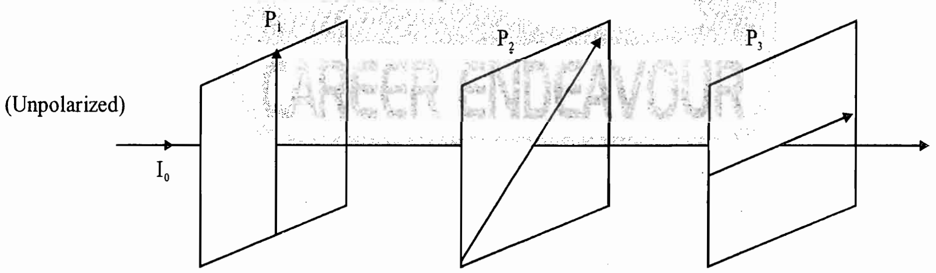
32. An electromagnetic wave is incident on a water-air interface. The phase of the perpendicular component of the electric field, $E_\perp$, of the reflected wave into the water is found to remain the same for ail angles of incidence. The phase of the magnetic field H. **[NET June 2012]**

(a) does not change (b) changes by $3\pi/2$ (c) changes by $\pi/2$ (d) changes by π

33. In a Young's double slit interference expriment, the slits are at a distance 2L from each other and the screen is at a distance D from the slits. If a glass slab of refractive index μ and thickness d is placed in the path of one of the beams, the minimum value of d for the central fringe to be dark is **[NET Dec. 2011]**

(a) $\dfrac{\lambda D}{(\mu-1)\sqrt{D^2+L^2}}$ (b) $\dfrac{\lambda D}{(\mu-1)L}$ (c) $\dfrac{\lambda}{(\mu-1)}$ (d) $\dfrac{\lambda}{2(\mu-1)}$

34. Consider three polarizers P_1, P_2 and P_3 placed along an axis as shown in the Figure. **[NET Dec. 2011]**



The pass axis of P_1 and P_3 are at right angles to each other while the pass axis of P_2 makes an angle θ with that of P_1. A beam of unpolarized light of intensity I_0 is incident on P_1 as shown. The intensity of light emerging from P_3 is **[NET Dec. 2011]**

(a) 0 (b) $\dfrac{I_0}{2}$ (c) $\dfrac{I_0}{8}\sin^2 2\theta$ (d) $\dfrac{I_0}{4}\sin^2 2\theta$

35. Given any arbitrary electric and magnetic field $\vec{E}(\vec{r},t)$ and $\vec{E}(\vec{r},t)$, it is always possible to choose the scalar potential $\phi(\vec{r},t)$ and vector potential $\vec{A}(\vec{r},t)$ such that **[NET 2008]**

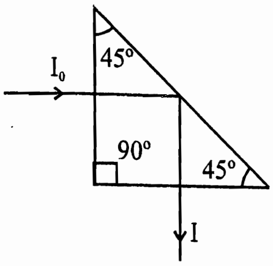
(a) $\vec{A}(\vec{r},t)$ is identically zero.

(b) $\phi(\vec{r},t)$ is identically zero.

(c) $|\vec{A}(\vec{r},t)|$ is any given non-zero constant.

(d) The conditions $\nabla.\vec{A}=0$ and $\frac{1}{c}\frac{\partial\phi}{\partial t}+\nabla.\vec{A}=0$ are simultaneously satisfied.

36. Circularly polarized light with intensity I_0 is incident normally on a glass prism as shown in the figure. The index of refraction of glass is 1.5. The intensity I of light emerging from the prism is: **[NET June 2011]**



(a) I_0 (b) $0.96\ I_0$ (c) $0.92\ I_0$ (d) $0.88\ I_0$.

37. In an interference pattern formed by two coherent sources, the maximum and the minimum of the intensities are $9I_0$ and I_0, respectively. The intensities of the individual waves are **[GATE 2014]**

(a) $3I_0$ and I_0 (b) $4I_0$ and I_0 (c) $5I_0$ and $4I_0$ (d) $9I_0$ and I_0

38. Which of the following is Lorentz Invarient **[NET June 2009]**

(a) $E^2-B^2C^2$ (b) $\vec{E}\times\vec{B}$ (c) E^2-BC^2 (d) $\vec{E}\times\vec{C}$

39. If $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields respectively, identify the Lorentz invariant quantity from the following. **[NET June 2009]**

(a) $\vec{E}^2\ \vec{B}$ (b) $|\vec{E}\times\vec{B}|^2$ (c) $\vec{E}^2-c^2\vec{B}^2$ (d) $\vec{E}^2+c^2\vec{B}^2$

40. Which of the following quantities is Lorentz invariant?

(a) $|E\times B|^2$ (b) $|E|^2-|B|^2$ (c) $|E|^2+|B|^2$ (d) $|E|^2\,|B|^2$

41. An observer in an initial frame that a point P the electric field vanishes but the magnetic field does not. This implies that in any other initial frame the electric field $\vec{E}$ and the magnetic field $\vec{B}$ satisfy **[JEST 2012]**

(a) $|\vec{E}|^2=|B|^2$ (b) $\vec{E}.\vec{B}=0$ (c) $\vec{E}\times\vec{B}=0$ (d) $\vec{E}=0$

42. An electron is moving with an acceleration 10 m/s^2 whereas a positron is moving with an acceleration 20 m/s^2. The ratio of the power radiated by the positron to that of electron is:
(a) 2 (b) 1/2 (c) 1/4 (d) 4

43. Consider the electric and magnetic fields of an accelerating charge. How should the fields vary with r (the retarded distance) for the poynting vector to remain finite at arbitrary large distances?

(a) $1/r$ (b) $1/r^2$ (c) r (d) r^2

44. When a plane electromagnetic wave is incident angle i= 60°, the reflected light is completely polarized. The dielectric constant is

(a) 4/3 (b) $\sqrt{3}$ (c) 2 (d) 3

45. A dipole is formed by charges q and –q. The energy radiated by the dipole is maximum.
(a) Along the line joining the charges
(b) Along the line perpendicular to the dipole and passing midway between the dipole.
(c) Along the line inclined to the dipole axis at + 45°.
(d) At the centre of the dipole.

46. In a given frame of refrence, it is found that the electric field $\vec{E}(\vec{r},t)$ and the magnetic field $\vec{B}(\vec{r},t)$ are perpendicular to each other at all points, i.e., $\vec{E}(\vec{r},t).\vec{B}(\vec{r},t)=0$. If the fields observed in any other inertial frame are $\vec{E}'$ and $\vec{B}'$, then **[NET 2008]**

(a) $\vec{E}'//\vec{B}'$ at all points (b) $\vec{E}'.\vec{B}'<0$ at all points

(c) $\vec{E}'.\vec{B}'>0$ (d) $\vec{E}'.\vec{B}'=0$

47. Consider the following three independent cases : **[GATE 2005]**

(i) Particle A of charge +q moves in free space with a constant velocity $\upsilon\,(\upsilon <<$ speed of light$)$
(ii) Particle B of charge +q moves in free space in a circle of radius R with same speed υ as in case (i)
(iii) Particle C having charge –q moves as in case (ii)

If the powers radiated by A, B and C are P_A, P_B and P_C respectively, then :

(a) $P_A=0, P_B>P_C$ (b) $P_A=0, P_B=P_C$

(c) $P_A>P_B>P_C$ (d) $P_A=P_B=P_C$

48. The electromagnetic field due to a point charge must be described by Lienard Weichert potentials when
(a) the point charge is highly accelerated **[GATE 2007]**
(b) the electric and magnetic fields are not perpendicular
(c) the point charge is moving with velocity close to that of light.
(d) the calculation is done for the radiation zone, i.e. far away from the charge.

49. The electric and magnetic fields caused by an accelerated charged particle are found to scale as $E\propto r^{-n}$ and $B\propto r^{-m}$ at large distances. What are the values of *n* and *m*? **[JEST 2013]**

(a) $n=1, m=2$ (b) $n=2, m=1$ (c) $n=1, m=1$ (d) $n=2, m=2$

50. The ratio of cyclotron frequencies of an α-particle and a proton moving in the same magnetic field $\vec{B}$ is:

(a) 1/2 (b) 2 (c) 4 (d) 1/4

51. A circular loop of wire of radius a carries an oscillating current I of frequency ω. The electric and magnetic fields at distance r much larger than $a\,(r>>a)$ very as

(a) $E\propto\frac{1}{r},\ B\propto\frac{1}{r^2}$ (b) $E\propto\frac{1}{r^2},\ B\propto\frac{1}{r^2}$ (c) $E\propto\frac{1}{r},\ B\propto\frac{1}{r}$ (d) $E\propto\frac{1}{r^2},\ B\propto\frac{1}{r}$

52. An oscillating current $I(t)=I_0\exp(-i\omega t)$ flows in the direction of the y-axis through a thin metal sheet of area 1.0 cm^2 kept in the xy-plane. The rate of total energy radiated per unit area from the surfaces of the metal sheet at a distance of 100m is **[NET June 2013]**

(a) $I_0\omega/(12\pi\varepsilon_0c^3)$ (b) $I_0^2\omega^2/(12\pi\varepsilon_0c^3)$ (c) $I_0^2\omega/(12\pi\varepsilon_0c^3)$ (d) $I_0\omega^2/(24\pi\varepsilon_0c^3)$

53. A non-relativistic particle of mass *m* and charge *e*, moving with a velocity $\vec{v}$ and acceleration $\vec{a}$, emits radiation of intensity *I*. What is the intensity of the radiation emitted by a particle of mass *m*/2, charge 2*e*, velocity $\vec{v}/2$ and acceleration $2\vec{a}$? **[NET Dec. 2014]**

(a) 16 *I* (b) 8 *I* (c) 4 *I* (d) 2 *I*

54. In an electromagnetic field, which one of the following remains invariant under Lorentz transformation? **[GATE 2006]**

(a) $\vec{E}\times\vec{B}$ (b) $E^2-c^2B^2$ (c) B^2 (d) E^2

55. The electric (E) and magnetic (B) field amplitudes associated with an electromagnetic radiation from a point source behave at a distance r from the source as : **[GATE 2005]**

(a) E = constant, B = constant (b) $E\propto\frac{1}{r}, B\propto\frac{1}{r}$

(c) $E\propto\frac{1}{r^2}, B\propto\frac{1}{r^2}$ (d) $E\propto\frac{1}{r^3}, B\propto\frac{1}{r^3}$

56. The electric and the magnetic fields $\vec{E}(z,t)$ and $\vec{B}(z,t)$, respectively corresponding to the scalar potential $\phi(z,t)=0$ and vector potential $\vec{A}(z,t)=\hat{i}tz$ are **[GATE 2012]**

(a) $\vec{E}=\hat{i}z$ and $\vec{B}=-\hat{j}t$ (b) $\vec{E}=\hat{i}z$ and $\vec{B}=\hat{j}t$ (c) $\vec{E}=-\hat{i}z$ and $\vec{B}=-\hat{j}t$ (d) $\vec{E}=-\hat{i}z$ and $\vec{B}=\hat{j}t$

57. An electro-magnetic field is defined by a scalar potential $\phi(\vec{r},t)=0$ and vector potential $\vec{A}(\vec{r},t)=\frac{1}{4\pi\varepsilon_0}\frac{qt}{r^2}\hat{r}$.

The corresponding electric field is :

(a) $\vec{E}=0$ (b) $\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$ (c) $\vec{E}=-\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$ (d) $\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\theta}$

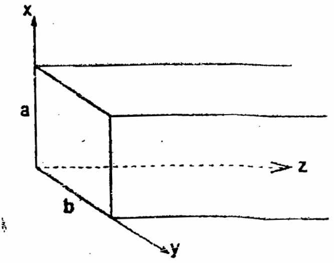
58. The angular distribution of the average power radiated from a short dipole antenna of length d, is dependent on d and polar angle θ. This dependence is proportional to

(a) $\sin^2\theta$ (b) $d\sin^2\theta$ (c) $d^2\sin^2\theta$ (d) $d\sin\theta$

59. A rod of length *L* carries a total charge *Q* distributed uniformly. If this is observed in a frame moving with a speed *v* along the rod, the charge per unit length (as measured by the moving observer) is

(a) $\frac{Q}{L}\left(1-\frac{v^2}{c^2}\right)$ (b) $\frac{Q}{L}\sqrt{1-\frac{v^2}{c^2}}$ (c) $\frac{Q}{L\sqrt{1-\frac{v^2}{c^2}}}$ (d) $\frac{Q}{L\left(1-\frac{v^2}{c^2}\right)}$ **[NET June 2015]**

60. The magnetic field of the TE_{11} mode of a rectangular waveguide of dimensions a×b as shown in the figure is given by $H_z=H_0\cos(0.3\pi x)\cos(0.4\pi y)$, where x and y are in cm. **[NET June 2011]**



A. The dimensions of the waveguide are

(a) a = 3.33 cm, b = 2.50 cm (b) a = 0.40 cm, b = 0.30 cm

(c) a = 0.80 cm, b = 0.60 cm (d) a = 1.66 cm, b = 1.25 cm

B. The entire range of frequencies f for which the TE_{11} mode will propagate is:

(a) 6.0 GHz < f < 7.5 GHz (b) 7.5 GHz < f < 9.0 GHz

(c) 7.5 GHz < f < 12.0 GHz (d) 7.5 GHz < f

61. Electromagnetic waves are propagating along a hollow, metallic waveguide whose cross-section is a square of side W. The minimum frequency of the electromagnetic waves is :

(a) c/W (b) 2c/W (c) $\pi c / W$ (d) $\sqrt{2}\,\pi c / W$ **[GATE 2003]**

62. For a wave propagating in an air filled metallic rectangular waveguide, the frequency and the wave vector are related as (c is the speed of light and ω_c is the cut off frequency)

(a) $c^2k^2 = \omega^2 + \omega_c^2$ (b) $c^2k^2 = \omega^2$ (c) $c^2k^2 = \omega^2 - \omega_c^2$ (d) $c^2k^2 = \sqrt{\omega^4 - \omega_c^4}$

63. An electromagnetic wave propagating along the z-direction in a rectangular waveguide in the TE mode is characterized by

(a) $E_z = 0, H_z \neq 0$ (b) $H_z = 0, E_z \neq 0$ (c) $E_z = 0, H_z = 0$ (d) $E_z \neq 0, H_z \neq 0$

64. Consider a rectangular wave guide with transverse dimensions 2 m × 1 m driven with an angular frequency $\omega = 10^9$ rad/s. Which transverse electric (TE) modes will propagate in this wave guide ? **[NET June 2015]**

(a) TE_{10}, TE_{01} and TE_{20} (b) TE_{10}, TE_{11} and TE_{20}
(c) TE_{01}, TE_{10} and TE_{11} (d) TE_{01}, TE_{10} and TE_{22}

65. In a distortionless transmission line the attenuation constant α is frequency independent while the propagation constant depends linearly on frequency. If R, L, C and G denote the transmission line parameters, for the distortion, less line

(a) $RG = \frac{1}{\sqrt{LC}}$ (b) $\frac{R}{L} = \frac{C}{G}$ (c) $R = \sqrt{\frac{L}{C}}$ (d) $\frac{R}{L} = \frac{G}{C}$

66. Sunlight is incident on the surface of the sea at an angle θ. A sailor viewing through a rotating polaroid can almost block the reflected light by adjusting the angle of the polaroid. The value of θ is about (Refractive index of water is 1.33):

(a) 57° (b) 53° (c) 49° (d) 41°

67. The values of the electric and magnetic fields in a particular reference frame (in Gaussian units) are $\boldsymbol{E} = 3\hat{x} + 4\hat{y}$ and $\boldsymbol{B} = 3\hat{z}$, respectively. An inertial observer moving with respect to this frame measures the magnitude of the electric field to be $|\boldsymbol{E}'| = 4$. The magnitude of the magnetic field $|\boldsymbol{B}'|$ measured by him is

(a) 5 (b) 9 (c) 0 (d) 1

68. A wavelength has a square cross-section of side $2a$. For the TM modes of wavevector k, the transverse electromagnetic modes are obtained in terms of a function $\psi(x, y)$ which obeys the equation

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \left(\frac{\omega^2}{c^2} - k^2\right)\right]\psi(x, y) = 0$$

with the boundary condition $\psi(\pm a, y) = \psi(x, \pm a) = 0$. The frequency ω of the lowest mode is given by

(a) $\omega^2 = c^2\left(k^2 + \frac{4\pi^2}{a^2}\right)$ (b) $\omega^2 = c^2\left(k^2 + \frac{\pi^2}{a^2}\right)$

(c) $\omega^2 = c^2\left(k^2 + \frac{\pi^2}{2a^2}\right)$ (d) $\omega^2 = c^2\left(k^2 + \frac{\pi^2}{4a^2}\right)$

69. Two coherent monochromatic light beams of intensities I and 4I are superimposed. The maximum and minimum possible internsities in the resulting beam are

(a) 3I and I (b) 5I and 3I (c) 7I and 5I (d) 9I and I

70. An unpolarized ray of light in a medium with refractive index n_1 is incident on a plane glass surface with refractive index $n_2 (n_2 > n_1)$. The intensity of the reflected light is minimum when the angle of incidence i is given by

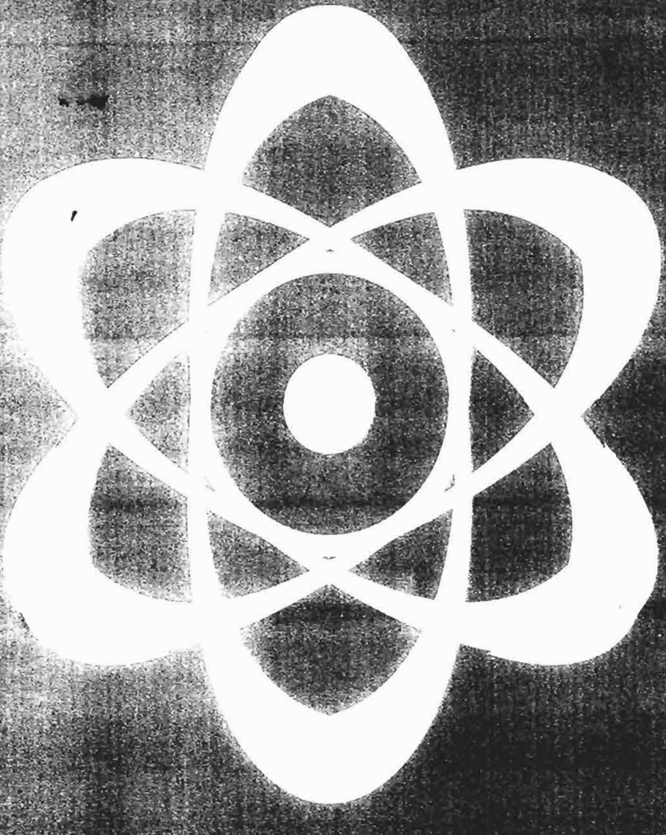
(a) $\tan(i) = n_1 / n_2$ (b) $\tan(i) = n_2 / n_1$

(c) $\tan(i) = 1 / n_1 n_2$ (d) $\tan(i) = n_1 n_2$

ELECTRONICS

CSIR-UGC-NET Dec. 2016 | GATE 2017

NET | GATE | TIFR | JEST



Previous Years' Questions

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ELECTRONICS

Previous Years' Questions

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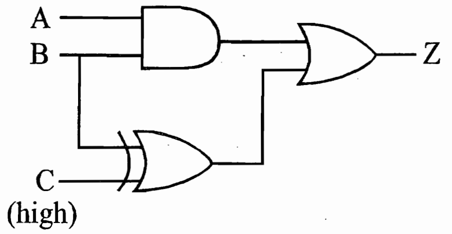
CSIR-UGC-NET Previous Years' Questions

1. Consider the digital circuit shown below in which the input C is always high (1). **[NET June 2011]**



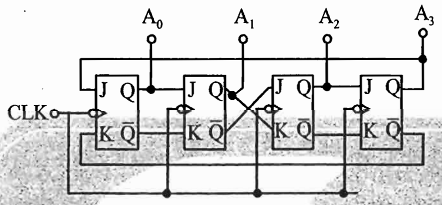
The truth table for the circuit can be written as

A	B	Z
0	0	
0	1	
1	0	
1	1	

The entries in the Z column (vertically) are

(a) 1011 (b) 0100 (c) 1111 (d) 1010

2. A counter consists of four flip-flops connected as shown in the figure. **[NET Dec. 2011]**

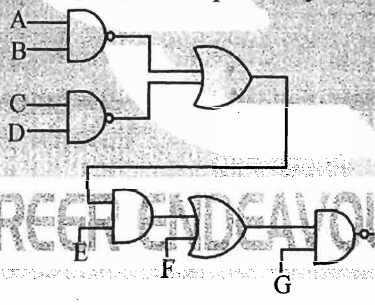


If the counter is initialized as $A_0\,A_1\,A_2\,A_3 = 0110$, the state after the next clock pulse is

(a) 1000 (b) 0001 (c) 0011 (d) 1100

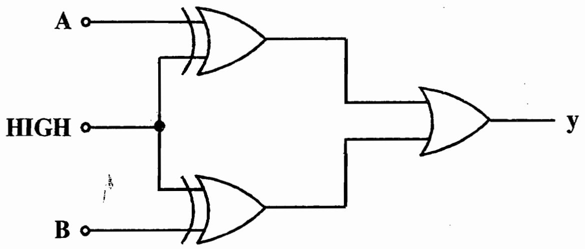
3. The output 0, of the given circuit in cases I and II, where **[NET June 2012]**

Case-I: A, B = 1; C, D = 0; E, F = 1 and G = 0
Case-II: A,B = 0; C, D = 0; E, F = 0 and G = 1 are respectively



(a) 1, 0 (b) 0, 1 (c) 0, 0 (d) 1, 1

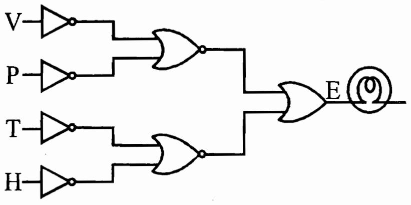
4. The logic circuit shown in the figure below **[NET Dec. 2012]**



implements the Boolean expression

(a) $y = \overline{A.B}$ (b) $y = \bar{A}.\bar{B}$ (c) $y = A.B$ (d) $y = A + B$

5. Four digital outputs V, P, T and H monitor the speed v, tyre pressure p, temperature t and relative humidity h of a car. These outputs switch from 0 to 1 when the values of the parameters exceed 85 km/hr, 2 bar, 40°C and 50%, respectively. A logic circuit that is used to switch ON a lamp at the output E is shown below. **[NET June 2013]**



Which of the following conditions will switch the lamp ON?

(a) $v < 85\text{km/hr}, p < 2\text{bar}, t > 40^\circ\text{C}, h > 50\%$

(b) $v < 85\text{km/hr}, p < 2\text{bar}, t > 40^\circ\text{C}, h < 50\%$

(c) $v > 85\text{km/hr}, p < 2\text{bar}, t > 40^\circ\text{C}, h < 50\%$

(d) $v > 85\text{km/hr}, p < 2\text{bar}, t < 40^\circ\text{C}, h > 50\%$

6. If one of the inputs of a J-K flip flop is high and the other is low, then the outputs Q and $\bar{Q}$ **[NET Dec. 2013]**

(a) oscillate between low and high in race-around condition

(b) toggle and the circuit acts like a T flip flop

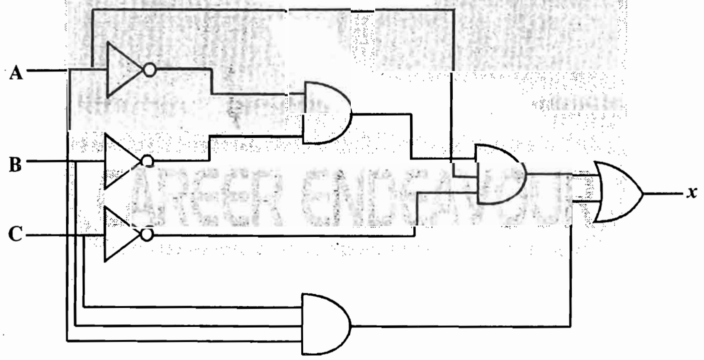
(c) are opposite to the inputs

(d) follow the inputs and the circuit acts like an R-S flip flop

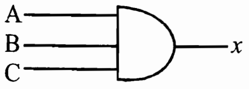
7. A 4-variable switching function is given by $f = \Sigma(5,7,8,10,13,15) + d(0,1,2)$, where d is the do-not-care-condition. The minimized form of f in sum of products (SOP) form is **[NET Dec. 2013]**

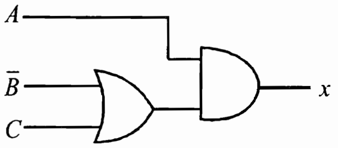
(a) $\bar{A}\bar{C} + \bar{B}\bar{D}$ (b) $A\bar{B} + C\bar{D}$ (c) $AD + BC$ (d) $\bar{B}\bar{D} + BD$

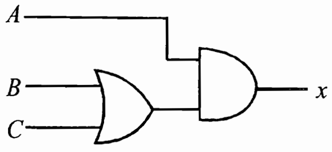
8. For the logic circuit shown in the figure below **[NET June 2014]**

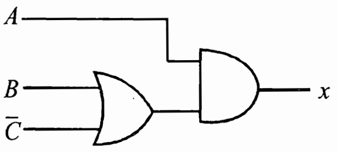


a simplified equivalent circuit is

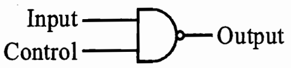
(a)


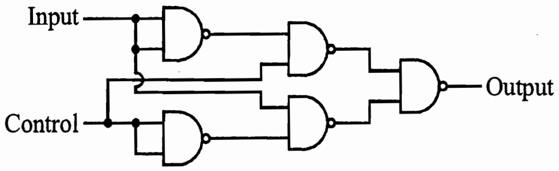
(b)


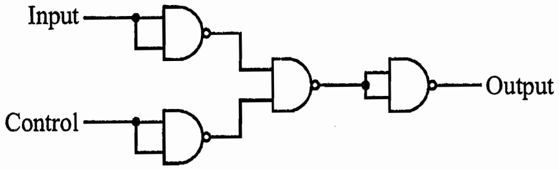
(c)


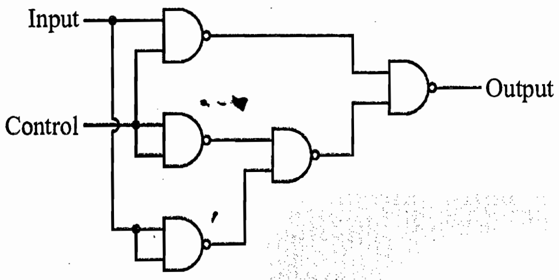
(d)


9. Which of the following circuits behaves as a controlled inverter? **[NET June 2015]**

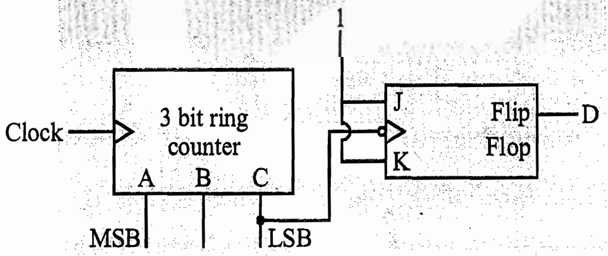
(a)

Input
Control
Output

(b)

Input
Control
Output

(c)

Input
Control
Output

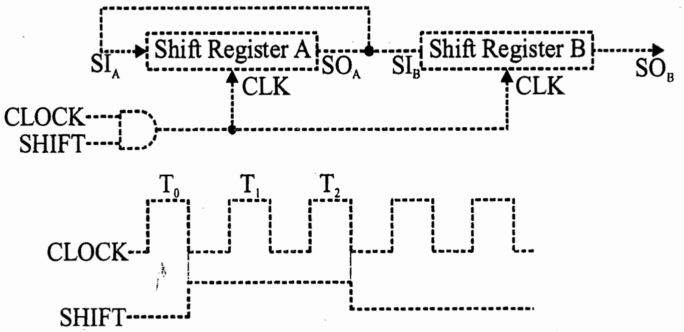
(d)

Input
Control
Output

10. For the logic circuit given below, the decimal count sequence and the modulus of the circuit corresponding to A B C D are **[NET June 2015]**


1
Clock
3 bit ring counter
A
B
C
MSB
LSB
J
K
Flip Flop
D

(a) $8 \to 4 \to 2 \to 1 \to 9 \to 5$ (mod 6)
(b) $8 \to 4 \to 2 \to 9 \to 5 \to 3$ (mod 6)
(c) $2 \to 5 \to 9 \to 1 \to 3$ (mod 5)
(d) $8 \to 5 \to 1 \to 3 \to 7$ (mod 5)

11. In the schematic figure given below, the initial values of 4 bit shift registers A and B are 1011 and 0010 respectively. The values of SO_A and SO_B after the pulse T_2 are respectively. **[NET Dec. 2015]**

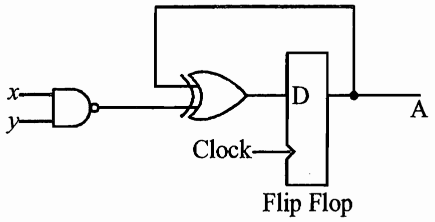

Shift Register A
Shift Register B
SI_A
SO_A
SI_B
SO_B
CLK
CLK
CLOCK
SHIFT
T_0
T_1
T_2
CLOCK
SHIFT

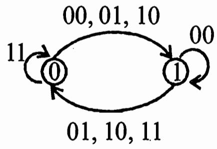
(a) 1110 and 1001
(b) 1101 and 1001
(c) 1101 and 1100
(d) 1110 and 1100

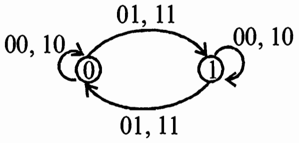
12. Two data sets A and B consist of 60 and 10 readings of a voltage measured using voltmeters of resolution of 1mV and 0.5mV respectively. The uncertainty in the mean voltage obtained from the data sets A and B are U_A and U_B, respectively. If the uncertainty of the mean of the combined data sets is U_{AB}, then which of the following statements is correct? **[NET Dec. 2015]**

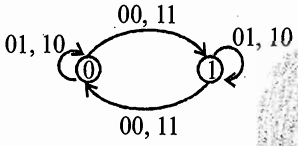
(a) $U_{AB} < U_A$ and $U_{AB} > U_B$
(b) $U_{AB} < U_A$ and $U_{AB} < U_B$
(c) $U_{AB} > U_A$ and $U_{AB} < U_B$
(d) $U_{AB} > U_A$ and $U_{AB} > U_B$

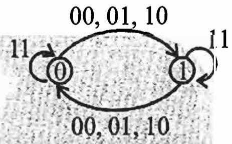
13. The state diagram corresponding to the following circuit is **[NET Dec. 2015]**


x
y
D
A
Clock
Flip Flop

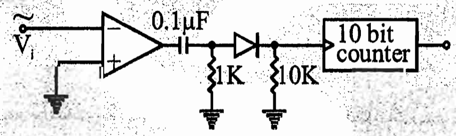
(a)

00, 01, 10
11
00
0
1
01, 10, 11

(b)

01, 11
00, 10
00, 10
0
1
01, 11

(c)

00, 11
01, 10
01, 10
0
1
00, 11

(d)

00, 01, 10
11
11
0
1
00, 01, 10

14. A sinusoidal signal of peak to peak amplitude 1V and unknown time period is input to the following circuit for 5 seconds duration. If the counter measures a value $(3E8)_H$ in hexadecimal then the time period of the input signal is **[NET Dec. 2015]**


~
V_i
−
+
0.1μF
1K
10K
10 bit counter

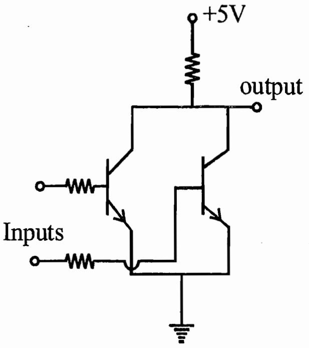
(a) 2.5 ms (b) 4 ms (c) 10 ms (d) 5 ms

ANSWER KEY

1. (a)	**2. (c)**	**3. (d)**	**4. (a)**	**5. (a)**	**6. (d)**	**7. (d)**
8. (a)	**9. (b)**	**10. (b)**	**11. (d)**	**12. (b)**	**13. (d)**	**14. (d)**

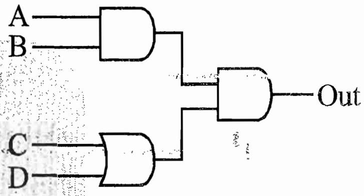
GATE Previous Years' Questions

1. The circuit shown in figure below acts as a **[GATE 1997]**



(a) NOR gate (b) NAND gate
(c) AND gate (d) XOR gate

2. If the output of the logic circuit shown in the figure is 1, the input could be **[GATE 2000]**



(a) A = 1, B = 1, C = 1, D = 0
(b) A = 1, B = 1, C = 0, D = 0
(c) A = 1, B = 0, C = 1, D = 1
(d) A = 0, B = 1, C = 1, D = 1.

3. Which of the following options is true for a two input XOR gate? **[GATE 2002]**

	Input		**Output**
	A	B	
(a)	0	1	1
(b)	1	0	0
(c)	0	0	1
(d)	1	1	1

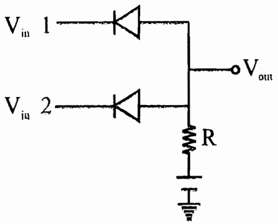
4. Which of the given relations between the Boolean variables P and Q is NOT correct? (In the notation used here, P′ denotes NOT P and Q′ denotes NOT Q) **[GATE 2003]**

(a) $PQ' + PQ = P$ (b) $(PQ)' = P' + Q'$ (c) $PQ' = (P' + Q)'$ (d) $PQ' + Q = P$

5. A half-adder is a digital circuit with **[GATE 2004]**

(a) Three inputs and one output (b) Three inputs and two outputs
(c) Two inputs and one output (d) Two input and two outputs.

6. The circuit shown can be used as **[GATE 2005]**

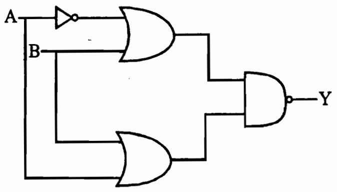


(a) NOR gate (b) OR gate (c) NAND gate (d) AND gate

7. The Boolean expression: $B(A+B) + A.(\bar{B}+A)$ can be realized using minimum number of

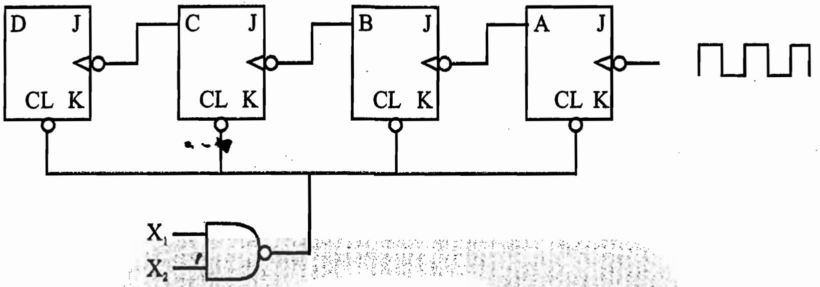
(a) 1 AND gate (b) 2 AND gates (c) 1 OR gate (d) 2 OR gates. **[GATE 2005]**

8. In the given digital logic circuit, A and B form the input. The output Y is: **[GATE 2006]**



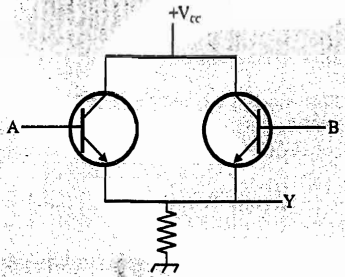
(a) $Y = \bar{A}$ (b) $Y = A\bar{B}$ (c) $Y = A + B$ (d) $Y = \bar{B}$

9. A ripple counter designed with JK flip-flops provided with CLEAR (CL) input is shown in the figure. In order that this circuit functions as a MOD-12 counter, the NAND gate input (X_1 and X_2) should be



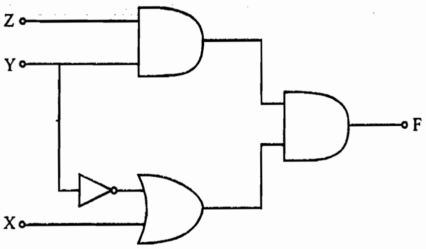
(a) A and C (b) A and D (c) B and D (d) C and D **[GATE 2006]**

10. The circuit shown in the figure function as **[GATE 2006]**



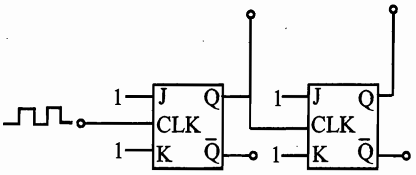
(a) an OR gate (b) an AND gate (c) a NOR gate (d) a NAND gate

11. Identify the function F generated by the logic network shown **[GATE 2007]**



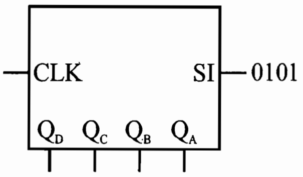
(a) $F = (X + Y)Z$ (b) $F = Z + Y + \bar{Y}X$ (c) $F = ZY(Y + X)$ (d) $F = XYZ$

12. In the circuit shown, the ports Q_1 and Q_2 are in the state $Q_1 = 1, Q_2 = 0$. The circuit is now subjected to two complete clock pulses. The state of these ports now becomes **[GATE 2007]**



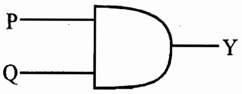
(a) $Q_2 = 1, Q_1 = 0$ (b) $Q_2 = 0, Q_1 = 1$ (c) $Q_2 = 1, Q_1 = 1$ (d) $Q_2 = 0, Q_1 = 0$

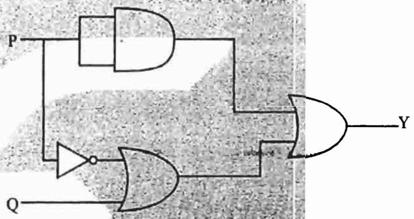
13. The registers Q_D, Q_C, Q_B and Q_A shown in the figure are initially in the state 1010 respectively. An input sequence SI=0101 is applied. After two clock pulses, the state of the shift registers (in the same sequence $Q_D\ Q_C\ Q_B\ Q_A$) is: **[GATE 2007]**

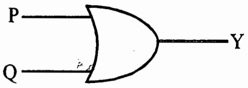


(a) 1001 (b) 0100 (c) 0110 (d) 1010

14. The simplest logic gate circuit corresponding to the Boolean expression, $Y = P + PQ$ is :

(a)


(b)


(c)


(d) None of these **[GATE 2008]**

15. An analog voltage V is converted into 2-bit binary number. The minimum number of comparators required and their reference voltages are **[GATE 2008]**

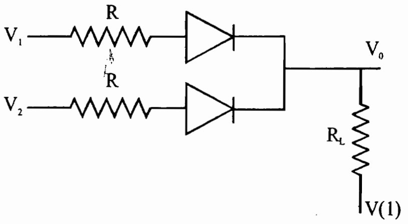
(a) $3, \left(\frac{V}{4}, \frac{V}{2}, \frac{3V}{4}\right)$

(b) $3, \left(\frac{V}{3}, \frac{2V}{3}, V\right)$

(c) $4, \left(\frac{V}{5}, \frac{2V}{5}, \frac{3V}{5}, \frac{4V}{5}\right)$

(d) $4, \left(\frac{V}{4}, \frac{V}{2}, \frac{3V}{5}, V\right)$

16. The following circuit (where $R_L >> R$) performs the operation of **[GATE 2008]**



(a) OR gate for a negative logic system

(b) NAND gate for a negative logic system.

(c) AND gate for a positive logic system.

(d) AND gate for a negative logic system.

Statement for Linked Q.17 and Q.18. **[GATE 2009]**

The Karnaugh map of a logic circuit is shown below:

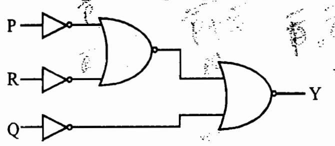
	$\bar{R}$	R
$\bar{P}\bar{Q}$	1	1
$\bar{P}Q$	1	
PQ		
$P\bar{Q}$	1	1

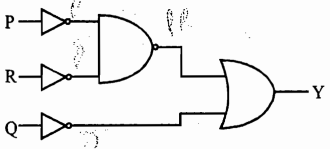
$\bar{Q} + \overline{RP}$

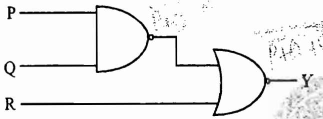
17. The minimized logic expression for the above map is:

(a) $Y = \bar{P}\bar{R} + \bar{Q}$ (b) $Y = \bar{Q}.PR$ (c) $Y = \bar{Q} + PR$ (d) $Y = Q.\overline{PR}$

18. The corresponding logic implementation using gates is given as:

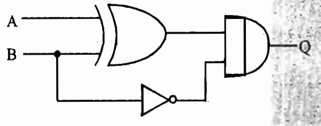
(a) 

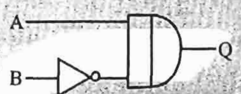
(b) 

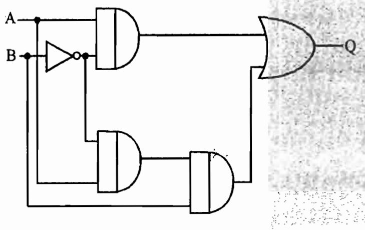
(c) 

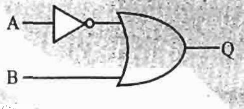
(d) None of the above

19. For any set of inputs, A and B, the following circuits give the same output Q, except one. Which one is it? **[GATE 2010]**

(a) 

(b) 

(c) 

(d) 

20. The following Boolean expression **[GATE 2011]**

$Y = A \bullet \bar{B} \bullet \bar{C} \bullet \bar{D} + \bar{A} \bullet B \bullet \bar{C} \bullet D + \bar{A} \bullet \bar{B} \bullet C \bullet D + \bar{A} \bullet \bar{B} \bullet \bar{C} \bullet D + \bar{A} \bullet B \bullet C \bullet D + A \bullet \bar{B} \bullet \bar{C} \bullet D$

can be simplified to

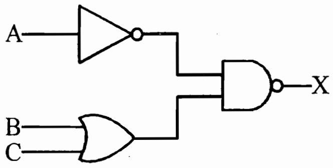
(a) $\bar{A} \bullet \bar{B} \bullet C + A \bullet \bar{D}$ (b) $\bar{A} \bullet B \bullet \bar{C} + A \bullet \bar{D}$ (c) $A \bullet \bar{B} \bullet \bar{C} + \bar{A} \bullet D$ (d) $A \bullet \bar{B} \bullet C + \bar{A} \bullet D$

21. The minimum number of flip-flops required to construct a mod-75 counter is _________ **[GATE 2014]**

22. Which one of the following DOES NOT represent an exclusive OR operation for inputs A and B?

(a) $(A+B)\overline{AB}$ (b) $A\bar{B} + B\bar{A}$ (c) $(A+B)(\bar{A}+\bar{B})$ (d) $(A+B)AB$ **[GATE 2015]**

23. For the digital circuit given below, the output X is **[GATE 2016]**



(a) $\overline{\bar{A}+B \cdot C}$ (b) $\overline{\bar{A} \cdot (B+C)}$ (c) $\bar{A} \cdot (B+C)$ (d) $A + \overline{(B \cdot C)}$

ANSWER KEY

1. (a)	**2.** (a)	**3.** (a)	**4.** (d)	**5.** (d)	**6.** (d)	**7.** (c)
8. (d)	**9.** (d)	**10.** (a)	**11.** (d)	**12.** (d)	**13.** (a)	**14.** (d)
15. (a)	**16.** (a)	**17.** (a)	**18.** (d)	**19.** (d)	**20.** (c)	
21. (6.99 – 7.01)		**22.** (d)	**23.** (b)			

$\overline{\overline{P}+Q}$ $= P\overline{Q}$ $\overline{PQ+\overline{Q}}$

PQ

$\overline{\overline{P}+Q}+R$

$\overline{\overline{P}.\overline{Q}+R}$

$\overline{\overline{P}.\overline{Q}}.\overline{R}$

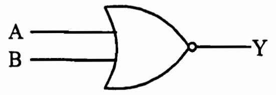
$\overline{\overline{P}+\overline{Q}}.\overline{R}$

$(P+Q)\overline{R}$

$PR + QR$

TIFR Previous Years' Questions

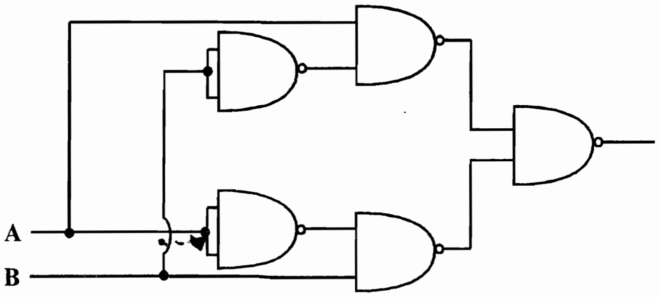
1. Consider the circuit shown below. **[TIFR 2012]**



The minimum number of NAND gates required to design this circuit is

(a) 6 (b) 5 (c) 4 (d) 3

2. The circuit shown below uses only NAND gates. Find the final output? **[TIFR 2013]**



(a) A XOR B (b) A AND B (c) A OR B (d) A NOR B.

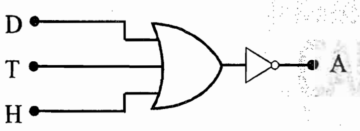
3. A control circuit needs to be designed to save on power consumption by an air-conditioning unit A in a windowless room with a single door. The room is fitted with the following devices: **[TIFR 2014]**

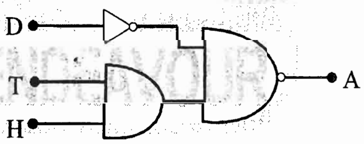
(1) A temperature sensor T, which is enabled (T = 1) whenever the temperature falls below a pre-set value;

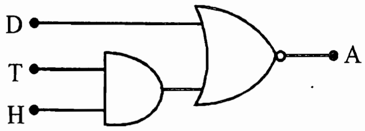
(2) A humidity sensor H, which is enabled (H = 1) whenever the humidity falls below a certain pre-set value;

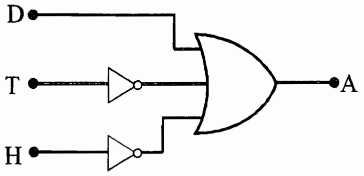
(3) A sensor D on the door, which is triggered (D = 1) whenever the door opens

Which of the following logical circuits will turn the air-conditioning unit off (A = 0) whenever the door is opened or when the both temperature and humidity are below their pre-set values?

(a)


(b)


(c)


(d)


4. To measure the voltage in the range 0 – 5 V with a precision of 5 mV, the minimum number of bits required in a digital voltmeter is **[TIFR 2015]**

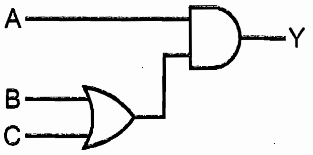
(a) 9 (b) 10 (c) 11 (d) 12

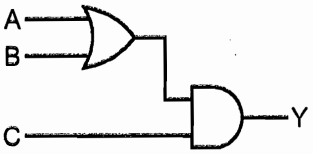
5. In a digital circuit for three input signals (A, B and C) the final output (Y) should be such that for inputs

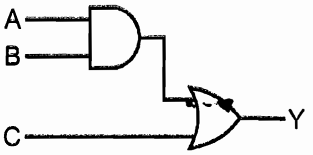
A	B	C
0	0	0
0	0	1
0	1	0

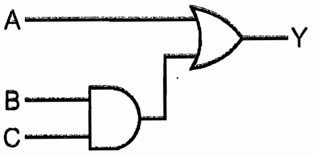
[TIFR 2016]

the output (Y) should be low and for all other cases it should be high. Which of the following digital circuits will give such output?

(a)


(b)


(c)


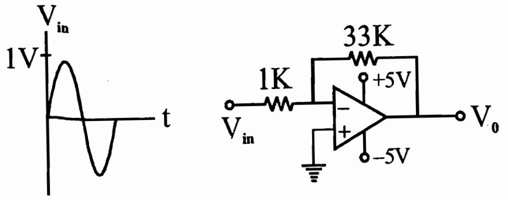
(d)


ANSWER KEY

1. (c) **2. (a)** **3. (c)** **4. (b)** **5. (d)**

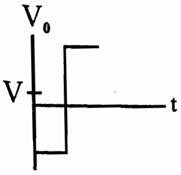
CSIR-UGC-NET Previous Years' Questions

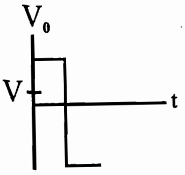
1. For the OP-AMP circuit shown in the figure below, which is the correct output waveform?

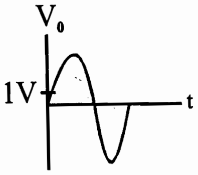


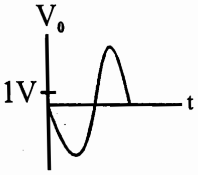
V_{in}
1V
t
33K
1K
+5V
V_{in}
V_0
–5V

[NET Dec. 2010]

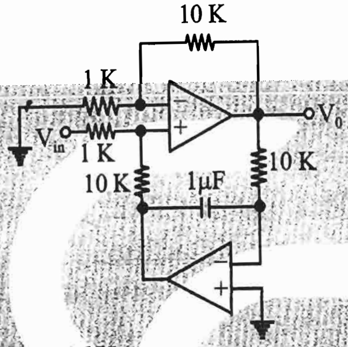
(a)


(b)


(c)


(d)


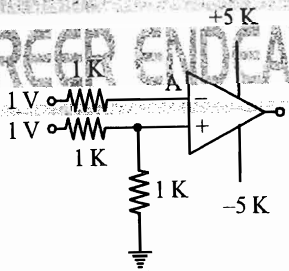
2. A time varying signal V_{in} is fed to an op-amp circuit with output signal V_0 as shown in the figure below. The circuit implements a **[NET June 2011]**



10 K
1 K
V_{in}
1 K
V_0
10 K
1μF
10 K

(a) High pass filter with cutoff frequency 16 Hz.
(b) High pass filter with cutoff frequency 100 Hz
(c) Low pass filter with cutoff frequency 16 Hz
(d) Low pass filter with cutoff frequency 100 Hz.

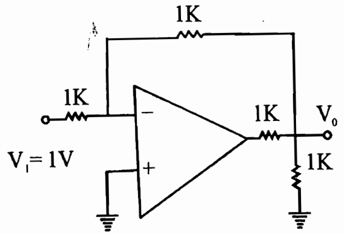
3. In the operational amplifier circuit below, the voltage at point A is **[NET Dec. 2011]**



+5 K
1K
A
1 V
1 V
1 K
1 K
–5 K

(a) 1.0 V (b) 0.5 V (c) 0 V (d) -5.0V

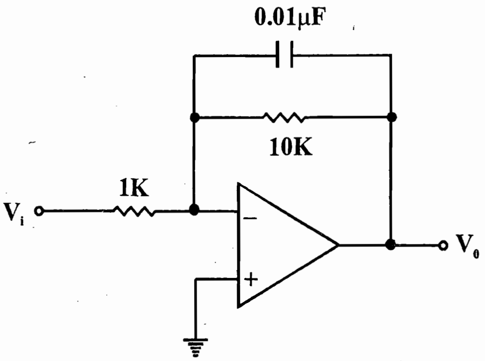
4. In the op-amp circuit shown in the figure below, the input voltage V_i is 1V. The value of the output V_0 is:



1K
1K
1K
V_0
V_i= 1V
1K

(a) –0.33 V (b) –0.50 V **[NET June 2012]**
(c) –1.00 V (d) –0.25 V

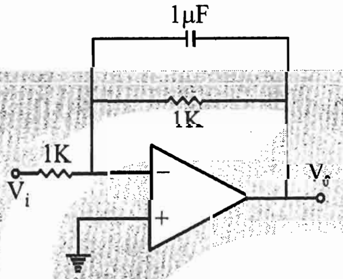
5. In the op-amp circuit shown in the figure, V_i is a sinusoidal input signal of frequency 10 Hz and V_0 is the output signal. **[NET Dec. 2012]**



The magnitude of the gain and the phase shift, respectively, are close to the values

(a) $5\sqrt{2}$ and $\frac{\pi}{2}$ (b) $5\sqrt{2}$ and $\frac{-\pi}{2}$ (c) 10 and zero (d) 10 and π

6. Consider the op-amp circuit shown in the figure. **[NET Dec. 2013]**



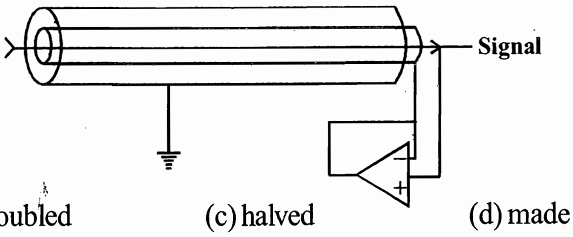
If the input is a sinusoidal wave $V_i = 5\sin(1000t)$, then the amplitude of the output V_0 is

(a) $\frac{5}{2}$ (b) 5 (c) $\frac{5\sqrt{2}}{2}$ (d) $5\sqrt{2}$

7. An op-amp based voltage follower **[NET June 2014]**

(a) is useful for converting a low impedance source into a high impedance source
(b) is useful for converting a high impedance source into a low impedance source
(c) has infinitely high closed loop output impedance
(d) has infinitely high closed loop gain

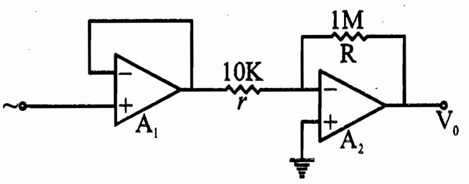
8. The inner shield of a triaxial conductor is driven by an (ideal) op-amp follower circuit as shown. The effective capacitance between the signal-carrying conductor and ground is **[NET June 2014]**



(a) unaffected (b) doubled (c) halved (d) made zero

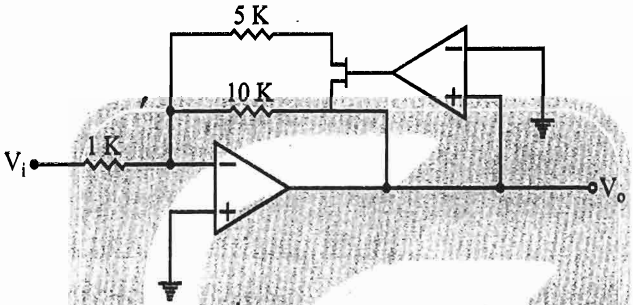
9. Consider the amplifier circuit comprising of the two op-amps A_1 and A_2 as shown in the figure

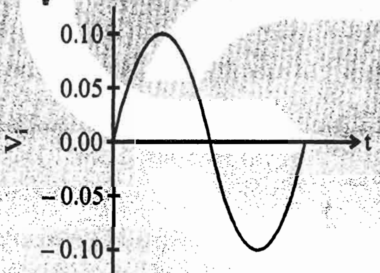
If the input ac signal source has an impedance of $50k\Omega$, which of the following statements is true?



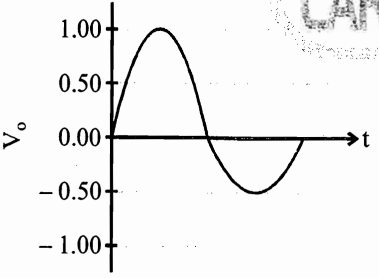
(a) A_1 is required in the circuit because the source impedance is much greater than r **[NET Dec. 2014]**
(b) A_1 is required in the circuit because the source impedance is much less than R
(c) A_1 can be eliminated from the circuit without affecting the overall gain
(d) A_1 is required in the circuit if the output has to follow the phase of the input signal

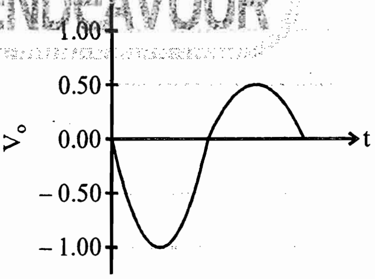
10. For the circuit and the input sinusoidal waveform shown in the figures below, which is the correct waveform at the output? **[NET June 2015]**

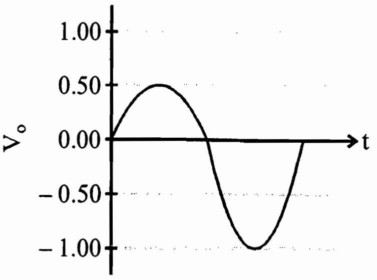


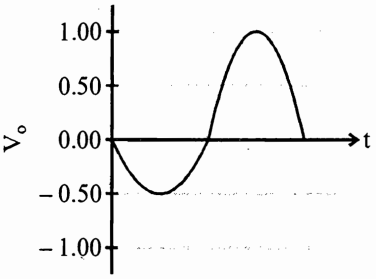


(The time scales in all the plots are the same).

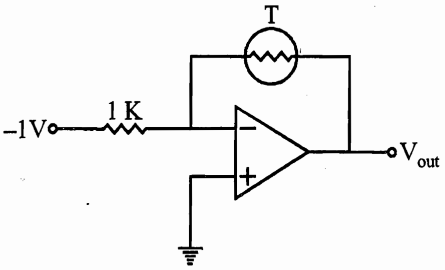
(a)


(b)


(c)


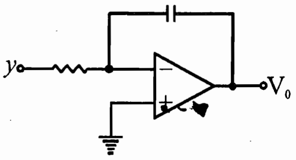
(d)


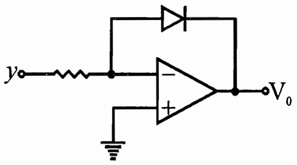
11. In the circuit given below, the thermistor has a resistance 3 kΩ at 25°C. Its resistance decreases by 150Ω per °C upon heating. The output voltage of the circuit at 30°C is **[NET June 2015]**

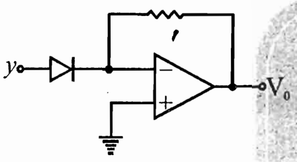


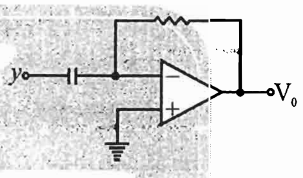
(a) –3.75 V (b) –2.25 V (c) 2.25 V (d) 3.75 V

12. If the parameters y and x are related by $y = \log(x)$, then the circuit that can be used to produce an output voltage V_0 varying linearly with x is **[NET Dec. 2016]**

(a) 

(b) 

(c) 

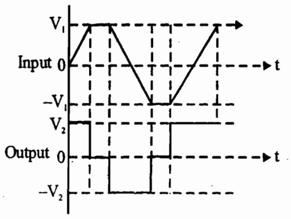
(d) 

ANSWER KEY

1. (a) 2. (a) 3. (d) 4. (c) 5. (b) 6. (c) 7. (b)
8. (d) 9. (a) 10. (b) 11. (c) 12. (c)

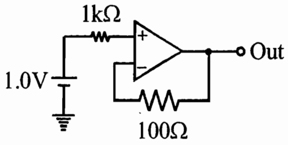
GATE Previous Years' Questions

1. The circuit for which the input and output waveforms are shown below is: **[GATE 1996]**



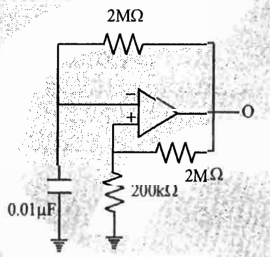
(a) Clipping circuit (b) Integrator (c) Differentiation (d) Schmitt trigger.

2. The output of the circuit on the right will be **[GATE 2000]**



(a) 1V (b) 11V (c) −10V (d) 0 V

3. A bistable multivibrator with a saturation voltage ±5V is shown in the diagram. The positive and negative threshold at the inverting terminal for which the multivibrator will switch to the other state are **[GATE 2003]**

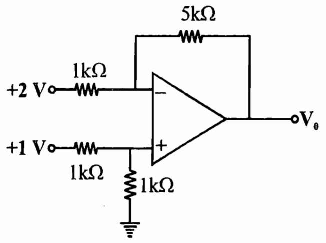


(a) ±5/11 V (b) ±10/11 V (c) ±5 V (d) ±11V

4. The inverting input terminal of an operational amplifier (OP-AMP) is shorted with the output terminal apart from being grounded. A voltage signal v_i is applied to the non-inverting input terminal of the op-amp. Under this configuration, the op-amp functions as **[GATE 2004]**

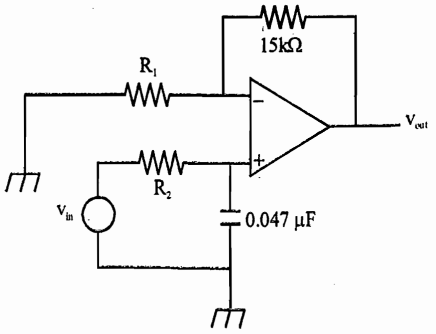
(a) An open loop inverter (b) A voltage to current converter.
(c) A voltage follower (d) An oscillator.

5. The output V_0 of the ideal OP-AMP circuit shown in the figure is: **[GATE 2005]**



(a) −7V (b) −5 V (c) 5 V (d) 7 V

6. The low-pass active filter shown in the figure has a cut-off frequency of 2kHz and a pass band gain of 1.5. The values of the resistors are **[GATE 2006]**

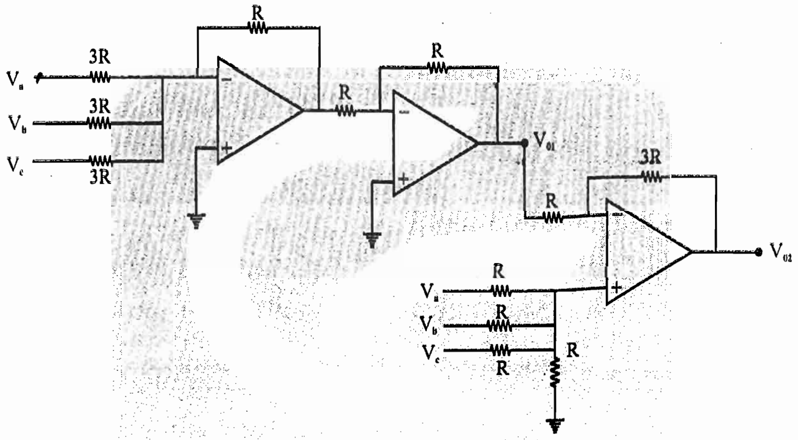


(a) $R_1 = 10k\Omega; R_2 = 1.3\Omega$ (b) $R_1 = 30k\Omega; R_2 = 1.3\Omega$

(c) $R_1 = 10k\Omega; R_2 = 1.7\Omega$ (d) $R_1 = 30k\Omega; R_2 = 1.7\Omega$

Statement for Linked Answer Q.7 and Q.8 **[GATE 2008]**

The following circuit contains three operational amplifiers and resistors.



7. The output voltage at the end of second operational amplifier V_{01} is:

(a) $V_{01} = 3(V_a + V_b + V_c)$ (b) $V_{01} = -\frac{1}{3}(V_a + V_b + V_c)$

(c) $V_{01} = \frac{1}{3}(V_a + V_b + V_c)$ (d) $V_{01} = \frac{4}{3}(V_a + V_b + V_c)$

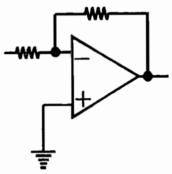
8. The output V_{02} (at the end of third OP-AMP) of the above circuit is:

(a) $V_{02} = 2(V_a + V_b + V_c)$ (b) $V_{02} = 3(V_a + V_b + V_c)$

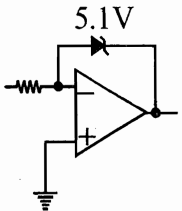
(c) $V_{02} = -\frac{1}{2}(V_a + V_b + V_c)$ (d) Zero.

9. In one of the following circuits, negative feedback does not operate for a negative input. Which one is it? The opamps are running from ±15V supplies. **[GATE 2010]**

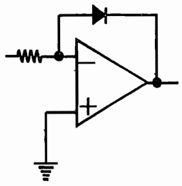
(a)



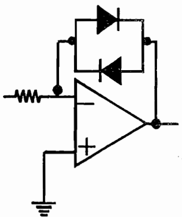
(b)



(c)

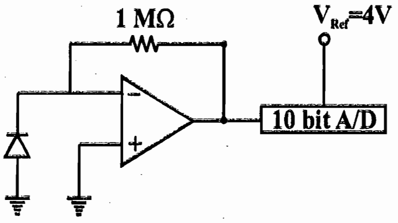


(d)



Statement for Linked Q.10 and Q.11 **[GATE 2010]**

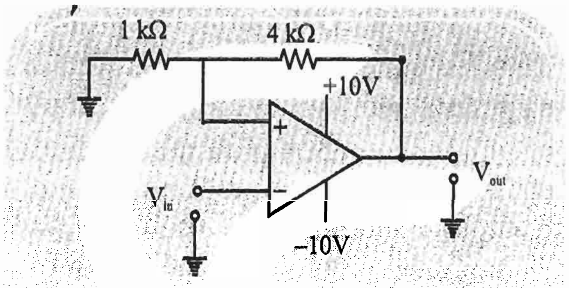
Shown in the figure is a circuit to measure light intensity and convert it to a digital signal. The photodiode P has a responsivity of 0.1 A per watt of incident light intensity. The Op-amp converts the induced photocurrent to a voltage which is digitized by the 10-bit A/D converter with a reference voltage of 4V.



10. For a light intensity of 25 μW incident on the photodiode, the voltage output of the OP-amp is:
(a) 0.25 V (b) 1.0 V (c) 4.0 V (d) 2.5 V

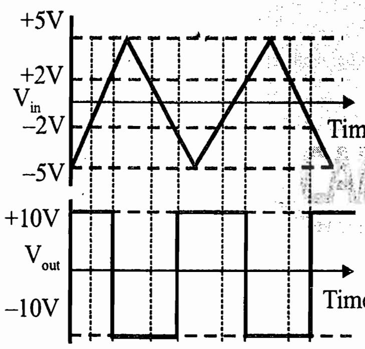
11. The range of light intensity which can be measured by this set up is:
(a) 100 nW to 100μW (b) 100 nW to 100μW
(c) 40 nW to 40μW (d) 40 nW to 40μW

12. Consider the following circuit **[GATE 2011]**

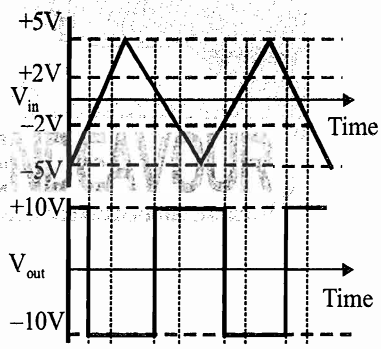


Which of the following represents the output V_{out} corresponding to the input V_{in}?

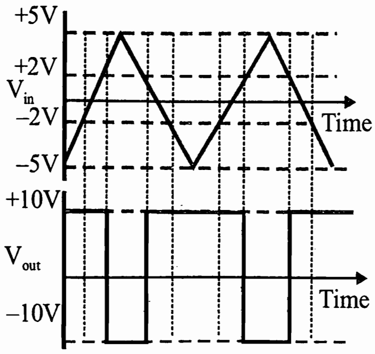
(a)



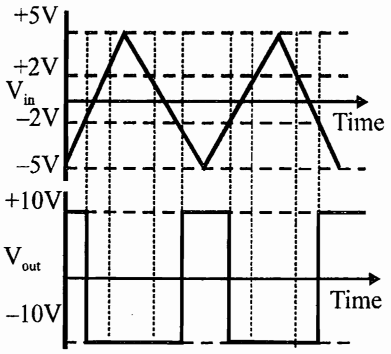
(b)



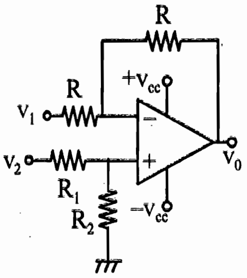
(c)



(d)

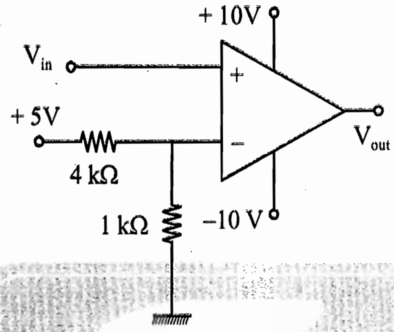


13. In the following circuit, for the output voltage to $V_0 = (-V_1 + V_2/2)$ the ratio R_1/R_2 is **[GATE 2012]**



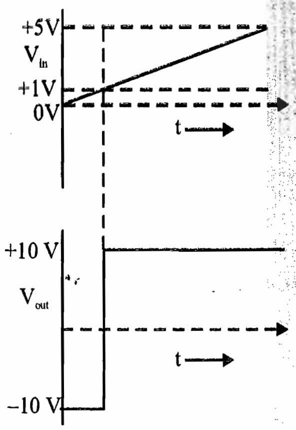
(a) 1/2 (b) 1 (c) 2 (d) 3

14. Consider the following OP-AMP circuit. **[GATE 2012]**

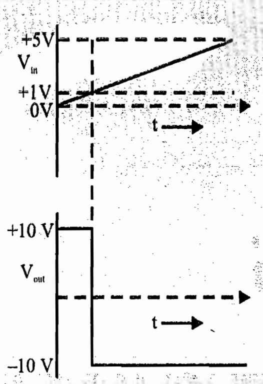


Which one of the following correctly represents the output V_{out} corresponding to the input V_{in}?

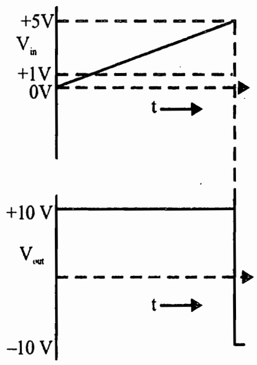
(a)



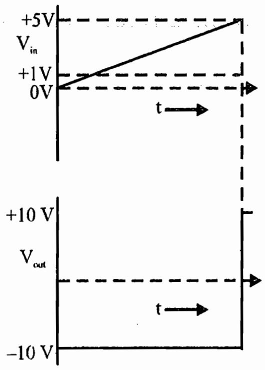
(b)



(c)

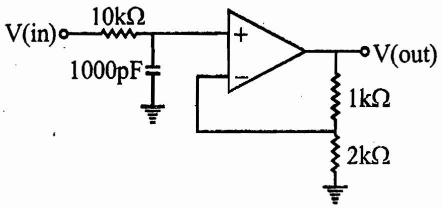


(d)



Statement for Linked Answer Q. 15 and Q. 16 : **[GATE 2013]**

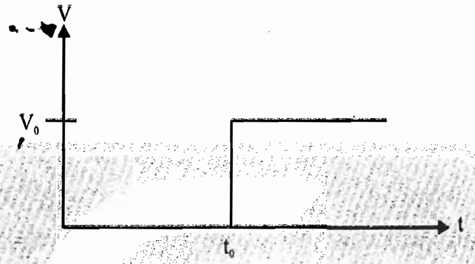
Consider the following circuit



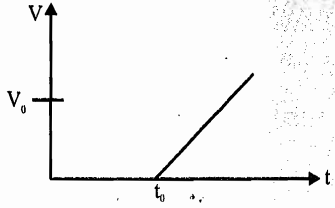
15. For this circuit the frequency above which the gain will decrease by 20*dB* per decade is
(a) 15.9 kHz (b) 1.2 kHz (c) 5.6 kHz (d) 22.5 kHz

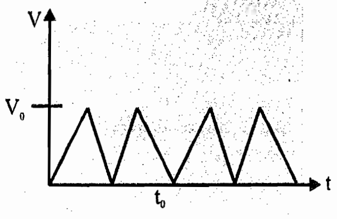
16. At 1.2 kHz the closed loop gain is
(a) 1 (b) 1.5 (c) 3 (d) 0.5

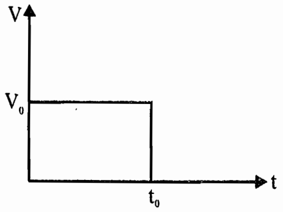
17. The input given to be an ideal OP-AMP integrator circuit is **[GATE 2014]**

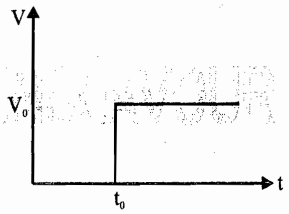


The correct output of the integrator circuit is

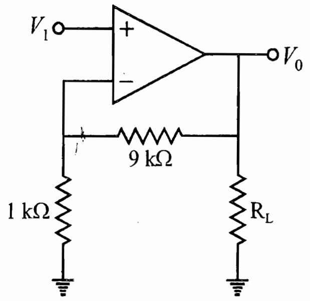
(a)


(b)


(c)


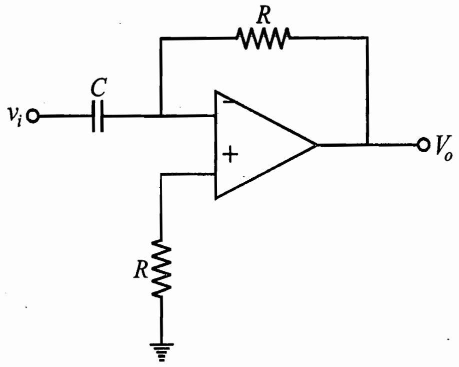
(d)


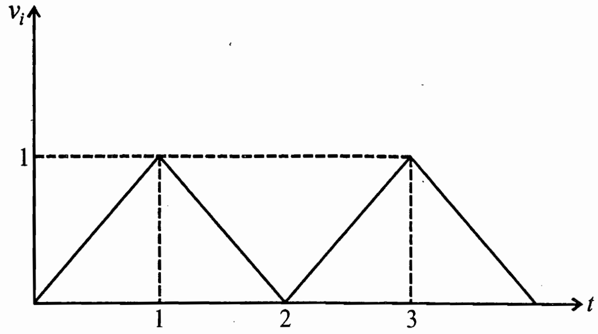
18. In the given circuit, if the open loop gain $A = 10^5$, the feedback configuration and the closed loop gain A_f are



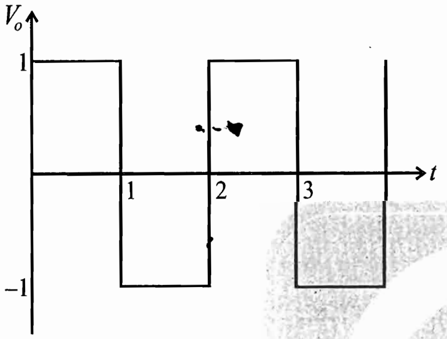
(a) series-shunt, $A_f = 9$ (b) series-series, $A_f = 10$ **[GATE 2015]**
(c) series-shunt, $A_f = 10$ (d) shunt-shunt, $A_f = 10$

19. Consider the circuit shown in the figure, where RC = 1. For an input signal V_i shown below, choose the correct V_0 from the options: **[GATE 2015]**

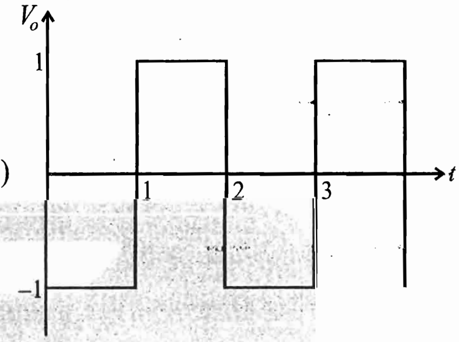




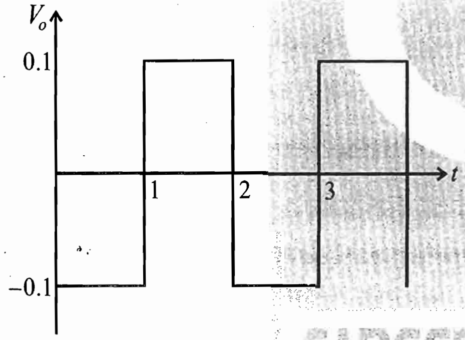
(a)



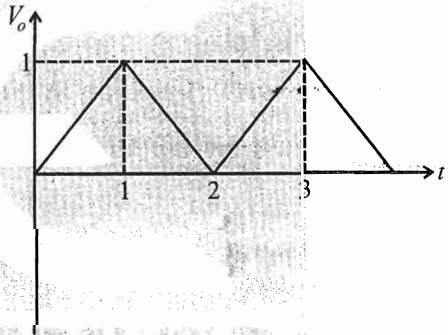
(b)



(c)



(d)

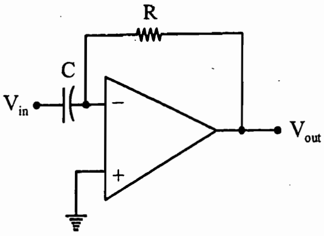


ANSWER KEY

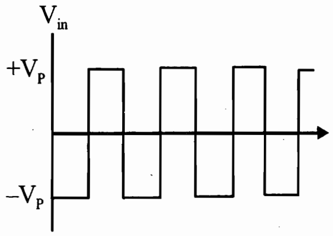
1. (c) **2. (a)** **3. (a)** **4. (c)** **5. (a)** **6. (c)** **7. (c)**
8. (b) **9. (c)** **10. (d)** **11. (b)** **12. (a)** **13. (d)** **14. (a)**
15. (a) **16. (b)** **17. (b)** **18. (c)** **19. (b)**

TIFR Previous Years' Questions

1. Consider the following circuit : **[TIFR 2012]**

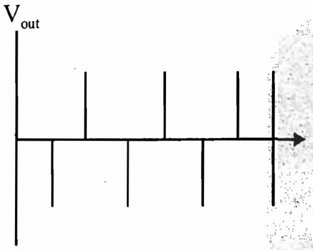


If the waveform given below is fed in at V_{in}

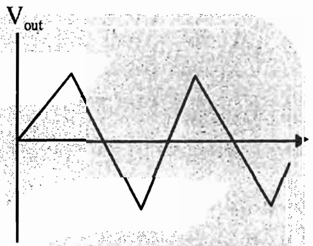


then the waveform at the output V_{out} will be

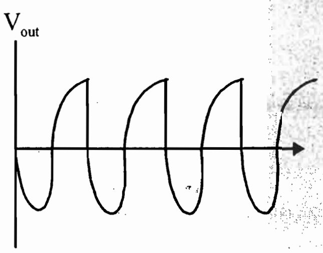
(a)



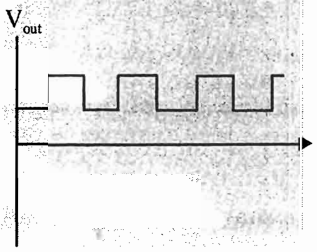
(b)



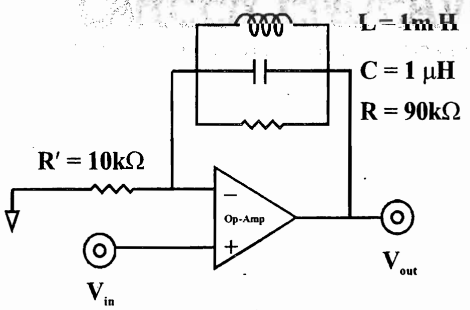
(c)



(d)



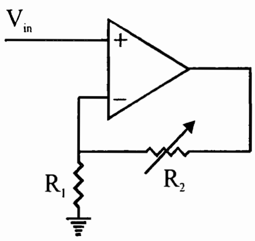
2. An input of 1.0 V DC is given to the ideal Op-Amp circuit depicted below. What will be the output voltage?



[TIFR 2013]

(a) 10.0 V (b) −9.0 V (c) 1.0 V (d) 0 V

3. In the following circuit, the resistance R_2 is doubled. **[TIFR 2014]**



It follows that the current through R_2.

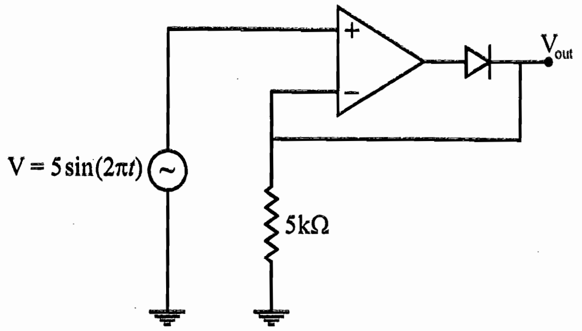
(a) remains the same

(b) is halved

(c) is doubled

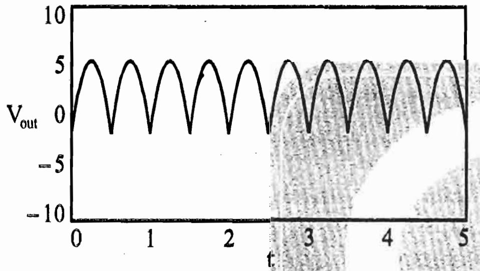
(d) is quadrupled.

4. In the circuit shown below, the op-amp is powered by a bipolar supply of ±10 V. **[TIFR 2015]**

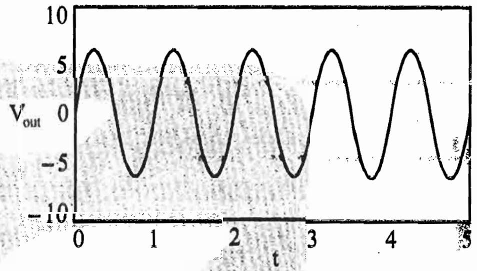


Which one of the following graphs represents V_{out} correctly?

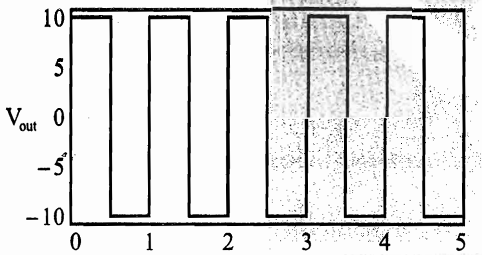
(a)



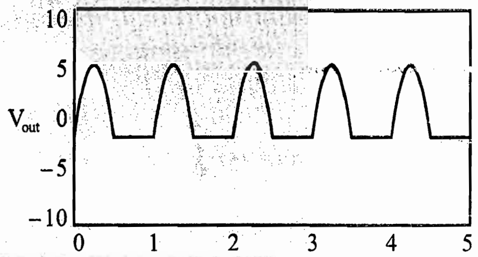
(b)



(c)



(d)

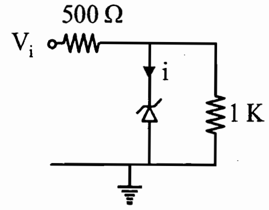


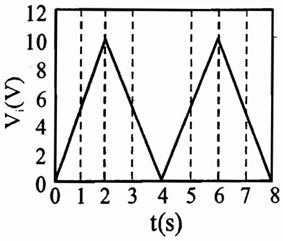
ANSWER KEY

1. (a) **2. (c)** **3. (a)** **4. (d)**

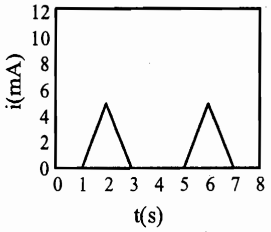
CSIR-UGC-NET Previous Years' Questions

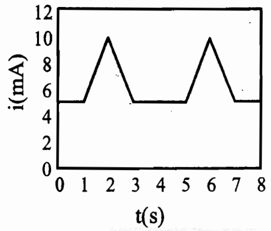
1. The figure below shows a voltage regulator utilizing a Zener diode of breakdown voltage 5V and a positive triangular wave input of amplitude 10V. **[NET Dec. 2011]**

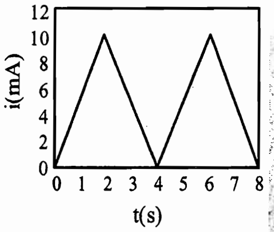


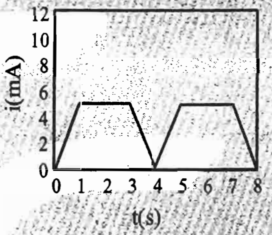


For $V_i > 5V$, the Zener regulates the output voltage by channeling the excess current through it self. Which of the following waveforms shows the current 'i' passing through the Zener diode?

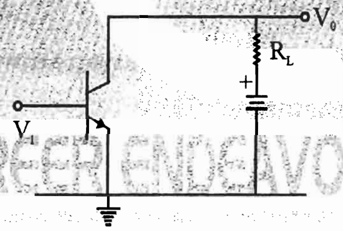
(a)


(b)


(c)


(d)


2. The transistor in the given circuit has $h_{fc} = 35\Omega$ and $h_{ie} = 1000\Omega$. If the load resistance $R_L = 1000\Omega$, the voltage and current gain are, respectively. **[NET June 2012]**



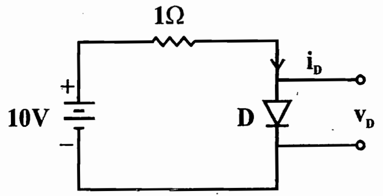
(a) –35, +35 (b) 35, 35 (c) 35, 0.97 (d) 0.98, 35

3. A live music broadcast consists of a radio-wave of frequency 7 MHz, amplitude-modulated by a microphone output consisting of signals with a maximum frequency of 10 KHz. The spectrum of modulated output will be zero outside the frequency band **[NET Dec. 2012]**

(a) 7.00 MHz to 7.01 MHz (b) 6.99 MHz to 7.01 MHz
(c) 6.99 MHz to 7.00 MHz (d) 6.995 MHz to 7.005 MHz

4. A diode D as shown in the circuit as an i–v relation which can be proximated by **[NET Dec. 2012]**

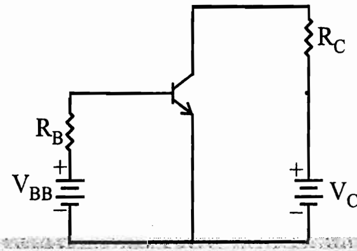
$$i_D = \begin{cases} v_D^2 + 2v_D, & \text{for } v_D > 0 \\ 0, & \text{for } v_D \le 0 \end{cases}$$

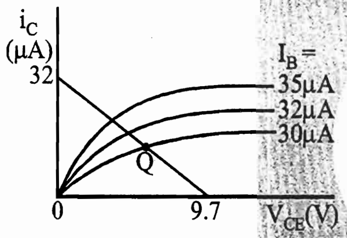


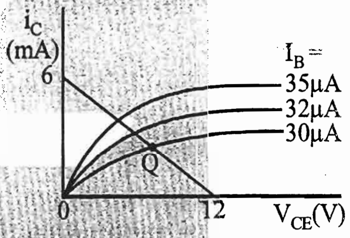
The value of v_D in the circuit is:

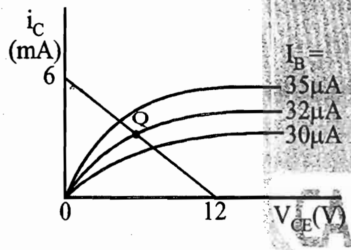
(a) $\left(-1+\sqrt{11}\right)$V (b) 8V (c) 5V (d) 2V

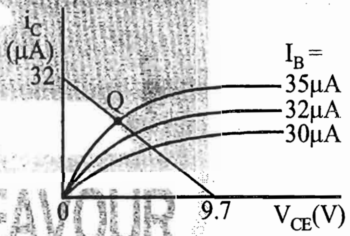
5. A silicon transistor with built-in voltage 0.7V is used in the circuit shown, with $V_{BB} = 9.7V$, $R_B = 300k\Omega$, $V_{CC} = 12V$ and $R_C = 2k\Omega$. Which of the following figures correctly represents the load line and the quiescent Q point? **[NET June 2013]**



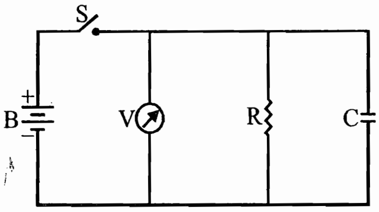
(a)


(b)


(c)


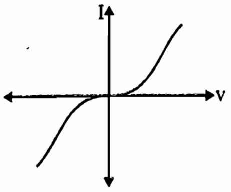
(d)


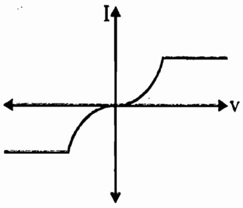
6. The insulation resistance R of an insulated cable is measured by connecting it in parallel with a capacitor C, a voltmeter, and battery B as shown. The voltage across the cable dropped from 150V to 15V, 1000 seconds after the switch S is closed. If the capacitance of the cable is 5 μF then its insulation resistance is approximately **[NET June 2013]**

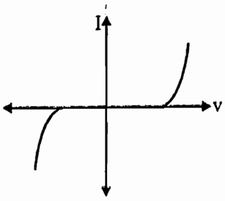


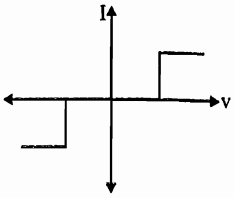
(a) $10^9\Omega$ (b) $10^8\Omega$ (c) $10^7\Omega$ (d) $10^6\Omega$

7. Two identical Zener diodes are placed back to back in series and are connected to a variable DC power supply. The best representation of the I-V characteristics of the circuit is **[NET Dec. 2013]**

(a)


(b)


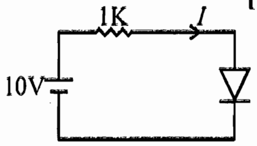
(c)


(d)


8. An RC network produces a phase-shift of 30°. How many such RC networks should be cascaded together and connected to a Common Emitter amplifier so that the final circuit behaves as an oscillator?

(a) 6 (b) 12 (c) 9 (d) 3 **[NET June 2014]**

9. The I-V characteristics of the diode in the circuit is given by **[NET Dec. 2014]**



$$I = \begin{cases} (V-0.7)/500 & \text{for } V \geq 0.7 \\ 0 & \text{for } V < 0.7 \end{cases}$$

where V is measured in volts and I is measured in amperes.

The current I in the circuit is

(a) 10.0 mA (b) 9.3 mA (c) 6.2 mA (d) 6.7 mA

10. A junction is made between a metal of work function W_M, and a doped semiconductor of work function W_S with $W_M > W_S$. If the electric field at the interface has to be increased by a factor of 3, then the dopant concentration in the semiconductor would have to be **[NET Dec. 2014]**

(a) increased by a factor of 9 (b) decreased by a factor of 3

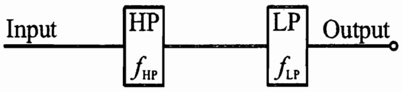
(c) increased by a factor of 3 (d) decreased by a factor of $\sqrt{3}$

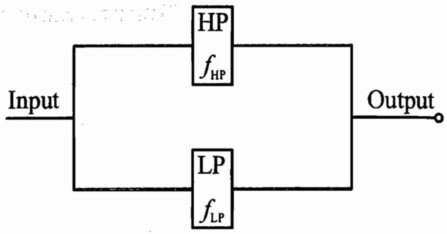
11. A large MOS transistor consists of N individual transistors connected in parallel. If the only form of noise in each transistor is $1/f$ noise, then the equivalent voltage noise spectral density for the MOS transistor is **[NET Dec. 2014]**

(a) $1/N$ times that of a single transistor (b) $1/N^2$ times that of a single transistor

(c) N times that of a single transistor (d) N^2 times that of a single transistor

12. Consider a Low Pass (LP) and a High Pass (HP) filter with cut-off frequencies f_{LP} and f_{HP}, respectively, connected in series or in parallel configurations as shown in the Figures A and B below.

(A)


(B)


Which of the following statements is correct? **[NET Dec. 2014]**

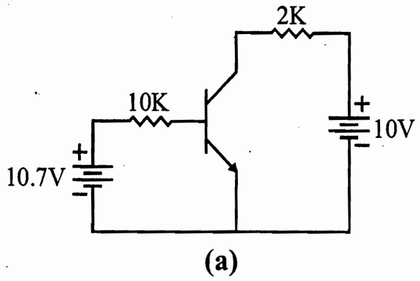
(a) For $f_{HP} < f_{LP}$, A acts as a Band Pass filter and B acts as a Band Reject filter

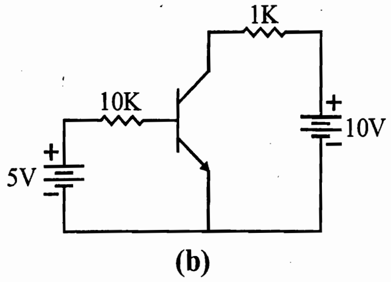
(b) For $f_{HP} > f_{LP}$, A stops the signal from passing through and B passes the signal without filtering

(c) For $f_{HP} < f_{LP}$, A acts as a Band Pass filter and B passes the signal without filtering

(d) For $f_{HP} > f_{LP}$, A passes the signal without filtering and B acts as a Band Reject filter

13. Consider the circuits shown in Figures (a) and (b) below. **[NET June 2015]**





If the transistors in Figures (a) and (b) have current gain (β_{dc}) of 100 and 10 respectively, then they operate in the

(a) active region and saturation region respectively
(b) saturation region and active region respectively
(c) saturation region in both cases
(d) active region in both cases

14. If the reverse bias voltage of a silicon varactor is increased by a factor of 2, the corresponding transition capacitance **[NET Dec. 2015]**

(a) increases by a factor of $\sqrt{2}$
(b) increases by a factor of 2
(c) decreases by a factor of $\sqrt{2}$
(d) decreases by a factor of 2

ANSWER KEY

1. (a)	**2. (a)**	**3. (d)**	**4. (b)**	**5. (b)**	**6. (b)**	**7. (c)**
8. ()	**9. (c)**	**10. (a)**	**11. (a)**	**12. (c)**	**13. (b)**	**14. (c)**

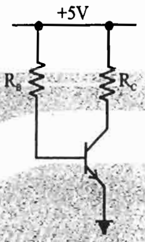
GATE Previous Years' Questions

1. A current amplifier is characterised by **[GATE 1996]**
(a) low input impedance and high output impedance
(b) high input impedance and low output impedance.
(c) low impedance at both input and output terminals.
(d) high impedance at both input and output terminals.

2. Which of the following characteristics DOES NOT belong to a common collector transistor amplifer?
(a) Low voltage gain [≈] (b) High current gain. **[GATE 1996]**
(c) High input impedance (d) High output impedance.

3. An amplifier has a voltage gain of 500 and an input impedance 20 KΩ, without any feedback. Now a negative feedback with $\beta = 0.1$ is applied. Its gain and input impedance with feedback will respectively be
(a) 9.8 and 392 K ohms (b) 9.8 and 1020 K ohms **[GATE 1996]**
(c) 50 and 1020 K ohms (d) 50 and 2 K ohms

4. In the circuit shown below, $R_B = 1\text{ k}\Omega$ and $R_C = 100\Omega$. If the transistor $\beta(h_{FE})$ is 100, the current through R_C will be

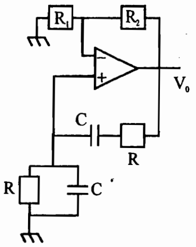


(a) ≈ 0.43 A (b) ≈ 50 mA **[GATE 2000]**
(c) zero (d) Oscillating between 0 and 50 mA.

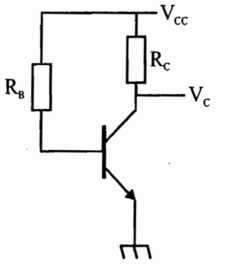
5. In an n-p-n transistor, the leakage current consists of **[GATE 2001]**
(a) Electrons moving from the base to the emitter
(b) Electrons moving from the collector to the base
(c) Electrons moving from the collector to the emitter.
(d) Electrons moving from the base to the collector.

6. A field effect transistor is a **[GATE 2004]**
(a) Unipolar device (b) Special type of bipolar junction transistor.
(c) Unijunction device (d) Device with low input impedance

7. What should be the values of the components R and R_2 **[GATE 2004]**
such that the frequency of the Wien Bride oscillator is 300 Hz?
[Given: $C = 0.01\ \mu F$ and $R_1 = 12\text{ k}\Omega$]
(a) $R = 48\text{k}\Omega$ and $R_2 = 12\text{k}\Omega$ (b) $R = 26\text{k}\Omega$ and $R_2 = 24\text{k}\Omega$
(c) $R = 530\text{ k}\Omega$ and $R_2 = 1\text{ M}\Omega$ (d) $R = 53\text{k}\Omega$ and $R_2 = 24\text{k}\Omega$



8 Calculate the collector voltage (v_c) of the transistor circuit shown in the figure **[GATE 2004]**
[Given: $\alpha = 0.96$, $I_{CBO} = 20\mu A$, $V_{BE} = 0.3V$, $R_B = 100\text{k}\Omega$, $V_{CC} = +10V$ and $R_C = 2.2\text{k}\Omega$]



(a) 3.78 V (b) 3.82 V (c) 4.72 V (d) 9.7 V

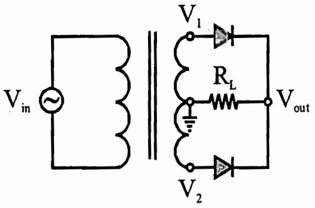
9. Which of the following statements is correct for a common emitter amplifier circuit? **[GATE 2004]**

(a) The output is taken from the emitter
(b) There is 180° phase shift between input and output voltages
(c) There is no phase shift between input and output voltages
(d) Both p-n juntions are forward biased.

10. A junction field effect transistor behaves as a **[GATE 2005]**

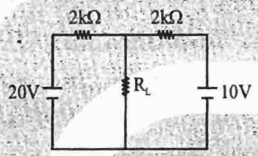
(a) Voltage controlled current source (b) Voltage controlled voltage source
(c) Current controlled voltage source (d) Current controlled current source

11. For the rectifier circuit shown in the figure, the sinusoidal voltage (V_1 or V_2) at the output of the transformer has a maximum value of 10V. The load resistance R_L is 1 kΩ. If I_{av} is the average current through the resistor R_L the circuit corresponds to a. **[GATE 2005]**



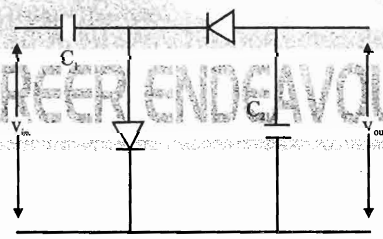
(a) Full wave rectifier with $I_{av} = 20/\pi$ mA (b) Half wave rectifier with $I_{av} = 20/\pi$ mA
(c) Half wave rectifier with $I_{av} = 10/\pi$ mA (d) Full wave rectifier with $I_{av} = 10/\pi$ mA

12. In the circuit shown in the figure the Thevenin voltage V_{Th} and Thevenin resistance R_{Th} as seen by the load resistance R_L (=1kΩ) are respectively. **[GATE 2005]**



(a) 15V, 1 kΩ (b) 30 V, 4 kΩ
(c) 20 V, 2 kΩ (d) 10V, 5 kΩ.

13. A sinusidal input voltage v_{in} of frequency ω is fed to the circuit shown in the figure, where $C_1 >> C_2$. If v_{in} is the peak value of the input voltage, then output voltage (v_{out}) is: **[GATE 2006]**



(a) $2v_{in}$ (b) $2v_0 \sin \omega t$ (c) $\sqrt{2}v_{in}$ (d) $\frac{v_{in}}{2}\sin \omega t$

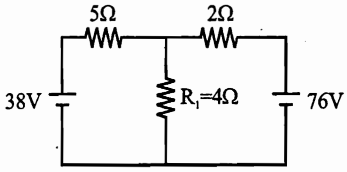
14. The high input impedance of field effect transistor (FET) amplifier is due to **[GATE 2006]**

(a) The pinch-off voltage (b) Its very low gate current.
(c) The source and drain being far apart (d) The geometry of the FET.

15. The largest analog output voltge from a 6-bit digital to analog converter (DAC) which produces 1.0 V output for a digital input of 010100, is: **[GATE 2006]**

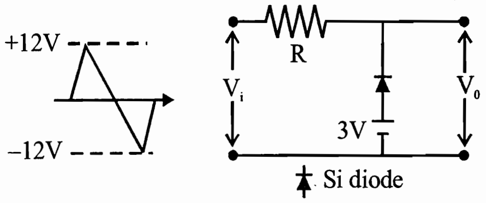
(a) 1.6 V (b) 2.9 V (c) 3.15 V (d) 5.0 V

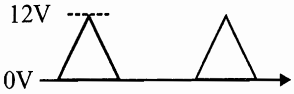
16. For the circuit shown, the potential difference (in Volts) across R_L is: **[GATE 2007]**

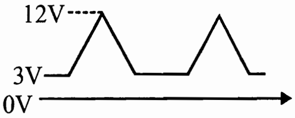

5Ω
2Ω
38V
R_l=4Ω
76V

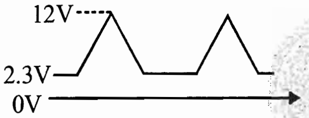
(a) 48 (b) 52 (c) 56 (d) 65

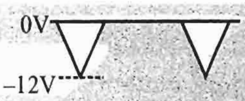
17. When an input voltage V_i, of the form shown, is applied to the circuit given below, the output voltage V_0 is of the form. **[GATE 2007]**


+12V
−12V
R
V_i
V_0
3V
Si diode

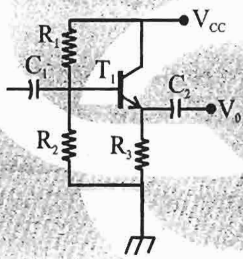
(a)


(b)


(c)


(d)


18. The circuit shown below **[GATE 2009]**

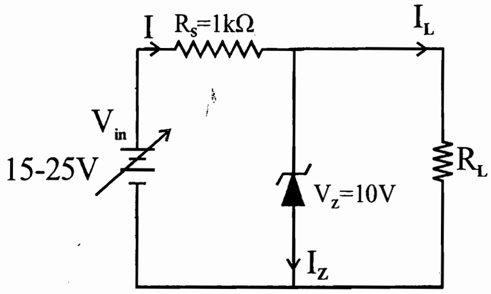

V_cc
R_1
C_1
T_1
C_2
V_0
R_2
R_3

(a) Is a common-emitter amplifier (b) Uses a pnp transistor
(c) Is an oscillator (d) Has a voltage gain less than one.

19. An amplifier of gain 1000 is made into a feedback amplifier by feeding 9.9% of its output voltage in series with the input opposing. If f_L = 20 Hz and f_H = 200 kHz for the amplifier without feedback, then due to the feedback. **[GATE 2009]**

(a) The gain decreasse by 10 times (b) The outputresistance increases by 10 times
(c) The f_H increases by 100 times (d) The input resistance decreases by 100 times.

20. Pick the correct statement based on the below circuit. **[GATE 2009]**


I
R_S=1kΩ
I_L
V_in
15-25V
V_Z=10V
R_L
I_Z

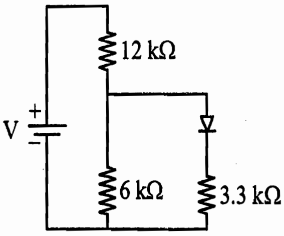
(a) The maximum Zener current, $I_{Z(max)}$, when $R_L = 10k\Omega$ is 15mA

(b) The maximum Zener current, $I_{z(min)}$, when $R_L = 10k\Omega$ is 5mA

(c) With $V_{in} = 20V, I_L = I_Z$, when $R_L = 2k\Omega$

(d) The power dissipated across the Zener when $R_L = 10k\Omega$ and $V_{in} = 20V$ is 100 mW.

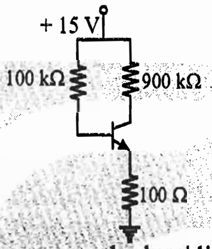
21. In the following circuit, the voltage drop across the ideal diode in forward bias condition is 0.7 V.



The current passing through the diode is **[GATE 2012]**

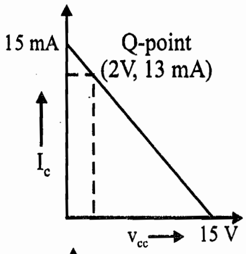
(a) 0.5 mA (b) 1.0 mA (c) 1.5 mA (d) 2.0 mA

22. Consider the following circuit in which the current gain β_{dc} of the transistor is 100. **[GATE 2012]**

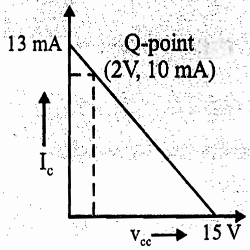


Which one of the following correctly represents the load line (collector current I_c with respect to collectr-emitter voltage V_{CE}) and Q-point of this circuit?

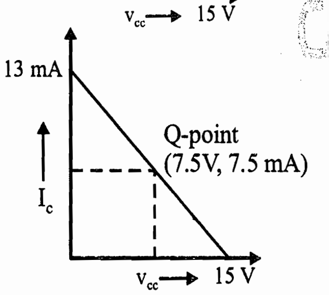
(a)



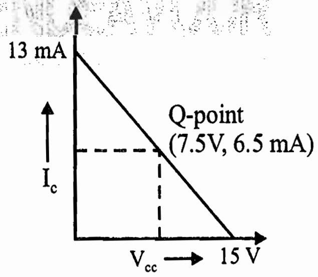
(b)



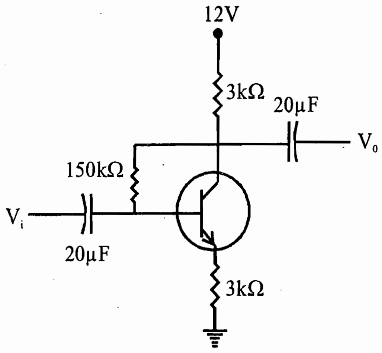
(c)



(d)



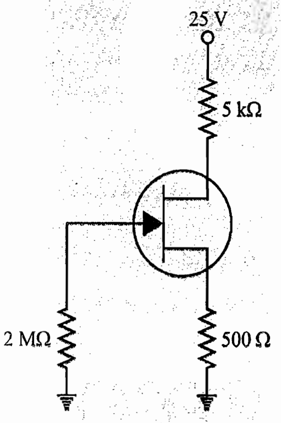
23. The current gain of the transistor in the following circuit is $\beta_{dc} = 100$. The value of collector current I_C is ____________ mA. **[GATE 2014]**



24. In order to measure a maximum of 1V with a resolution of 1mV using a n-bit A/D converter, working under the principle of ladder network, the minimum value of n is ______________________ **[GATE 2014]**

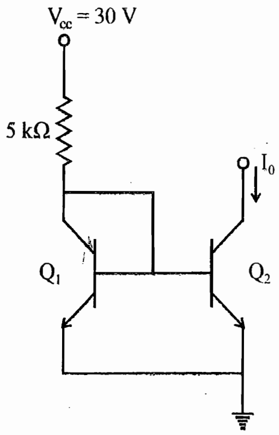
25. A low pass filter is formed by a resistance R and a capacitance C. At the cut-off angular frequency $\omega_c = \frac{1}{RC}$, the voltage gain and the phase of the output voltage relative to the input voltage respectively, are
(a) 0.71 and 45° (b) 0.71 and –45° (c) 0.5 and –90° (d) 0.5 and 90° **[GATE 2014]**

26. In the given circuit, the voltage across the source resistor is 1 V. The drain voltage (in V) is _______



[GATE 2015]

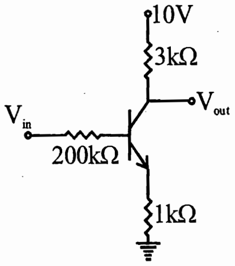
27. In the simple current source shown in the figure, Q_1 and Q_2 are identical transistors with current gain $\beta = 100$ and $V_{BE} = 0.7\ V$ **[GATE 2015]**



The current I_0 (in mA) is_______(upto two decimal places)

28. The number density of electrons in the conduction band of a semiconductor at a given temperature is 2×10^{19} m^{-3}. Upon lightly doping this semiconductor with donor impurities, the number density of conduction electrons at the same temperature becomes 4×10^{20} m^{-3}. The ratio of majority to minority charge carrier concentration is ________________. **[GATE 2016]**

29. For the transistor shown in the figure, assume $V_{BE} = 0.7$V and $\beta_{dc} = 100$. If $V_{in} = 5$V, V_{out} (in Volts) is _________. (Give your answer upto one decimal place). **[GATE 2016]**

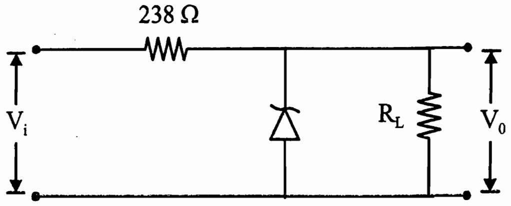


ANSWER KEY

1. (a)
2. (d)
3. (b)
4. (b)
5. (a)
6. (a)
7. (d)
8. (b)
9. (b)
10. ( )
11. (a)
12. (a)
13. (a)
14. (b)
15. (c)
16. (d)
17. (c)
18. (d)
19. (c)
20. (a)
21. (b)
22. (a)
23. (1.4–1.7)
24. (9.99–10.01)
25. (b)
26. ( )
27. (15)
28. (400)
29. (5.5 to 5.9)

TIFR Previous Years' Questions

1. The voltage regulator circuit shown in the figure has been made with a Zener diode rated at 15V, 200 mW. It is required that the circuit should dissipate 150 mW power across the fixed load resistor R_L

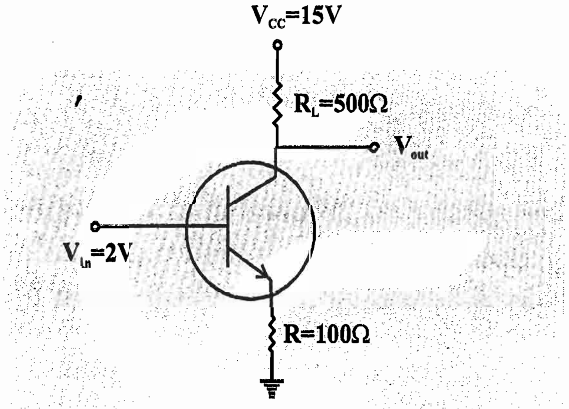


For stable operation of this circuit, the input voltage V_i must have a range **[TIFR 2012]**

(a) 17.5 V, -20.5 V (b) 15.5 V, -20.5 V
(c) 15.5 V, -22.5 V (d) 17.5 V, -22.5 V

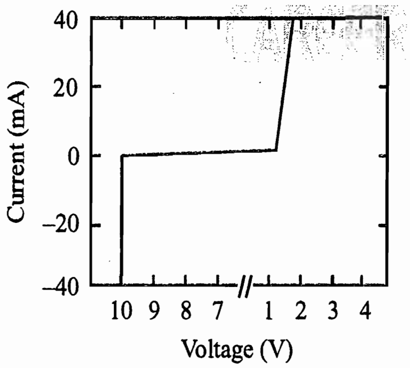
2. The circuit depicted on the right has been made with a silicon n-p-n transistor. **[TIFR 2013]**

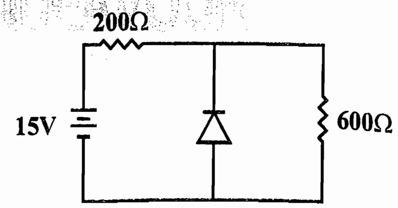
Assuming that there will be a 0.7V drop across a forward-biased silicon p-n junction, the power dissipated across the transistor will be, approximately,



(a) 53 mW (b) 94 mW (c) 17 mW (d) 67 mW

3. The figure on the right shows the current-voltage characteristics of a diode over a range of voltage and current where it is safe to operate the diode. **[TIFR 2013]**

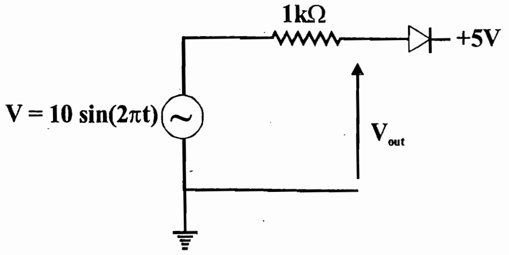




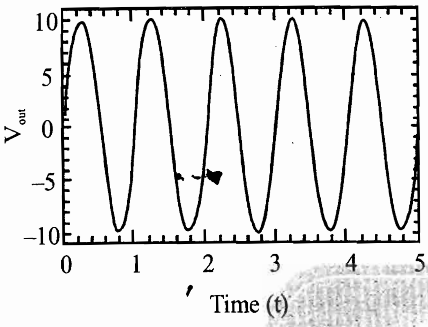
When this diode is used in the circuit on the extreme right, the approximate current, in mA, through the diode will be

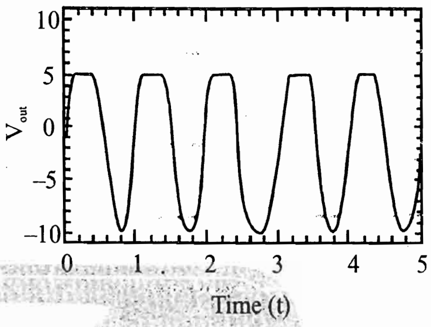
(a) 0 (b) 8.3 (c) 16.7 (d) 25

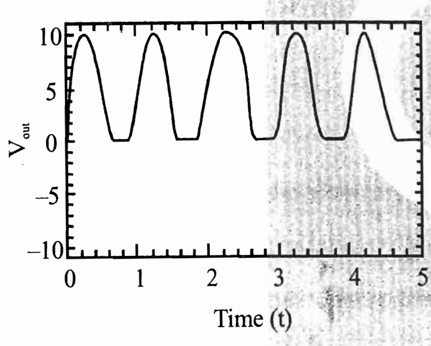
4. Consider the following circuit **[TIFR 2014]**

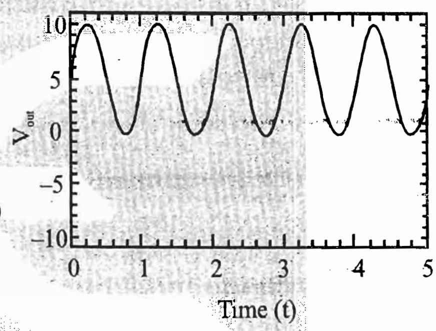


Which of the graphs given below is a correct representation of V_{out}?

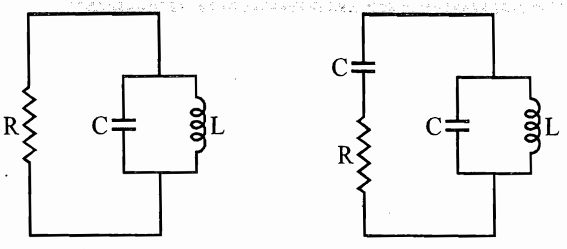
(a)


(b)


(c)


(d)


5. Two LCR circuits (A) and (B) are shown below where $C_c \ll C$. At time $t = 0$, a charge Q is put on the capacitor C. **[TIFR 2015]**



(A) (B)

Which of the following statements is correct ?

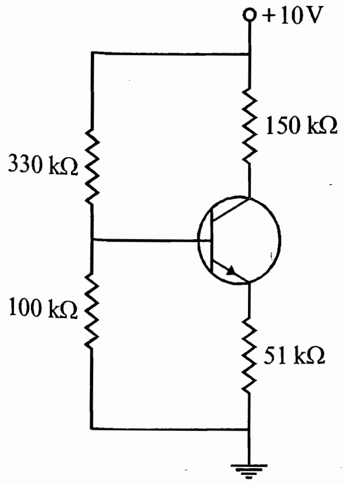
(a) The charge Q will decay faster in (A)

(b) The charge Q will decay faster in (B)

(c) The charge Q will decay at the same rate in (A) and (B)

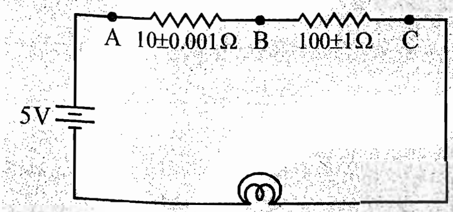
(d) The relative decay rates cannot be predicted without knowing the exact values of L, C, R and C_c

6. All resistors in the circuit on the right have a tolerance of ± 5%. **[TIFR 2015]**



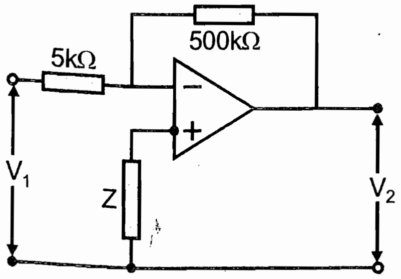
Assuming a diode drop of 0.7 V, which of the following is the lowest possible value of the collector voltage ?
(a) 3.1 V (b) 4.1 V (c) 4.7 V (d) 5.2 V

7. You are given the following circuit and two instruments : a voltmeter and an ammeter both with 0.001% accuracy in their readings. **[TIFR 2015]**



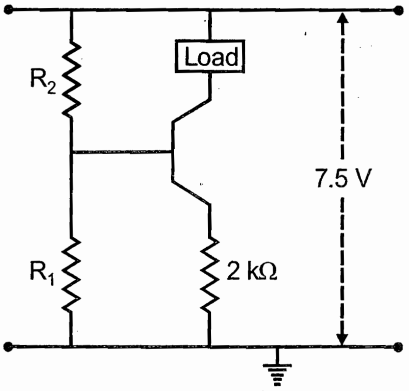
Which of the following methods will result in the most accurate reading for the current without interrupting the current in the circuit ?
(a) Use voltmeter to measure voltage across points B and C
(b) Use the ammeter to measure current at point B
(c) Use voltmeter to measure voltage across points A and B
(d) Use voltmeter to measure voltage across points A and C

8. In the generalized operational amplifier circuit shown on the right, the amp. has a very high input impedance ($Z > 50\ M\Omega$) and an open gain of 1000 for the frequency range under consideration. Assuming that the op. amp. draws negligible current, the voltage ratio V_2/V_1 is approximately **[TIFR 2016]**



(a) –190 (b) –190 (c) –90 (d) 80

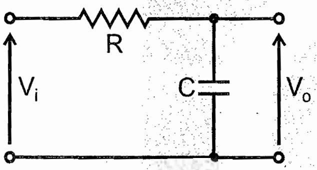
9. In the transistor circuit shown on the right, assume that the voltage drop between the base and the emitter is 0.5V. **[TIFR 2016]**

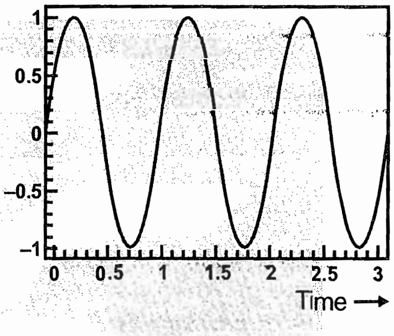


What will be the ratio of the resistances R_2/R_1, in order to make this circuit function as a source of constant current, $I = 1$ mA ?

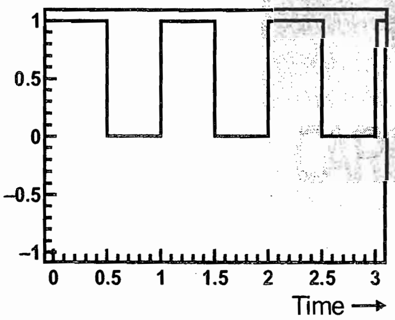
(a) 4.5 (b) 3.0 (c) 2.5 (d) 2.0

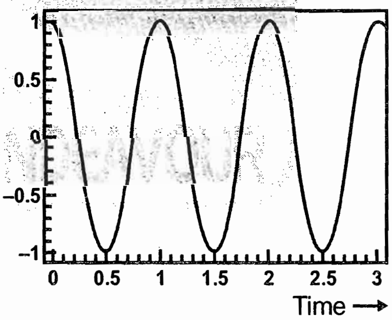
10. For the circuit depicted on the right, the input voltage V_i is a simple sinusoid as shown below, where the time period is much smaller compared to the time constant of this circuit. **[TIFR 2016]**

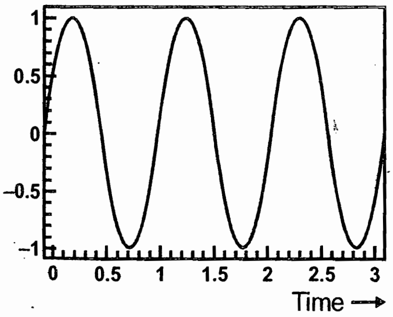


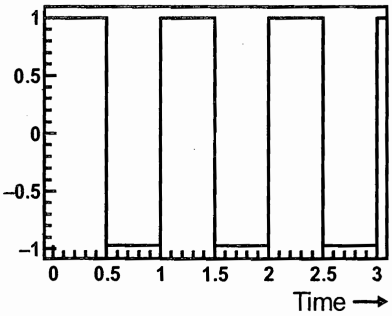


The voltage V_o across C is best represented by

(a)


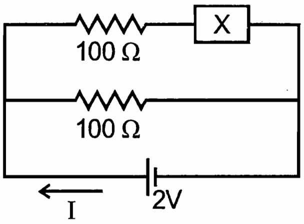
(b)


(c)


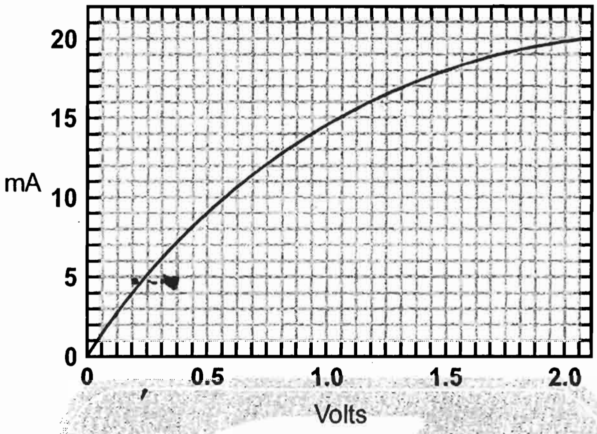
(d)


11. Consider a sawtooth waveform which rises linearly from 0 Volt to 1 Volt in 10 ns and then decays linearly to 0V over a period of 100 ns. Find the r.m.s. voltage in units of milliVolt? **[TIFR 2016]**

12. The circuit shown below contains an unknown device X.



The current voltage characteristic of the device X were determined and are shown in the plot given below.



Determine the current I (in mA) flowing through the device X. **[TIFR 2016]**

ANSWER KEY

1. (a)	**2. (b)**	**3. (b)**	**4. (b)**	**5. (a)**	**6. (b)**	**7. (c)**
8. (c)	**9. (d)**	**10. (c)**	**11. (577)**	**12. (12)**		

MATHEMATICAL PHYSICS

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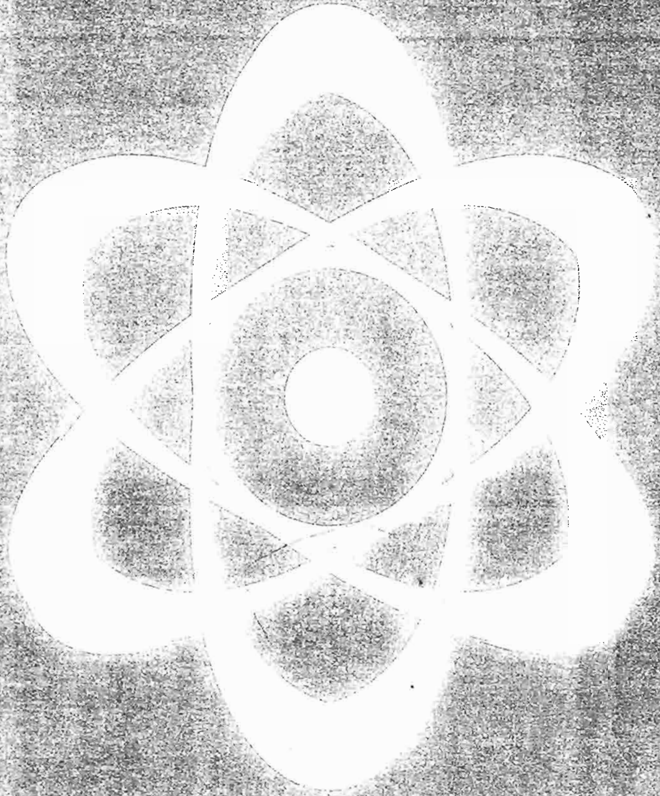
NET | GATE | TIFR | JEST



Previous Years' Questions

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MATHEMATICAL PHYSICS

SECTION: VECTOR ALGEBRA & VECTOR CALCULUS

CSIR PREVIOUS YEAR QUESTIONS

1. Let $\vec{a}$ and $\vec{b}$ be two distinct three-dimensional vectors. Then the component of $\vec{b}$ that is perpendicular to $\vec{a}$ is given by **[CSIR June - 2011]**

(a) $\dfrac{\vec{a}\times\left(\vec{b}\times\vec{a}\right)}{a^2}$ (b) $\dfrac{\vec{b}\times\left(\vec{a}\times\vec{b}\right)}{b^2}$ (c) $\dfrac{\left(\vec{a}.\vec{b}\right)\vec{b}}{b^2}$ (d) $\dfrac{\left(\vec{b}.\vec{a}\right)\vec{a}}{a^2}$

2. The equation of the plane that is tangent to the surface $xyz = 8$ at the point $(1, 2, 4)$ is **[CSIR Dec - 2011]**

(a) $x + 2y + 4z = 12$ (b) $4x + 2y + z = 12$
(c) $x + 4y + 2z = 12$ (d) $x + y + z = 7$

3. A vector perpendicular to any vector that lies on the plane defined by $x + y + z = 5$, is **[CSIR June - 2012]**

(a) $\vec{i}+\vec{j}$ (b) $\vec{j}+\vec{k}$ (c) $\vec{i}+\vec{j}+\vec{k}$ (d) $2\vec{i}+3\vec{j}+5\vec{k}$

4. The unit normal vector at the point $\left(\dfrac{a}{\sqrt{3}},\dfrac{b}{\sqrt{3}},\dfrac{c}{\sqrt{3}}\right)$ on the surface of the ellipsoid $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$, is **[CSIR Dec - 2012]**

(a) $\dfrac{bc\hat{i}+ca\hat{j}+ab\hat{k}}{\sqrt{a^2b^2+b^2c^2+c^2a^2}}$ (b) $\dfrac{a\hat{i}+b\hat{j}+c\hat{k}}{\sqrt{a^2+b^2+c^2}}$

(c) $\dfrac{b\hat{i}+c\hat{j}+a\hat{k}}{\sqrt{a^2+b^2+c^2}}$ (d) $\dfrac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

5. A unit vector $\hat{n}$ on the xy-plane is at an angle of 120° with respect to $\hat{i}$. The angle between the vectors $\vec{u}=a\hat{i}+b\hat{n}$ and $\vec{v}=a\hat{n}+b\hat{i}$ will be 60° if **[CSIR June - 2013]**

(a) $b=\sqrt{3}\,a/2$ (b) $b=2a/\sqrt{3}$ (c) $b = a/2$ (d) $b = a$

6. If $\vec{A}=yz\,\hat{i}+xz\,\hat{j}+xy\,\hat{k}$, then the integral $\oint\limits_C \vec{A}.d\vec{l}$ (where C is along the perimeter of a rectangular area bounded by $x = 0$, $x = a$ and $y = 0$, $y = b$) is **[CSIR Dec - 2013]**

(a) $\dfrac{1}{2}\left(a^3+b^3\right)$ (b) $\pi\left(ab^2+a^2b\right)$ (c) $\pi\left(a^3+b^3\right)$ (d) 0

7. If $\vec{A} = yz\,\hat{i} + zx\,\hat{j} + xy\,\hat{k}$ and C is the circle of unit radius in the plane defined by z = 1, with the centre on the z-axis, then the value of the integral $\oint_C \vec{A}.d\vec{l}$ is **[CSIR June - 2014]**

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 0

8. Let $\vec{r}$ be the position vector of any point in three dimensional space and $r = |\vec{r}|$. Then **[CSIR Dec - 2014]**

(a) $\vec{\nabla}.\vec{r} = 0$ and $\vec{\nabla}\times\vec{r} = \frac{\vec{r}}{r}$ (b) $\vec{\nabla}.\vec{r} = 0$ and $\vec{\nabla}^2\vec{r} = 0$

(c) $\vec{\nabla}.\vec{r} = 3$ and $\vec{\nabla}^2\vec{r} = \frac{\vec{r}}{r^2}$ (d) $\vec{\nabla}.\vec{r} = 3$ and $\vec{\nabla}\times\vec{r} = 0$

GATE PREVIOUS YEAR QUESTIONS

9. If S is the closed surface enclosing a volume V and $\hat{n}$ is the unit normal vector to the surface and $\vec{r}$ is the positive vector, then the value of the following integral $\iint_S \vec{r}.\hat{n}dS$ is: **[GATE 2001]**

(a) V (b) 2V (c) 0 (d) 3V

10. Consider the set of vectors $\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$ **[GATE 2001]**

(a) The three vectors are orthonormal (b) The three vectors are linearly independent

(c) The three vectors cannot form a basis in a three-dimensional real vector space.

(d) $\frac{1}{\sqrt{2}}(1,1,0)$ can be written as the linear combination of $\frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$.

11. If $\vec{A} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$, then $\nabla^2\vec{A}$ equals **[GATE 2001]**

(a) 1 (b) 3 (c) 0 (d) -3

12. A vector $\vec{A} = (5x+2y)\hat{i} + (3y-z)\hat{j} + (2x-az)\hat{k}$ is solenoidal if the constant a has a value: **[GATE 2002]**

(a) 4 (b) –4 (c) 8 (d) –8

13. Which of the following vectors is orthogonal to the vector $(a\hat{i} + b\hat{j})$, where a and b (a≠b) are constants, and $\hat{i}$ and $\hat{j}$ are unit orthogonal vectors? **[GATE 2002]**

(a) $-b\hat{i} + a\hat{j}$ (b) $-a\hat{i} + b\hat{j}$ (c) $-a\hat{i} - b\hat{j}$ (d) $-b\hat{i} - a\hat{j}$

14. The unit vector normal to the surface $3x^2 + 4y = z$ at the ponit (1, 1, 7) is: **[GATE 2002]**

(a) $\left(-6\hat{i} + 4\hat{j} + \hat{k}\right)/\sqrt{53}$ (b) $\left(4\hat{i} + 6\hat{j} - \hat{k}\right)/\sqrt{53}$

(c) $\left(6\hat{i} + 4\hat{j} - \hat{k}\right)/\sqrt{53}$ (d) $\left(4\hat{i} + 6\hat{j} + \hat{k}\right)/\sqrt{53}$

Common data for Q.15 and Q.16 **[GATE 2003]**

Consider the vector $\vec{V} = \frac{\vec{r}}{r^3}$

15. The surface integral of this vector over the surface of a cube of size and centered at the origin.
(a) 0 (b) 2π (c) $2\pi\ a^3$ (d) 4π

16. Which one of the following is NOT correct?
(a) Value of the line integral of this vector around any closed curve is zero
(b) This vector can be written as the gradient of some scalar function
(c) The line integral of this vector from point P to point Q is independent of the path taken.
(d) This vector can represent the magnetic field of some current distribution

17. The curl of the vector $\vec{A} = z\,\hat{i} + x\,\hat{j} + y\,\hat{k}$ is given by: **[GATE 2003]**
(a) $\hat{i}+\hat{j}+\hat{k}$ (b) $\hat{i}-\hat{j}+\hat{k}$ (c) $\hat{i}+\hat{j}-\hat{k}$ (d) $-\hat{i}-\hat{j}-\hat{k}$

18. The two vectors $\vec{p} = \hat{i},\ \vec{q} = (\hat{i}+\hat{j})\ /\ \sqrt{2}$ are: **[GATE 2003]**
(a) related by a rotation (b) related by a reflection through the xy-plane
(c) related by an inversion (d) not linearly independent

19. For the function $\phi = x^2y + xy$ the value of $\left|\vec{\nabla}\phi\right|$ at x= y = 1 is **[GATE 2004]**
(a) 5 (b) $\sqrt{5}$ (c) 13 (d) $\sqrt{13}$

20. If a vector field $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$, then $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{F}\right)$ is **[GATE 2005]**
(a) 0 (b) $\hat{i}$ (b) $2\hat{j}$ (d) $3\hat{k}$

21. Given the four vectors, $u_1 = \begin{pmatrix}1\\2\\1\end{pmatrix}, u_2 = \begin{pmatrix}3\\-5\\1\end{pmatrix}, u_3 = \begin{pmatrix}2\\4\\-8\end{pmatrix}, u_4 = \begin{pmatrix}3\\6\\-12\end{pmatrix}$, the linearly dependent pair is: **[GATE 2005]**
(a) u_1, u_2 (b) u_1, u_3 (c) u_1, u_4 (d) u_3, u_4

22. The unit normal to the curve $x^3\,y^2 + xy = 17$ at the point (2, 0) is: **[GATE 2005]**
(a) $\frac{\left(\hat{i}+\hat{j}\right)}{\sqrt{2}}$ (b) $-\hat{i}$ (c) $-\hat{j}$ (d) $\hat{j}$

23. A vector field is defined everywhere as $\vec{F} = \frac{y^2}{L}\hat{i} + z\hat{k}$. The net flux of $\vec{F}$ associated with a cube of side L, with one vertex at the origin and sides along the positive X, Y, and Z axes, is: **[GATE 2007]**
(a) L^3 (b) $4L^3$ (c) $8L^3$ (d) $10L^3$

24. If $\vec{r} = x\hat{i} + y\hat{j}$, then: **[GATE 2007]**
(a) $\vec{\nabla}.\vec{r} = 0$ and $\vec{\nabla}\left|\vec{r}\right| = \vec{r}$ (b) $\vec{\nabla}.\vec{r} = 2$ and $\vec{\nabla}\left|\vec{r}\right| = \vec{r}$
(c) $\vec{\nabla}.\vec{r} = 2$ and $\vec{\nabla}\left|\vec{r}\right| = \frac{\vec{r}}{r}$ (d) $\vec{\nabla}.\vec{r} = 3$ and $\vec{\nabla}\left|\vec{r}\right| = \frac{\hat{r}}{r}$

25. Cosnider a vector $\vec{p} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ in the coordinate system $(\hat{i}, \hat{j}, \hat{k})$. The axes are rotated anti-clockwise about the Y axis by an angle of 60°. The vector $\vec{p}$ in the rotated coordinate system $(\hat{i}', \hat{j}', \hat{k}')$ is: **[GATE 2007]**

(a) $(1-\sqrt{3})\hat{i}' + 3\hat{j}' + (1+\sqrt{3})\hat{k}'$ (b) $(1+\sqrt{3})\hat{i}' + 3\hat{j}' + (1-\sqrt{3})\hat{k}'$

(c) $(1-\sqrt{3})\hat{i}' + (3+\sqrt{3})\hat{j}' + 2\hat{k}'$ (d) $(1-\sqrt{3})\hat{i}' + (3-\sqrt{3})\hat{j}' + 2\hat{k}'$

26. The curl of the vector field is $\vec{F}$ is $2\hat{x}$. Identity the appropriate vector field $\vec{F}$ from the choices given below: **[GATE 2008]**

(a) $\vec{F} = 2z\hat{x} + 3z\hat{y} + 5y\hat{z}$ (b) $\vec{F} = 3z\hat{y} + 5y\hat{z}$

(c) $\vec{F} = 3x\hat{y} + 5y\hat{z}$ (d) $\vec{F} = 2x\hat{y} + 5y\hat{z}$

27. The value of the contour integral $\left|\int_C \vec{r} \times d\vec{\theta}\right|$, for a circle C of radius r with centre at the origin is **[GATE 2009]**

(a) $2\pi r$ (b) $\frac{r^2}{2}$ (c) πr^2 (d) r

28. An electrostatic field $\vec{E}$ exists in a given region R. Choose the wrong statement: **[GATE 2009]**

(a) Circulation of $\vec{E}$ is zero.

(b) $\vec{E}$ can always be expressed as the gradient of a scalar field.

(c) The potential difference between any two arbitary points in the region R is zero.

(d) The work done in a closed path lying entirely in R is zero.

29. Consider the set of vectors in three-dimensional real vector space $\mathbf{R}^3$, $S = \{(1,1,1),(1,-1,1),(1,1,-1)\}$. Whch of the following statement is true? **[GATE 2009]**

(a) S is not a linearly independent set (b) S is a basis for $\mathbf{R}^3$

(c) The vectors in S are orthogonal

(d) An orthogonal set of vectors cannot be generated from S

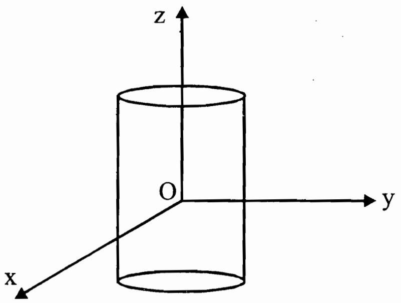
30. If a force $\vec{F}$ is derivable from a potential function $V(r)$, where r is the distance from the origin of the coordinate system, it follows that **[GATE 2011]**

(a) $\vec{\nabla} \times \vec{F} = 0$ (b) $\vec{\nabla}.\vec{F} = 0$ (c) $\vec{\nabla} V = 0$ (d) $\nabla^2 V = 0$

31. The unit vector normal to the surface $x^2 + y^2 - z = 1$ at the point P(1, 1, 1) is: **[GATE 2011]**

(a) $\frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$ (b) $\frac{2\hat{i} + \hat{j} - \hat{k}}{\sqrt{6}}$ (c) $\frac{\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{6}}$ (d) $\frac{2\hat{i} + 2\hat{j} - \hat{k}}{3}$

32. Consider a cylinder of height h and radius a, closed at both ends, centered at the origin. Let $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ be the position vector and $\hat{n}$ a unit vector normal to the surface. The surface integral $\int_s \vec{r}.\hat{n}ds$ over the closed surface of the cylinder is: **[GATE 2011]**



(a) $2\pi a^2(a+h)$ (b) $3\pi a^2 h$ (c) $2\pi a^2 h$ (d) Zero

33. Identify the CORRECT statement for the following vectors $\vec{a} = 3\hat{i} + 2\hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j}$ **[GATE 2012]**

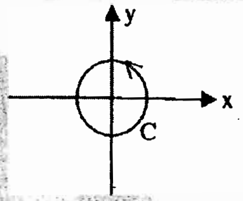
(a) The vectors $\vec{a}$ and $\vec{b}$ are linearly independent

(b) The vectors $\vec{a}$ and $\vec{b}$ are linearly dependent

(c) The vectors $\vec{a}$ and $\vec{b}$ are orthogonal

(d) The vectors $\vec{a}$ and $\vec{b}$ are normalized

34. Given $\vec{F} = \vec{r} \times \vec{B}$, where $\vec{B} = B_0(\hat{i} + \hat{j} + \hat{k})$ is a constant vector and $\vec{r}$ is the position vector. The value of $\oint_C \vec{F}.d\vec{r}$, where C is a circle of unit radius centered at origin is, **[GATE 2012]**



(a) 0 (b) $2\pi B_0$ (c) $-2\pi B_0$ (d) 1

35. If $\vec{A}$ and $\vec{B}$ are constant vectors, then $\nabla\left[\vec{A}.\left(\vec{B} \times \vec{r}\right)\right]$ is **[GATE 2013]**

(a) $\vec{A}.\vec{B}$ (b) $\vec{A} \times \vec{B}$ (c) $\vec{r}$ (d) zero

36. The unit vector perpendicular to the surface $x^2 + y^2 + z^2 = 3$ at the point (1, 1, 1) is

[GATE 2014]

(a) $\frac{\hat{x} + \hat{y} - \hat{z}}{\sqrt{3}}$ (b) $\frac{\hat{x} - \hat{y} - \hat{z}}{\sqrt{3}}$ (c) $\frac{\hat{x} - \hat{y} + \hat{z}}{\sqrt{3}}$ (d) $\frac{\hat{x} + \hat{y} + \hat{z}}{\sqrt{3}}$

37. Four forces are given below in Cartesian and spherical polar coordinates. **[GATE 2015]**

(i) $\vec{F}_1 = K\exp\left(\frac{-r^2}{R^2}\right)\hat{r}$ (ii) $\vec{F}_2 = K\left(x^3\hat{y} - y^3\hat{z}\right)$

(iii) $\vec{F}_3 = K\left(x^3\hat{x} + y^3\hat{y}\right)$ (iv) $\vec{F}_4 = K\left(\frac{\hat{\phi}}{r}\right)$

where K is a constant. Identify the correct option.

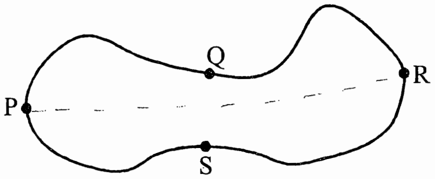
(a) (iii) and (iv) are conservative but (i) and (ii) are not

(b) (i) and (ii) are conservative but (iii) and (iv) are not

(c) (ii) and (iii) are conservative but (i) and (iv) are not

(d) (i) and (iii) are conservative but (ii) and (iv) are not

38. Given that magnetic flux through the closed loop PQRSP is f. If $\int_P^R \vec{A}\cdot d\vec{l} = \phi_1$ along PQR, the value of $\int_P^R \vec{A}\cdot d\vec{l}$ along PSR is **[GATE 2015]**



(a) $\phi - \phi_1$ (b) $\phi_1 - \phi$ (c) $-\phi_1$ (d) ϕ_1

39. The direction of $\vec{\nabla} f$ for a scalar field $f(x,y,z) = \frac{1}{2}x^2 - xy + \frac{1}{2}z^2$ at the point $P(1,1,2)$ is

[GATE 2016]

(a) $\frac{\left(-\hat{j}-2\hat{k}\right)}{\sqrt{5}}$ (b) $\frac{\left(-\hat{j}+2\hat{k}\right)}{\sqrt{5}}$ (c) $\frac{\left(\hat{j}-2\hat{k}\right)}{\sqrt{5}}$ (d) $\frac{\left(\hat{j}+2\hat{k}\right)}{\sqrt{5}}$

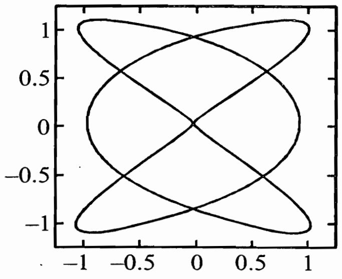
TIFR PREVIOUS YEAR QUESTIONS

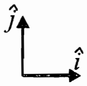
40. A two-dimensional vector $\vec{A}(t)$ is given by $\vec{A}(t) = \hat{i}\sin 2t + \hat{j}\cos 3t$

Which of the following graphs best describes the locus of the tip of the vector, as t is varied from 0 to 2π?

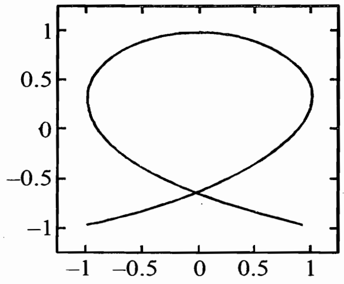
[TIFR 2013]

(a)

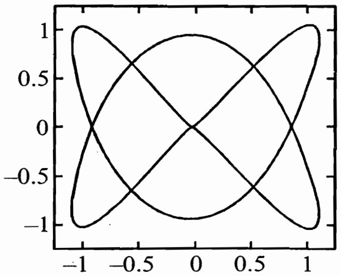




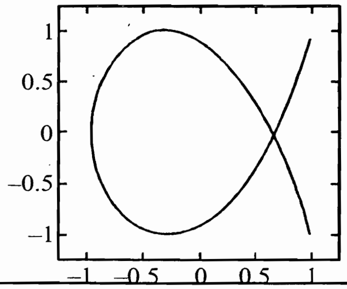
(b)



(c)



(d)



41. Consider the surface corresponding to the equation $4x^2+y^2+z=0$. A possible unit tangent to this surface at the point (1, 2, –8) is **[TIFR 2013]**

(a) $\frac{1}{\sqrt{5}}\hat{i}-\frac{2}{\sqrt{5}}\hat{j}$ (b) $\frac{1}{5}\hat{j}-\frac{4}{5}\hat{k}$

(c) $\frac{4}{9}\hat{i}-\frac{8}{9}\hat{j}+\frac{1}{9}\hat{k}$ (d) $-\frac{1}{\sqrt{5}}\hat{i}+\frac{3}{\sqrt{5}}\hat{j}-\frac{4}{\sqrt{5}}\hat{k}$

42. Which of the following vectors is parallel to the surface $x^2y+2xz=4$ at the point (2,-2,3)? **[TIFR 2015]**

(a) $-6\hat{i}-2\hat{j}+5\hat{k}$ (b) $6\hat{i}+2\hat{j}+5\hat{k}$ (c) $6\hat{i}-2\hat{j}+5\hat{k}$ (d) $6\hat{i}-2\hat{j}-5\hat{k}$

JEST PREVIOUS YEAR QUESTIONS

43. The vector field $xz\hat{i}+y\hat{j}$ in cylinderical polar coordinates is **[JEST 2013]**

(a) $\rho\left(z\cos^2\phi+\sin^2\phi\right)\hat{e}_\rho+\rho\sin\phi\cos\phi\left(1-z\right)\hat{e}_\phi$

(b) $\rho\left(z\cos^2\phi+\sin^2\phi\right)\hat{e}_\rho+\rho\sin\phi\cos\phi\left(1+z\right)\hat{e}_\phi$

(c) $\rho\left(z\sin^2\phi+\cos^2\phi\right)\hat{e}_\rho+\rho\sin\phi\cos\phi\left(1+z\right)\hat{e}_\phi$

(d) $\rho\left(z\sin^2\phi+\cos^2\phi\right)\hat{e}_\rho+\rho\sin\phi\cos\phi\left(1-z\right)\hat{e}_\phi$

MISCELLANEOUS QUESTIONS

44. Given that the co-ordinates of a particle are $y(t)=A\cos(2\omega t)$ and $x(t)=\sin(\omega t)$ the trajectory of the particle is a

(a) Circle (b) Ellipse (c) Hyperbola (d) Parabola

45. A point particle is moving in the (x,y) plane on a trajectory given in polar coordinates by the equation $r\sin\left(\theta+\frac{\pi}{4}\right)=5$. The trajectory of the particle is:

(a) a parabola (b) a straight line (c) a circle (d) a hyperbola

46. A point particle is moving the (x, y) plane on a trajectory given in polar coordinates by the equation

$$r^2-2r\sin\left(\theta+\frac{\pi}{4}\right)-3=0$$

The trajectory of the particle is

(a) a parabola (b) a straight line (c) a circle (d) a hyperbola

47. The normal to the surface given by the equation $z=\cos x\cosh y$ at the point $x=\frac{\pi}{2}$ and $y=0$ lies in

(a) (x,y) plane (b) (x, z) plane

(c) (y, z) plane (d) On the plane given by the equation $x+y+z=\frac{\pi}{2}+1$

48. A particle is moved quasi-statically from the point $(-3,0)$ to $(3,0)$, along a path $y = x^2 - 9$ in an external force field given by $\vec{F} = yi + 3yj$. Give that all physical quantities are in SI units, the magnitude of the work done on the particle is given by

(a) 36 J (b) 18 J (c) 9 J (d) 0

49. A point particle is moving in the (x, y) plane on a trajectory given in polar coordinates by the equation

$$25 + r^2 \cos 2\theta = 0$$

The trajectory of the particle is a

(a) parabola (b) circle (c) ellipse (d) hyperbola

50. The value of the line integral

$$\oint \frac{xdy - ydx}{x^2 + y^2}$$

along a circle of radius 3 centered at the origin in the counter clockwise direction is given by

(a) 0 (b) $\frac{3}{2\pi}$ (c) 2π (d) 6π

51. $\nabla^2\left(\frac{1}{r}\right)$ is:

(a) 0 (b) $-\delta(r)$ (c) $-4\pi\delta(r)$ (d) $4\pi\delta(r)$

52. $\iiint \nabla^2\left(\frac{1}{r}\right)dV \quad r \neq 0$ is:

(a) 0 (b) -4π (c) 4π (d) 1

53. The value of t for which three vevtors $[(1-t),0,0],[1,(1-t),0]$ & $[1,1,(1-t)]$ are linearly dependent is

(a) 1 (b) 0 (c) 2 (d) –1

54. The value of $\oint \vec{A} \cdot \vec{d\ell}$ along a square loop of side L in a uniform field $\vec{A}$ is:

(a) 0 (b) 2LA (c) 4LA (d) L^2A

55. The value of $\oint_S \frac{\vec{r} \cdot d\vec{S}}{r^3}$, where $\vec{r}$ is the position vector and S is a closed surface enclosing the origin, is:

(a) 0 (b) π (c) 4π (d) 8π

56. If $\vec{A}(t)$ is a vector of constant magnitude, which of the following is true?

(a) $\frac{d\vec{A}}{dt} = 0$ (b) $\frac{d^2\vec{A}}{dt^2} = 0$ (c) $\frac{d\vec{A}}{dt} \cdot \vec{A} = 0$ (d) $\frac{d\vec{A}}{dt} \times \vec{A} = 0$

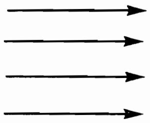
57. The function $r^n\ \hat{r}$ $(r > 0)$, where r refers to spherical coordinate system, is:

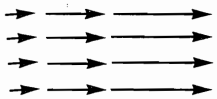
(a) an irrotational vector for $n = -2$ (b) a solenoidal vector for $n = -2$
(c) an irrotational vector for $n = -3$ (d) a solenoidal vector for $n = -3$

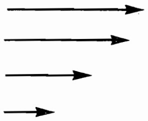
58. Two sets of vectors, one with m elements and the other with n elements $(m < n)$ span the same linear vector space. If k is the dimension of the vector space, then:

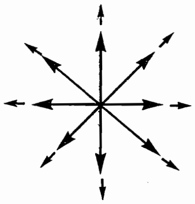
(a) $k \leq m$ (b) $k \geq m$ (c) $k = m + n$ (d) $k = n$

59. Identify the vector field given below which has a finite curl

(a)


(b)


(c)


(d)


60. Given any three non-zero vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$, their triple product $\vec{A}.\left(\vec{B}\times\vec{C}\right)$ vanishes if

(a) They are perpendicular to each other. (b) Any two of them are perpendicular

(c) Any two of them are parallel. (d) They are non-coplanar.

61. The necessary and sufficient condition that $\oint_c \vec{A}.d\vec{r}=0$, for any closed curve C is

(a) $\vec{\nabla}.\vec{A}=0$ (b) $\vec{\nabla}\times\vec{A}=0$ (c) $\vec{\nabla}\vec{\nabla}\times\vec{A}=0$ (d) $\vec{\nabla}\times\vec{\nabla}\times\vec{A}=0$

62. Any arbitrary vector in a three dimensional Cartesian space can be expressed as a linear combination of the following number of linearly independent vectors:

(a) Arbitrary number (b) 1 (c) 2 (d) 3

63. The line integral of $\vec{A}$ vanishes about every closed path. Then $\vec{A}$ must be equal to

(a) Curl of a vector function (b) Gradient of a scalar function

(c) Gradient of a vector function (d) Zero

64. When the fluid is incompressible, the equation of continuity can be reduced to $\frac{\partial\rho}{\partial t}=0=\rho\left(\vec{\nabla}.\vec{v}\right)$. Since the density ρ is constant in this case. Here $\vec{v}$ is the velocity of a typical particle of the fluid. Further, if the flow is irrotational, then the equation can be rewritten as

(a) $\vec{\nabla}.\vec{\nabla}\phi=0$ (b) $\vec{\nabla}\times\vec{\nabla}\phi=0$ (c) $\vec{\nabla}\phi=0$ (d) $\vec{v}=\text{constant}$

65. Given the vector $\vec{A}\left(y,-x,0\right)$, the line integral $\oint_c \vec{A}.\vec{dl}$, where C is a circle of radius 5 units with its centre at the origin, (correct to the first decimal place) is

(a) 172.8 (b) 157.1 (c) –146.3 (d) 62.8

66. The value of the integral $I=\int_s \vec{r}.d\vec{s}$ where S is the surface enclosing the volume V is

(a) 3 (b) V (c) 3V (d) 0

ANSWER KEY

1. (a)	2. (b)	3. (c)	4. (a)	5. (c)	6. (d)	7. (d)
8. (d)	9. (d)	10. (b)	11. (c)	12. (c)	13. (a)	14. (c)
15. (d)	16. (d)	17. (a)	18. (a)	19. (d)	20. (a)	21. (d)
22. (d)	23. (a)	24. (c)	25. (a)	26. (b)	27. (a)	28. (c)
29. (b)	30. (a)	31. (d)	32. (b)	33. (a)	34. (c)	35. (b)
36. (d)	37. (d)	38. (b)	39. (b)	40. (c)	41. (a)	42. (c)
43. (a)	44. (d)	45. (b)	46. (c)	47. (b)	48. (a)	49. (d)
50. (c)	51. (c)	52. (a)	53. (a)	54. (a)	55. (c)	56. (c)
57. (b)	58. (b)	59. (c)	60. (c)	61. (b)	62. (d)	63. (b)
64. (b)	65. (b)	66. (c)				

CSIR-UGC-NET/JRF DEC 2016

MATHEMATICAL PHYSICS

SECTION: MATRIX ALGEBRA

CSIR PREVIOUS YEAR QUESTIONS

Consider the matrix $M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ **[CSIR June - 2011]**

1. The eigenvalues of M are

(a) 0,1,2 (b) 0,0,3 (c) 1,1,1 (d) -1,1,3

2. The exponential of M simplifies (I is the 3 x 3 identity matrix)

(a) $e^M = I + \left(\frac{e^3 - 1}{3}\right) M$ (b) $e^M = I + M + \frac{M^2}{2!}$

(c) $e^M = I + 3^3 M$ (d) $e^M = (e-1)M$

3. A 3 x 3 matrix M has $Tr[M] = 6, Tr[M^2] = 26, Tr[M^3] = 90$. Which of the following can be possible set of eigenvalues of M? **[CSIR Dec - 2011]**

(a) {1,1,4} (b) {-1,0,7} (c) {-1,3,4} (d) {2,2,2}

4. The eigenvalues of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$ **[CSIR June - 2012]**

(a) (1,4,9) (b) (0,7,7) (c) (0,1,13) (d) (0,0,14)

5. The eiegenvalues of the anti-symmetric matrix $A = \begin{pmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{pmatrix}$ where n_1, n_2, n_3 are components of an unit vector, are **[CSIR June - 2012]**

(a) 0, -i, i (b) 0, 1, -1 (c) 0, 1+i, -1-i (d) 0, 0, 0

6. A 2 x 2 matrix 'A' has eigenvalues $e^{i\pi/5}$ and $e^{i\pi/6}$. The smallest value of 'n' such that $A^n = I$

[CSIR Dec - 2012]

(a) 20 (b) 30 (c) 60 (d) 120

7. Given a 2 x 2 unitary matrix satisfying $U'U = UU' = I$ with $\det U = e^{i\varphi}$, one can construct a unitary matrix V $\left(V'V = VV' = I\right)$ with det $V = 1$ from it by **[CSIR Dec - 2012]**

(a) Multiplying U by $e^{-i\varphi/2}$ (b) Multiplying a single element of U by $e^{-i\varphi}$

(c) Multiplying any row or column by $e^{-i\varphi/2}$ (d) Multiplying U by $e^{-i\varphi}$

8. Consider a $n \times n$ $(n > 1)$ matrix A, in which A_{ij} is the product of the indices i and j (namely $A_{ij} = ij$) The matrix A **[CSIR Dec - 2013]**

(a) has one degenerate eigenvalue with degeneracy $(n-1)$

(b) has two degenerate eigenvalues with degeneracies 2 and $(n-2)$

(c) has one degenerate eigenvalues with degeneracy n

(d) does not have any degenerate eigenvalues

9. Consider the matrix

$$M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$$

The eigenvalues of M are **[CSIR June - 2014]**

(a) -5,-2,7 (b) -7,0,7 (c) -4i,2i,2i (d) 2,3,6

10. The matrices

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

satisfy the commutation relation **[CSIR June - 2014]**

(a) $[A,B] = B + C, [B,C] = 0, [C,A] = B + C$

(b) $[A,B] = C, [B,C] = A, [C,A] = B$

(c) $[A,B] = B, [B,C] = 0, [C,A] = A$

(d) $[A,B] = C, [B,C] = 0, [C,A] = B$

11. The column vector $\begin{pmatrix} a \\ b \\ a \end{pmatrix}$ is a simultaneous eigenvector of $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ if

[CSIR Dec-2014]

(a) $b = 0$ and $a = 0$ (b) $b = a$ and $b = -2a$

(c) $b = 2a$ and $b = -a$ (d) $b = a/2$ and $b = -a/2$

GATE PREVIOUS YEAR QUESTIONS

12. For any operator A, $i(A^{+} - A)$ is: **[GATE 2001]**

(a) Hermitian (b) Anti-Hermitian (c) Unitary (d) Orthogonal

13. If two matrices A and B can be diagonalized simultaneously, which of the following is true? **[GATE 2002]**

(a) $A^2B = B^2A$ (b) $A^2B^2 = B^2A$ (c) $AB = BA$ (d) $AB^2AB = BABA^2$

14. Which one of the following matrices is the inverse of the matrix $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$? **[GATE 2002]**

(a) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

15. A 3×3 matrix has eigenvalues $0, 2+i$ and $2-i$. Which one of the following statements is correct? **[GATE 2003]**

(a) The matrix is Hermitian (b) The matrix is unitary

(c) The inverse of the matrix exists (d) The determinant of the matrix is zero

16. A real traceless 4×4 unitary matrix has two eigen values - 1 and 1 . The other eigenvalues are: **[GATE 2004]**

(a) zero and +2 (b) −1 and +1 (c) zero and +1 (d) +1 and +1

17. The eigenvalues of the matrix $\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ are: **[GATE 2004]**

(a) +1 and +1 (b) zero and +1 (c) zero and +2 (d) −1 and +1

18. Eigen values of the matrix $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2i \\ 0 & 0 & 2i & 0 \end{pmatrix}$ are: **[GATE 2005]**

(a) −2, −1, 1, 2 (b) −1, 1, 0, 2 (c) 1, 0, 2, 3 (d) −1, 1, 0, 3

19. The determinant of a3 × 3 real symmetric matrix is 36. If two of its eigen values are 2 and 3 then the third eigenvlaue is: **[GATE 2005]**

(a) 4 (b) 6 (c) 8 (d) 9

20. A linear transformation T, defined as $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \end{pmatrix}$, transform a vector $\underset{\sim}{x}$ from a three-dimensional real space to a two-dimensional real space. The transformation matrix T is: **[GATE 2006]**

(a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Common data for Q. 21 and Q. 22:

One of the eigen values of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is 5. **[GATE 2006]**

21. The other two eigen values are:
(a) 0 and 0 (b) 1 and 1 (c) 1 and -1 (d) -1 and -1

22. The normalized eigen vector corresponding to the eigen value 5 is:
(a) $\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

23. The eigenvalues of a matrix are i, $-2i$ and $3i$. The matrix is: **[GATE 2007]**
(a) unitary (b) anti-unitary (c) hermitian (d) anti-hermitian

24. The eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are: **[GATE 2007]**
(a) 6, 1 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (b) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
(c) 6, 1 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (d) 2, 5 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

25. For arbitary matrices E, F, G and H, if $EF - FE=0$, then Trace $(EFGH)$ is equal to **[GATE 2008]**
(a) Trace $(HGFE)$ (b) Trace (E).Trace (F).Trace (G).Trace (H)
(c) Trace $(GFEH)$ (d) Trace $(EGHF)$

26. An unitary matrix $\begin{bmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{bmatrix}$ is given, where a, b, c, d, α and β are real. The inverse of the matrix is **[GATE 2008]**
(a) $\begin{bmatrix} ae^{i\alpha} & -ce^{i\beta} \\ b & d \end{bmatrix}$ (b) $\begin{bmatrix} ae^{i\alpha} & ce^{i\beta} \\ b & d \end{bmatrix}$ (c) $\begin{bmatrix} ae^{-i\alpha} & b \\ ce^{-i\beta} & d \end{bmatrix}$ (d) $\begin{bmatrix} ae^{-i\alpha} & ce^{-i\beta} \\ b & d \end{bmatrix}$

27. The eigenvalues of the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ are **[GATE 2008]**
(a) $\frac{1}{2}\left(\sqrt{3} \pm i\right)$ when $\theta = 45^0$ (b) $\frac{1}{2}\left(\sqrt{3} \pm i\right)$ when $\theta = 30^0$
(c) ± 1 since, the matrix is unitary (d) $\frac{1}{2}(1 \pm i)$ when $\theta = 30^0$

28. The eigenvalues of the matrix $A=\begin{bmatrix}0 & i\\ i & 0\end{bmatrix}$ are **[GATE 2009]**

(a) real and distinct (b) complex and distinct

(c) complex and coinciding (d) real and coinciding

29. The eigen values of the matrix $\begin{pmatrix}2 & 3 & 0\\ 3 & 2 & 0\\ 0 & 0 & 1\end{pmatrix}$ are **[GATE 2010]**

(a) 5, 2, -2 (b) -5, -1, 1 (c) 5,1,-1 (d) -5, 1, 1

30. Two matrices A and B are said to be similar if $B=P^{-1}AP$ for some invertible matrix P. Which of the following statements is **NOT TRUE**? **[GATE 2011]**

(a) Det A = Det B (b) Trace of A = Trace of B

(c) A and B have the same eigenvectors (d) A and B have the same eigenvalues.

31. A 3×3 matrix has elements such that its trace is 11 and its determinant is 36. The eigenvalues of the matrix are all known to be positive integers. The largest eigenvalue of the matrix is: **[GATE 2011]**

(a) 18 (b) 12 (c) 9 (d) 6

32. The eigenvalues of the matrix $\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$ are **[GATE 2012]**

(a) 0,1,1 (b) $0,-\sqrt{2},\sqrt{2}$ (c) $\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0$ (d) $\sqrt{2},\ \sqrt{2},0$

33. The degenerate eigenvalues of the matrix $M=\begin{pmatrix}4 & -1 & -1\\ -1 & 4 & -1\\ -1 & -1 & 4\end{pmatrix}$ is: ....

(your answer should be an integer) **[GATE 2013]**

34. The matrix $A=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1+i\\ 1-i & -1\end{bmatrix}$ is **[GATE 2014]**

(a) orthogonal (b) symmetric (c) anti-symmetric (d) unitary

TIFR PREVIOUS YEAR QUESTIONS

35. The matrix $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ **[TIFR 2010]**

can be relateed by a similarity transformation to the matrix.

(a) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

36. Consider the matrix, $M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$

A 3-dimensional basis formed by eigenvectors of M is **[TIFR 2011]**

(a) $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

(c) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ (d) $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

37. The trace of the real 4 × 4 matrix $U = \exp(A)$, where $A = \begin{pmatrix} 0 & 0 & 0 & \pi/4 \\ 0 & 0 & -\pi/4 & 0 \\ 0 & \pi/4 & 0 & 0 \\ -\pi/4 & 0 & 0 & 0 \end{pmatrix}$

is equal to **[TIFR 2011]**

(a) $2\sqrt{2}$ (b) $\pi/4$ (c) $\exp(i\varphi)$ for $\varphi = 0, \pi$ (d) zero

38. Two different 2 × 2 matrices A and B are found to have the same eigenvalues. It is then correct to state that $A = SBS^{-1}$ where S can be a **[TIFR 2012]**

(a) traceless 2 × 2 matrix (b) Hermitian 2 × 2 matrix

(c) unitary 2 × 2 matrix (d) arbitrary 2 × 2 matrix

39. The product MN of two Hermitian matrices M and N is anti-Hermitian. It follows that **[TIFR 2014]**

(a) $\{M, N\} = 0$ (b) $[M, N] = 0$ (c) $M^{\dagger} = N$ (d) $M^{\dagger} = N^{-1}$

40. If the eigenvalues of a symmetric 3×3 matrix A are 0, 1, 3 and the corresponding eigenvectors can be written as **[TIFR 2016]**

$$\begin{bmatrix}1\\1\\1\end{bmatrix},\begin{bmatrix}1\\0\\-1\end{bmatrix},\begin{bmatrix}1\\-2\\1\end{bmatrix}$$

respectively, then the matrix A^4 is

(a) $\begin{bmatrix}41 & -81 & 40\\-81 & 0 & -81\\40 & -81 & 41\end{bmatrix}$ (b) $\begin{bmatrix}-82 & -81 & 79\\-81 & 81 & -81\\79 & -81 & 83\end{bmatrix}$

(c) $\begin{bmatrix}14 & -27 & 13\\-27 & 54 & -27\\13 & -27 & 14\end{bmatrix}$ (d) $\begin{bmatrix}14 & -13 & 27\\-13 & 54 & -13\\27 & -13 & 14\end{bmatrix}$

JEST PREVIOUS YEAR QUESTIONS

41. For an $N\times N$ matrix consisting of all ones, **[JEST 2012]**

(a) all eigenvalues = 1 (b) all eigenvalues = 0

(c) the eigenvalues are 1, 2, ..., N (d) one eigenvalue = N, the others = 0

42. The coordinate transformation

$$x' = 0.8x + 0.6y, \qquad y' = 0.6x - 0.8y$$

represents **[JEST 2013]**

(a) a translation (b) a proper rotation (c) a reflection (d) none of the above.

43. Given a matrix $M = \begin{pmatrix}2 & 1\\1 & 2\end{pmatrix}$, which of the following represents $\cos(\pi M/6)$? **[JEST 2016]**

(a) $\frac{1}{2}\begin{pmatrix}1 & 2\\2 & 1\end{pmatrix}$ (b) $\frac{\sqrt{3}}{4}\begin{pmatrix}1 & -1\\-1 & 1\end{pmatrix}$ (c) $\frac{\sqrt{3}}{4}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$ (d) $\frac{1}{2}\begin{pmatrix}1 & \sqrt{3}\\\sqrt{3} & 1\end{pmatrix}$

MISCELLANEOUS QUESTIONS

44. The eigenvalues of the matrix $\begin{bmatrix}0 & 16\\16 & 0\end{bmatrix}$ are :

(a) 1 and –1 (b) 16 and 16 (c) 16 and –16 (d) 1 and 256

45. Consider the matrix, $A = \begin{pmatrix}0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\\1 & 0 & 0 & 0\end{pmatrix}$

the eigenvalues of A are, $\left(i=\sqrt{-1}\right)$

(a) $\{1,-i,-1,i\}$ (b) $\{1,-1,0,0\}$ (c) $\{1,1,1,1\}$ (d) $\{1,1,i,-i\}$

46. Let P be a $n\times n$ diagonalizable matrix which satisfies the equations

$$P^2=P, Tr(P)=n-1$$

Det (P) is :

(a) n (b) 0 (c) 1 (d) $n-1$

47. Let M be a 3×3 Hermitian matrix which satisfies the matrix equation

$$M^2-5M+6I=0$$

Where I refers to the identity matrix. Which of the following are possible eigenvalues of M

(a) {1, 2, 3} (b) {2, 2, 3} (c) {2, 3, 5} (d) 5, 5, 6}

48. Given the three matrices

$$\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}\text{ and }\left[\sigma_i,\sigma_j\right]\equiv\sigma_i\sigma_j-\sigma_j\sigma_i,$$

then $\left[\sigma_1,\left[\sigma_2,\sigma_3\right]\right]+\left[\sigma_2,\left[\sigma_3,\sigma_1\right]\right]+\left[\sigma_3,\left[\sigma_1,\sigma_2\right]\right]$ is

(a) $\sigma_1^2+\sigma_2^2+\sigma_3^2$ (b) $\sigma_1+\sigma_2+\sigma_3$ (c) 0 (d) Identity

49. Given the three matrices

$$\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\sigma_2=\begin{pmatrix}0&i\\-i&0\end{pmatrix},\sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

Which of the following statements is true for all positive integers n and $i=1,2,3$?

(a) $\sigma_i^n=I$ (b) $\sigma_i^n=\sigma_i$ (c) $\sigma_i^{2n}=I$ (d) $\sigma_i^{2n}=\sigma_i$

50. The trace of a 2×2 matrix is 1 and its determinate is 1. Which of the following has to be true ?

(a) One of the eigenvalues is 0 (b) One of the eigenvalue is 1

(c) Both of the eigenvalues are 1 (d) Neither of the eigenvalues is 1

51. Let M by a 3×3 Hermitain matrix which satisfies the matrix equation

$$M^2-7M+12I=0$$

where I refers to the identity matrix. What is the determinant of the matrix M given that the trace is 10 ?

(a) 27 (b) 36 (c) 48 (d) 64

52. Consider a 3×3 matrix of the form:

$$\begin{pmatrix}a_1^2&a_1a_2&a_1a_3\\a_1a_2&a_2^2&a_2a_3\\a_1a_3&a_2a_3&a_3^2\end{pmatrix}$$

The number of zero eigenvalues for this matrix is

(a) 0 (b) 1 (c) 2 (d) 3

53. Which one of the following is not hermitian?

(a) $\begin{bmatrix}1&0\\0&-1\end{bmatrix}$ (b) $\begin{bmatrix}0&-i\\i&0\end{bmatrix}$ (c) $\begin{bmatrix}0&1\\1&0\end{bmatrix}$ (d) $\begin{bmatrix}i&0\\0&1\end{bmatrix}$

54. If A is an antisymmetric matrix and A^T be the transpose of this matrix, then which one of the following relations does not hold good?

(a) $A^T = -A$ (b) $AA^T = A^TA$

(c) A^2 is an antisymmetric matrix (d) A^2 is a symmetirc matrix

55. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ are given by λ_1, λ_2 and λ_3. Which one of th following statement is

NOT TRUE?

(a) $\lambda_1 + \lambda_2 + \lambda_3 = 0$ (b) $\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = 0$

(c)All eigenvalues are real (d) Product of the eigenvalues is 1

56. The matrix $\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is:

(a) orthogonal (b) hermitian (c) symmetric (d) anti-symmetric

57. A given $(n \times n)$ nilpotent matrix A satisfies the equation $A^k = 0$ for $1 < k < n$. Therefore,

(a) Exactly k eigenvalues of A must be zero. (b) Exactly $(n-k)$ eigenvalue of A must be zero.

(c) Every eigenvalue of A is zero. (d) A can have $(n-1)$ non-zero eigenvalues.

58. If $L(x)$ is a linear differential operator and $y_1(x), y_2(x)$ are two arbitrary functions. Then

(a) $L(x)(y_1 + y_2) = Ly_1 + Ly_2$ (b) $L(x)(y_1 + y_2) = y_1Ly_2 + y_2Ly_1$

(c) $L(x)(y_1y_2) = Ly_1 + Ly_2$ (d) $L(x)(y_1 - y_2) = Ly_1 + Ly_2$

59. For which of the following values of a, the inverse of the matrix, $M = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 3 & 5 \\ 0 & 4 & a \end{pmatrix}$, does not exist?

(a) –4 (b) 4 (c) 0 (d) 1

60. Which one of the following matrices is orthogonal?

(a) $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ (b) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ (c) $\begin{pmatrix} -\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ (d) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix}$

61. The number of independent real parameters of a most general hermitian matrix of order 4 is:

(a) 4 (b) 8 (c) 16 (d) 32

62. The eigen values of the matrix, $\begin{bmatrix} 0 & 0 & -i \\ 0 & 2 & 0 \\ i & 0 & 0 \end{bmatrix}$ are

(a) Purely imaginary (b) Complex (c) Real (d) Zero

63. If a matrix A satisfies the condition $A^2 = I$ and $Tr(A) = 0$ (where I is a $n \times n$ identity matrix), then

(a) The determinant of A must be 0 (b) The determinant of A must be +1

(c) A must be odd dimensional (d) A must be even dimensional

64. The matrix A, defined by $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & a & b \\ 0 & -b & a \end{pmatrix}$ is orthogonal if

(a) $a = 1, b = -1$ (b) $a = 1/\sqrt{2}, b = -1/\sqrt{2}$

(c) $a = 1/\sqrt{2}, b = -i/\sqrt{2}$ (d) $a = 1,\ b = 1$

65. If A and B are two $n \times n$ matrices, then the trace of $C = [A, B]$ is

(a) $Tr(AB)$ (b) $Tr(A)\ Tr(B)$ (c) 0 (d) $Tr(A) + Tr(B)$

66. Consider the matrix $M = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$ where a and c are real and b may be complex. If M can be diagonalized by a matrix S, then S is

(a) diagonal (b) symmetric (c) orthogonal (d) unitary

67. The trace and the determinant of a 2×2 matrix are denoted by T and D. The eigenvalues of the matrix are given by

(a) $\frac{T \pm \sqrt{T^2 - 4D}}{2}$ (b) $\pm\frac{\sqrt{T^2 - 4D}}{2}$ (c) $\frac{T \pm D}{2}$ (d) $\pm\frac{T}{2}$

ANSWER KEY

1. (b)	**2. (a)**	**3. (c)**	**4. (d)**	**5. (a)**	**6. (c)**	**7. (a)**
8. (a)	**9. (b)**	**10. (d)**	**11. (b)**	**12. (a)**	**13. (c)**	**14. (c)**
15. (d)	**16. (b)**	**17. (c)**	**18. (a)**	**19. (b)**	**20. (a)**	**21. (c)**
22. (d)	**23. (d)**	**24. (a)**	**25. (a)**	**26. (d)**	**27. (b)**	**28. (b)**
29. (c)	**30. (c)**	**31. (d)**	**32. (b)**	**33. (5)**	**34. (d)**	**35. (d)**
36. (b)	**37. (a)**	**38. (c)**	**39. (a)**	**40. (c)**	**41. (d)**	**42. (c)**
43. (b)	**44. (c)**	**45. (a)**	**46. (b)**	**47. (b)**	**48. (c)**	**49. (c)**
50. (d)	**51. (b)**	**52. (c)**	**53. (d)**	**54. (c)**	**55. (c)**	**56. (a)**
57. (c)	**58. (a)**	**59. (b)**	**60. (b)**	**61. (a)**	**62. (c)**	**63. (d)**
64. (b)	**65. (c)**	**66. (d)**	**67. (a)**			

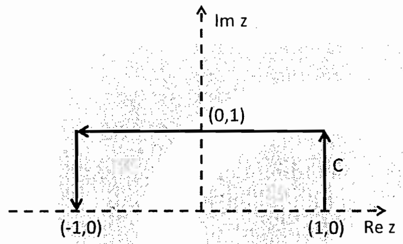
CSIR-UGC-NET/JRF DEC 2016

MATHEMATICAL PHYSICS

SECTION: COMPLEX ANALYSIS

CSIR PREVIOUS YEAR QUESTIONS

1. The value of the integral $\int_C dz\, z^2 e^z$, where C is an open contour in the complex z-plane as shown in figure below: **[CSIR June 2011]**



(a) $\frac{5}{e}+e$ (b) $e-\frac{5}{e}$ (c) $\frac{5}{e}-e$ (d) $-\frac{5}{e}-e$

2. Which of the following is an analytic function of the complex variable $z = x + iy$ in the domain $|z| < 2$? **[CSIR June 2011]**

(a) $(3+x-iy)^7$ (b) $(1+x+iy)^4(7-x-iy)^3$

(c) $(1-2x-iy)^4(3-x-iy)^3$ (d) $(x+iy-1)^{1/2}$

3. The first few terms in the Taylor series expansion of the function $f(x) = \sin x$ around $x=\frac{\pi}{4}$ are **[CSIR Dec 2011]**

(a) $\frac{1}{\sqrt{2}}\left[1+\left(x-\frac{\pi}{4}\right)+\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2+\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+\dots\dots\dots\right]$

(b) $\frac{1}{\sqrt{2}}\left[1+\left(x-\frac{\pi}{4}\right)+\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2-\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+\dots\dots\dots\right]$

(c) $\left[\left(x-\frac{\pi}{4}\right)-\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+\dots\dots\dots\right]$ (d) $\frac{1}{\sqrt{2}}\left[1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\dots\dots\right]$

4. The first few terms in the Laurent series for $\frac{1}{(z-1)(z-2)}$ in the region $1 \le |z| \le 2$ and around $z = 1$ is

[CSIR June 2012]

(a) $\frac{1}{2}\left[1+z+z^2+....\right]\left[1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}+.......\right]$ (b) $\frac{1}{1-z}+z-(1-z)^2+(1-z)^3+........$

(c) $\frac{1}{z^2}\left[1+\frac{1}{z}+\frac{1}{z^2}+......\right]\left[1+\frac{2}{z}+\frac{4}{z^2}+........\right]$ (d) $2(z-1)+5(z-1)^2+7(z-1)^3+......$

The value of the integral $\int_{-\infty}^{\infty}\frac{1}{t^2-R^2}\cos\left(\frac{\pi t}{2R}\right)dt$ **[CSIR June 2012]**

(a) $-\frac{2\pi}{R}$ (b) $-\frac{\pi}{R}$ (c) $\frac{\pi}{R}$ (d) $\frac{2\pi}{R}$

Let $u(x,y)=x+\frac{1}{2}(x^2-y^2)$ be the real part of an analytic function $f(z)$ of the complex variable $z=x+iy$. the imaginary part of $f(z)$ is **[CSIR June 2012]**

(a) $y+xy$ (b) xy (c) y (d) y^2-x^2

The taylor series expansion of the function $\ln(\cosh x)$, where x is real, about point x = 0 starts with the following terms: **[CSIR Dec 2012]**

(a) $-\frac{1}{2}x^2+\frac{1}{12}x^4+........$ (b) $\frac{1}{2}x^2-\frac{1}{12}x^4+........$

(c) $-\frac{1}{2}x^2+\frac{1}{6}x^4+........$ (d) $\frac{1}{2}x^2+\frac{1}{6}x^4+........$

The value of the integral $\oint_C\frac{z^3}{z^2-5z+6}dz$, where C is closed contour defined by the equation $2|z|-5=0$, traversed in the anti-clockwise direction, is **[CSIR Dec 2012]**

(a) $-16\pi i$ (b) $16\pi i$ (c) $8\pi i$ (d) $2\pi i$

With z = x+ iy, which of the following functions f (x,y) is NOT a complex analytic function of z? **[CSIR June 2013]**

(a) $(x+iy-8)^3(4+x^2-y^2+2ixy)^7$ (b) $(x+iy)^7(1-x-iy)^3$

(c) $(x^2-y^2+2ixy-3)^5$ (d) $(1-x+iy)^4(2+x+iy)^6$

0. Which of the following functions cannot be the real part of a complex analytic function of $z=x+iy$? **[CSIR Dec 2013]**

(a) x^2y (b) x^2-y^2 (c) x^3-3xy^2 (d) $3x^2y-y-y^3$

1. Given that the integral $\int_0^{\infty}\frac{dx}{y^2+x^2}=\frac{\pi}{2y}$, the value of $\int_0^{\infty}\frac{dx}{\left(y^2+x^2\right)^2}$ is **[CSIR Dec 2013]**

(a) $\frac{\pi}{y^3}$ (b) $\frac{\pi}{4y^3}$ (c) $\frac{\pi}{8y^3}$ (d) $\frac{\pi}{2y^3}$

12. If C is the contour defined by $|z|=\frac{1}{2}$, the value of the integral $\oint_C \frac{dz}{\sin^2 z}$ is **[CSIR June 2014]**

(a) ∞ (b) $2\pi i$ (c) 0 (d) πi

13. The principal value of the integral $\int_{-\infty}^{\infty} \frac{\sin(2x)}{x^3} dx$ is **[CSIR Dec 2014]**

(a) -2π (b) $-\pi$ (c) π (d) 2π

14. The Laurent series expansion of the function $f(z)=e^z+e^{1/z}$ about $z=0$ is

[CSIR Dec 2014]

(a) $\sum_{n=-\infty}^{\infty} \frac{z^n}{n!}$ for all $|z|<\infty$ (b) $\sum_{n=0}^{\infty}\left(z^n+\frac{1}{z^n}\right)\frac{1}{n!}$ only if $0<|x|<1$

(c) $\sum_{n=0}^{\infty}\left(z^n+\frac{1}{z^n}\right)\frac{1}{n!}$ for all $0<|z|<\infty$ (d) $\sum_{n=-\infty}^{\infty} \frac{z^n}{n!}$, only if $|z|<1$

15. Consider the function $f(z)=\frac{1}{z}\ln(1-z)$ of a complex variable $z=re^{i\theta}(r\geq 0,-\infty<\theta<\infty)$. The singularities of $f(z)$ are as follows: **[CSIR Dec 2014]**

(a) branch points at $z=1$ and $z=\infty$; and a pole at $z=0$ only for $0\leq\theta<2\pi$

(b) branch points at $z=1$ and $z=\infty$; and a pole at $z=0$ for all θ other than $0\leq\theta<2\pi$

(c) branch points at $z=1$ and $z=\infty$; and a pole at $z=0$ for all θ

(d) branch points at $z=0$, $z=1$ and $z=\infty$

16. The value of the integral $\int_{-\infty}^{\infty} \frac{1}{1+x^4} dx$ is **[CSIR June 2015]**

(a) $\pi/\sqrt{2}$ (b) $\pi/2$ (c) $\sqrt{2}\pi$ (d) 2π

17. The function $\frac{z}{\sin \pi z^2}$ of a complex variable z has **[CSIR Dec 2015]**

(a) a simple pole at 0 and poles of order 2 at $z=\pm\sqrt{n}$ for n = 1, 2, 3 ....

(b) a simple pole at 0 and poles of order 2 at $z=\pm\sqrt{n}$ and $z=\pm i\sqrt{n}$ for n = 1, 2, 3 ....

(c) poles of order 2 at $z=\pm\sqrt{n}$ for n = 0, 1, 2, 3 ....

(d) poles of order 2 at $z=\pm n$ for n = 0, 1, 2, 3 ....

18. The radius of convergence of the Taylor series expansion of the function $\frac{1}{\cosh(x)}$ around x = 0, is

[CSIR June 2016]

(a) ∞ (b) π (c) $\pi/2$ (d) 1

19. The value of the contour integral

$$\frac{1}{2\pi i}\oint_C \frac{e^{4z}-1}{\cosh(z)-2\sinh(z)}dz$$

around the unit circle C traversed in the anti-clockwise direction, is **[CSIR June 2016]**

(a) 0 (b) 2 (c) $-\frac{8}{\sqrt{3}}$ (d) $-\tanh\left(\frac{1}{2}\right)$

GATE PREVIOUS YEAR QUESTIONS

20. The value of the integral $\int_C z^{10}dz$, where C is the unit circle with the origin as the centre is: **[GATE 2001]**

(a) 0 (b) $z^{11}/11$ (c) $2\pi iz^{11}/11$ (d) 1/11

21. The value of the residue of $\frac{\sin z}{z^6}$ is **[GATE 2001]**

(a) $-\frac{1}{5!}$ (b) $\frac{1}{5!}$ (c) $\frac{2\pi i}{5!}$ (d) $-\frac{2\pi i}{5!}$

22. If a function $f(z) = u(x, y) + iv(x, y)$ of the complex variable $z = x + iy$, where x, y, u and v are real, is analytic in a domain D of z, then which of the following is true? **[GATE 2002]**

(a) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial x}$ (b) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$

(c) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial y}$ (d) $\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 v}{\partial x\partial y}$

23. The value of the integral $\int_C dz/z^2$, where z is a complex variable and C is the unit circle with the origin as its centre, is: **[GATE 2003]**

(a) 0 (b) $2\pi i$ (c) $4\pi i$ (d) $-4\pi i$

24. The inverse of the complex number $\frac{3+4i}{3-4i}$ is: **[GATE 2004]**

(a) $\frac{7}{25}+i\frac{24}{25}$ (b) $-\frac{7}{25}+i\frac{24}{25}$ (c) $\frac{7}{25}-i\frac{24}{25}$ (d) $-\frac{7}{25}-i\frac{24}{25}$

25. The value of $\int_C \frac{dz}{(z^2+a^2)}$, where C is a unit circle (anti clockwise) centered at the origin in the complex z-plane is: **[GATE 2004]**

(a) π for $a = 2$ (b) zero for $a = \frac{1}{2}$ (c) 4π for $a = 2$ (d) $\frac{\pi}{2}$ for $a = \frac{1}{2}$

26. The value of the integral $\int_C \frac{dz}{z+3}$ where C is a circle (anticlockwise) with $|z| = 4$, is:

[GATE 2005]

(a) 0 (b) π i (c) 2 π i (d) 4 π i

27. All solutions of the equation $e^z = -3$ are **[GATE 2005]**

(a) $in\pi \ln 3, n = \pm 1, \pm 2$............ (b) $\ln 3 + i(2n+1)\pi, n = 0, \pm 1, \pm 2$............

(c) $\ln 3 + i2n\pi, n = 0, \pm 1, \pm 2$............ (d) $i3n\pi, n = \pm 1, \pm 2$............

28. The value of $\oint_C \frac{e^{2z}}{(z+1)^4} dz$, where C is a circle defined by $|z| = 3$, is: **[GATE 2006]**

(a) $\frac{8\pi i}{3} e^{-2}$ (b) $\frac{8\pi i}{3} e^{-1}$ (c) $\frac{8\pi i}{3} e$ (d) $\frac{8\pi i}{3} e^{2}$

29. The contour integral $\oint \frac{dz}{z^2 + a^2}$ is to be evaluated on a circle of radius 2a centered at the origin. It will have contributions only from the points. **[GATE 2007]**

(a) $\frac{1+i}{\sqrt{2}} a$ and $-\frac{1+i}{\sqrt{2}} a$ (b) ia and $-ia$

(c) $ia, -ia, \frac{1-i}{\sqrt{2}} a$ and $-\frac{1-i}{\sqrt{2}} a$ (d) $\frac{1+i}{\sqrt{2}} a, -\frac{1+i}{\sqrt{2}} a, \frac{1-i}{\sqrt{2}} a$ and $-\frac{1-i}{\sqrt{2}} a$

30. If $I = \oint_C \ln z \, dz$, where C is the unit circle taken anticlockwise and lnz is the principal branch of the logarithmic function, which of the following is correct? **[GATE 2008]**

(a) $I = 0$ by residue theorem. (b) I is not defined since, lnz is branch cut.

(c) $I \neq 0$ (d) $\oint_C \ln(z^2)\, dz = 2I$

31. The value of $\int_{-i}^{i} \pi(z+1)\, dz$ is **[GATE 2008]**

(a) 0 (b) $2\pi i$ (c) $-2\pi i$ (d) $(-1+2i)\pi$

32. The value of the integral $\int_C \frac{e^z}{z^2 - 3z + 2} dz$, where the contour C is the circle $|z| = \frac{3}{2}$ is

[GATE 2009]

(a) $2\pi ie$ (b) πie (c) $-2\pi ie$ (d) $-\pi ie$

33. The value of the integral $\oint_C \frac{e^z \sin z}{z^2} dz$, where the contour C is the unit circle: $|z-2| = 1$, is

[GATE 2010]

(a) $2\pi i$ (b) $4\pi i$ (c) πi (d) 0

For the complex function, $f(z)=\dfrac{e^{\sqrt{z}}-e^{-\sqrt{z}}}{\sin(\sqrt{z})}$, which of the following statement is correct?

[GATE 2010]

(a) $z=0$ is a branch point.
(b) $z=0$ is a pole of order one
(c) $z=0$ is a removable singularity
(d) $z=0$ is an essential singularity

Common data for Q. 35 and Q. 36

Consider a function $f(z)=\dfrac{z\sin z}{(z-\pi)^2}$ of a complex variable z. **[GATE 2011]**

Which of the following statements is TRUE for the function $f(z)$?

(a) $f(z)$ is analytic everywhere in the complex plane

(b) $f(z)$ has a zero at $z=\pi$

(c) $f(z)$ has a pole of order 2 at $z=\pi$
(d) $f(z)$ has a simple pole at $z=\pi$.

Consider a counterclockwise circular contour $|z|=1$ about the origin. The integral $\oint f(z)\,dz$ over this contour is:

(a) $-i\pi$
(b) Zero
(c) $i\pi$
(d) $2i\pi$

The value of the integral $\oint_C e^{1/z}dz$, using the contour C of circle with unit radius $|z|=1$, is

[GATE 2012]

(a) 0
(b) $1-2\pi i$
(c) $1+2\pi i$
(d) $2\pi i$

For the function $f(z)=\dfrac{16z}{(z+3)(z-1)^2}$, the residue at the pole $z=1$...

(Your answer should be an integer) **[GATE 2013]**

The value of the integral $\oint_C \dfrac{z^2}{e^z+1}dz$, where C is the circle $|z|=4$, is **[GATE 2014]**

(a) $2\pi i$
(b) $2\pi^2 i$
(c) $4\pi^3 i$
(d) $4\pi^2 i$

Consider $w=f(z)=u(x,y)+iv(x,y)$ to be an analytic function in a domain D. Which one of the following options is NOT correct? **[GATE 2015]**

(a) $u(x,y)$ satisfies Laplace equation in D

(b) $v(x,y)$ satisfies Laplace equation in D

(c) $\int_{z_1}^{z_2} f(z)\,dz$ is dependent on the choice of the contour between z_1 and z_2 in D

(d) $f(z)$ can be Taylor expanded in D

Consider a complex function $f(z)=\dfrac{1}{z\left(z+\frac{1}{2}\right)\cos(z\pi)}$. Which one of the following statements is correct? **[GATE 2015]**

(a) $f(z)$ has simple poles at $z=0$ and $z=-\frac{1}{2}$

(b) $f(z)$ has a second order pole at $z=-\frac{1}{2}$

(c) $f(z)$ has infinite number of second order poles

(d) $f(z)$ has all simple poles

42. Which of the following is an analytic function of z everywhere in the complex plane? **[GATE 2016]**

(a) z^2 (b) $\left(z^*\right)^2$ (c) $|z|^2$ (d) $\sqrt{z}$

TIFR PREVIOUS YEAR QUESTIONS

43. If $z=x+iy$ then the function $f(x,y)=(1+x+y)(1+x-y)+a(x^2-y^2)-1+2iy(1-x-ax)$ where a is a real parameter, is analytic in the complex z plane if a is equal to **[TIFR 2013]**

(a) -1 (b) $+1$ (c) 0 (d) i

44. The integral $\int_0^{\infty}\frac{dx}{4+x^4}$ evalautes to **[TIFR 2014]**

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

45. The integral $\int_0^{2\pi}\frac{d\theta}{1-2a\cos\theta+a^2}$ where $0<a<1$, evaluates to **[TIFR 2015]**

(a) $\frac{2\pi}{1-a^2}$ (b) $\frac{2\pi}{1+a^2}$ (c) 2π (d) $\frac{4\pi}{1-a^2}$

46. The value of the integral $\oint_C \frac{\text{Sin } z}{z^6}dz$, where C is the circle with centre $z=0$ and radius 1 unit **[TIFR 2016]**

(a) $i\pi$ (b) $\frac{i\pi}{120}$ (c) $\frac{i\pi}{60}$ (d) $-\frac{i\pi}{6}$

JEST PREVIOUS YEAR QUESTIONS

47. The value of integral $\int_0^{\infty}\frac{\ln x}{\left(x^2+1\right)^2}dx$ is **[JEST 2012]**

(a) 0 (b) $-\frac{\pi}{4}$ (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

Compute $\lim\limits_{z\to 0}\dfrac{\mathrm{Re}\left(z^2\right)+\mathrm{Im}\left(z^2\right)}{z^2}$ **[JEST 2013]**

(a) the limit does not exist (b) 1

(c) –i (d) –1

The value of integral

$$I=\oint_c \frac{\sin z}{2z-\pi}\,dz$$

with c a is circle $|z|=2$, is **[JEST 2014]**

(a) 0 (b) $2\pi i$ [✓ (c) πi] (d) $-\pi i$

The value of limit $\lim\limits_{z\to i}\dfrac{z^{10}+1}{z^6+1}$ is equal to **[JEST 2014]**

(a) 1 (b) 0 (c) –10/3 [✓ (d) 5/3]

Given an analytic function $f(x,y)=\phi(x,y)+i\,\psi(x,y)$ where $\phi(x,y)=x^2+4x-y^2+2y$. If C is a constant, then which of the following relations is true? **[JEST 2015]**

(a) $\psi(x,y)=x^2y+4y+C$ (b) $\psi(x,y)=2xy-2x+C$

[✓ (c) $\psi(x,y)=2xy+4y-2x+C$] (d) $\psi(x,y)=x^2y-2x+C$

The value of the integral $\displaystyle\int_0^\infty \frac{\ln x}{\left(x^2+1\right)}dx$ is **[JEST 2016]**

(a) $\dfrac{\pi^2}{4}$ (b) $\dfrac{\pi^2}{2}$ (c) π^2 [✓ (d) 0]

MISCELLANEOUS QUESTIONS

Given $i=\sqrt{-1}$, then i^i is

(a) Purely real (b) Purely imaginary

(c) Of the form $x + iy$ with $x\neq 0, y\neq 0$ (d) Not defined

Let z_1 and z_2 be two non-zero complex numbers. If

$$|z_1+z_2|=|z_1|+|z_2|$$

Which of the following is true?

(a) $\mathrm{Re}\left(z_1\bar{z}_2\right)<0$, and $\mathrm{Im}\left(z_1\bar{z}_2\right)=0$ (b) $\mathrm{Re}\left(z_1\bar{z}_2\right)>0$, and $\mathrm{Im}\left(z_1\bar{z}_2\right)=0$

(c) $\mathrm{Re}\left(z_1\bar{z}_2\right)>0$, and $\mathrm{Im}\left(z_1\bar{z}_2\right)>0$ (d) $\mathrm{Re}\left(z_1\bar{z}_2\right)<0$, and $\mathrm{Im}\left(z_1\bar{z}_2\right)<0$

If 'z' is a complex number, them we can express $\sin^{-1}z$ as

(a) $-i\,\ell n\left(iz\pm\sqrt{1-z^2}\right)$ (b) $-i\,\ell n\left(z\pm\sqrt{z^2-1}\right)$ (c) $\ell n\left(z+\sqrt{z^2+1}\right)$ (d) $\ell n\left(z+\sqrt{z^2-1}\right)$

56. The value of $\oint_C \frac{dz}{(z-3)}$ if C is the circle $|z-2|=5$ is

(a) πi (b) $2\pi i$ (c) 0 (d) 2π

57. The complex function $f(z)=e^{\frac{1}{(z-1)^2}}$ has singular point at $z = 1$ which is:

(a) a pole of order 1
(b) a pole of order 2
(c) an isolated essential singularity
(d) a non-isolated essential singularity

58. The function tan (1/*z*) does not have a pole at *z* =

(a) $\pi/2$ (b) $2/\pi$ (c) $2/(3\pi)$ (d) $-2/(3\pi)$

59. The real part of a complex function *f(z)*, analytic in a region is given by $u(x, y) = x^2 - y^2$. If the function vainshes at z = 0,the imaginary part of the function is

(a) $2xy$ (b) $-2xy$ (c) $x^2 + y^2$ (d) $y^2 - x^2$

60. The residue of the function, $f(z)=\frac{z}{(2z+1)(5-z)}$ at $z=-1/2$ is:

(a) 1/11 (b) −1/11 (c) 1/22 (d) −1/22

61. Two complex numbers Z_1 and Z_2 are given to satisfy $|Z_1| = |Z_2|$ necessarily follows that:

(a) $Z_1 = Z_2$ (b) $Z_1 = e^{i\phi} Z_2$ (c) $Z_1 = Z_2$ (d) $Z_1 = Z_2^{-1}$

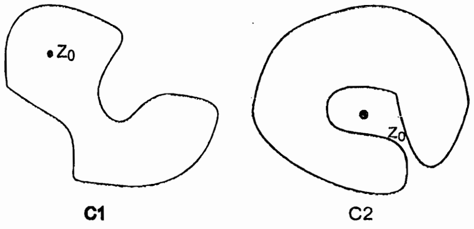
62. Two functions of a complex variable $z = x + iy$ are given as (i) $|z|$ and (ii) z^2. which of the following statements is correct?

(a) Both (i) and (ii) are analytic
(b) (i) is analytic and (ii) is nonanalytic
(c) Both are nonanalytic
(d) (i) is nonanalytic and (ii) is analytic

63. A function *f(Z)* of a complex variable *Z* is given as $f(Z) = Z\, exp\left(\frac{1}{Z}\right)$. At *Z* = 0.

(a) It has no singularity
(b) It has a simple pole and residue is 1
(c) It has an essential sigularity and residue is 1/2
(d) It has a pole of order 2 and residue is 1/6

64. A function *f(Z)* of a complex variable *Z* is analytic everywhere in the complex plane except at $Z = Z_0$. Two contours *C1* and *C2* around Z_0 are shown in the figure.



The values of the integrals $I_1 = \oint_{C1} \frac{f(Z)dZ}{Z-Z_0}, I_2 = \oint_{C2} \frac{f(Z)dZ}{Z-Z_0}$ *are*

(a) $I_1 = 0, I_2 = 2\pi i f(Z_0)$
(b) $I_1 = 2\pi i f(Z_0), I_2 = 0$
(c) $I_1 = \pi i f(Z_0), I_2 = \pi i f(Z_0)$
(d) $I_1 = 2\pi i f(Z_0), I_2 = 2\pi i f(Z_0)$

65. The value of the integral $\oint_C \frac{z^3}{(z-1)^2}dz$, where C is the circle $|z|=4$ is

(a) $6\pi i$ (b) $-6\pi i$ (c) $2\pi i$ (d) Zero

66. The leading term in Laurent expansion of $f(z)=\frac{1}{z(1-z)^2}$ around $z=0$ is

(a) $\frac{1}{z^3}$ (b) $\frac{1}{z^2}$ (c) $\frac{1}{z}$ (d) 1

67. The value of $\oint_c \frac{e^z(z+3)}{z-2}dz$ where c is a unit circle with centre at origin is:

(a) $5e^2$ (b) $\frac{5}{2\pi i}e^2$ (c) 0 (d) $10\pi ie^2$

68. If $f(z)=\frac{Z^2+a^2-2aZ}{(Z-a)^3(Z-b)}$, then $Z=a$ is:

(a) Pole of order 1 (b) Pole of order 2 (c) Pole of order 3 (d) An essential singularity.

69. Which one of the following functions is not analytic?

(a) $f(z)=|z|$ (b) $f(z)=z^2$ (c) $f(z)=\cos z$ (d) $f(z)=\sin z$

70. The value of the integral $\oint_c \frac{\cos \pi z}{z(z+1)}$ where C is the circle $|z-1|=3$ is:

(a) 0 (b) πi (c) $2\pi i$ (d) $4\pi i$

ANSWER KEY

1. (c)	**2. (b)**	**3. (b)**	**4. (b)**	**5. (b)**	**6. (a)**	**7. (b)**
8. (a)	**9. (d)**	**10. (a)**	**11. (b)**	**12. (c)**	**13. (a)**	**14. (c)**
15. (b)	**16.(a)**	**17. (b)**	**18. (c)**	**19. (c)**	**20. (a)**	**21. (b)**
22. (b)	**23. (a)**	**24. (d)**	**25. (b)**	**26. (c)**	**27. (b)**	**28. (a)**
29. (b)	**30. (a)**	**31. (b)**	**32. (c)**	**33. (a)**	**34. (c)**	**35. (d)**
36. (b)	**37. (d)**	**38. (3)**	**39. (c)**	**40. (c)**	**41. (b)**	**42. (a)**
43. (a)	**44. (d)**	**45. (a)**	**46. (c)**	**47. (b)**	**48. (a)**	**49. (c)**
50. (d)	**51. (c)**	**52. (d)**	**53. (a)**	**54. (b)**	**55. (a)**	**56. (b)**
57. (c)	**58. (a)**	**59. (a)**	**60. (d)**	**61. (b)**	**62. (d)**	**63. (c)**
64. (b)	**65. (a)**	**66. (c)**	**67. (c)**	**68. (a)**	**69. (a)**	**70. (d)**

CAREER ENDEAVOUR

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CSIR-UGC-NET/JRF DEC 2016

MATHEMATICAL PHYSICS

SECTION: DIFFERENTIAL EQUATIONS & SPECIAL FUNCTIONS

CSIR PREVIOUS YEAR QUESTIONS

1. Let $p_n(x)$ (where n = 0,1,2, ...) be ploynomial of order 'n' wirh real co-efficients defined in the interval $2 \le n \le 4$. If $\int_2^4 p_m(x)\,p_n(x)\,dx = \delta_{mn}$, then **[CSIR June -2011]**

(a) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{\frac{3}{2}}(-3-x)$ (b) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{3}(3+x)$

(c) $p_0(x) = \frac{1}{2}$ and $p_1(x) = \sqrt{\frac{3}{2}}(3-x)$ (d) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{\frac{3}{2}}(3-x)$

2. The genarating function $F(x,t) = \sum_{n=0}^{\infty} P_n(x)\,t^n$ for Legendre polynomials $P_n(x)$ is

$F(x,t) = \left(1-2xt+t^2\right)^{-1/2}$. The value of $P_3(-1)$ is **[CSIR Dec -2011]**

(a) 5/2 (b) 3/2 (c) 1 (d) -1

3. Let $x_1(t)$ and $x_2(t)$ be two linearly independent solutions of the differential equation

$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + f(t)x = 0$. Let, $w(t) = x_1(t)\frac{dx_2(t)}{dt} - x_2(t)\frac{dx_1(t)}{dt}$. If $w(0) = 1$, then $w(1)$ is given by

[CSIR Dec -2011]

(a) 1 (b) e^2 (c) $1/e$ (d) $1/e^2$

4. Let y(x) be a continuous real function in the range of 0 to 2π, satisfying the inhomogeneous differential equation: $\sin x\frac{d^2y}{dx^2} + \cos x\frac{dy}{dx} = \delta\left(x - \frac{\pi}{2}\right)$. The value of $\frac{dy}{dx}$ at the point $x = \frac{\pi}{2}$

[CSIR June -2012]

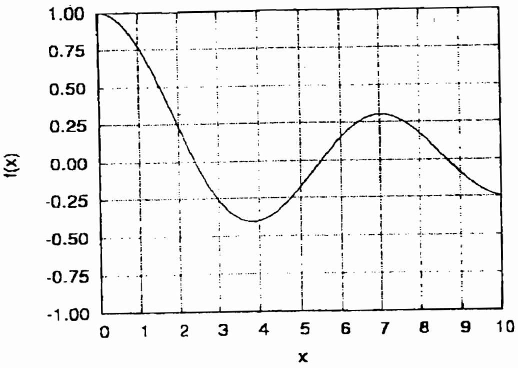
(a) is continuous (b) has a discontinuity of 3

(c) has a discontinuity of 1/3 (d) has a discontinuity of 1

5. A function $f(x)$ obeys the differential equation $\frac{d^2f}{dx^2} - (3-4i)f = 0$ and satisfies the condition $f(0) = 1$ and $f(x) \to 0$ as $x \to \infty$. The value of $f(\pi)$ is **[CSIR Dec -2012]**

(a) $e^{2\pi}$ (b) $e^{-2\pi}$ (c) $-e^{-2\pi}$ (d) $-e^{2\pi}$

6. The graph of the function f(x) shown below is best described by **[CSIR Dec -2012]**



(a) The bessel function $J_0(x)$ (b) $\cos x$

(c) $e^{-x}\cos x$ (d) $\dfrac{\cos x}{x}$

7. Given that $\sum_{n=0}^{\infty} H_n(x)\dfrac{t^n}{n!} = e^{-t^2+2tx}$, the value of $H_4(0)$ is **[CSIR June -2013]**

(a) 12 (b) 6 (c) 24 (d) -6

8. The solution of the differential equation $\dfrac{dx}{dt} = x^2$ with the initial condition $x(0)=1$ will blow up as t tend to **[CSIR June -2013]**

(a) 1 (b) 2 (c) 1/2 (d) ∞

9. Consider the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$$ **[CSIR June -2014]**

with initial conditions $x(0)=0$ and $\dot{x}(0)=1$. The solution $x(t)$ attains its maximum value when 't' is

(a) 1/2 (b) 1 (c) 2 (d) ∞

10. GIven, $\sum_{n=0}^{\infty} P_n(x)t^n = \left(1-2xt+t^2\right)^{-1/2}$ for $|t|<1$, the value of $P_5(-1)$ is **[CSIR June -2014]**

(a) 0.26 (b) 1 (c) 0.5 (d) -1

11. The function $f(x) = \sum_{n=0}^{\infty} \dfrac{(-1)^n}{n!(n+1)!}\left(\dfrac{x}{2}\right)^{2n+1}$ satisfies the differential equation **[CSIR Dec -2014]**

(a) $x^2\dfrac{d^2f}{dx^2} + x\dfrac{df}{dx} + \left(x^2+1\right)f = 0$ (b) $x^2\dfrac{d^2f}{dx^2} + 2x\dfrac{df}{dx} + \left(x^2-1\right)f = 0$

(c) $x^2\dfrac{d^2f}{dx^2} + x\dfrac{df}{dx} + \left(x^2-1\right)f = 0$ (d) $x^2\dfrac{d^2f}{dx^2} - x\dfrac{df}{dx} + \left(x^2-1\right)f = 0$

12. Consider the differential equation $\dfrac{d^2x}{dt^2} - 3\dfrac{dx}{dt} + 2x = 0$. If x = 0 at t = 0 and x = 1 at t = 1, the value of x at t =2 , will be **[CSIR June -2015]**

(a) e^2+1 (b) e^2+e (c) $e+2$ (d) $2e$

13. The solution of the differential equation $\frac{dx}{dt} = 2\sqrt{1-x^2}$, with initial condition $x = 0$ at $t = 0$ is

[CSIR Dec -2015]

(a) $x = \begin{cases} \sin 2t, & 0 \le t \le \frac{\pi}{4} \\ \sinh 2t, & t \ge \frac{\pi}{4} \end{cases}$

(b) $x = \begin{cases} \sin 2t, & 0 \le t < \frac{\pi}{2} \\ 1, & t \ge \frac{\pi}{2} \end{cases}$

(c) $x = \begin{cases} \sin 2t, & 0 \le t < \frac{\pi}{4} \\ 1, & t \ge \frac{\pi}{4} \end{cases}$

(d) $x = 1 - \cos 2t, \quad t \ge 0$

14. The Hermite polynomial $H_n(x)$ satisfies the differential equation

$$\frac{d^2H_n}{dx^2} - 2x\frac{dH_n}{dx} + 2nH_n(x) = 0$$

[CSIR Dec -2015]

The corresponding generating function $G(t,x) = \sum_{n=0}^{\infty} \frac{1}{n!} H_n(x) t^n$ satisfies the equation

(a) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2t\frac{\partial G}{\partial t} = 0$

(b) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} - 2t^2\frac{\partial G}{\partial t} = 0$

(c) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2\frac{\partial G}{\partial t} = 0$

(d) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2\frac{\partial^2 G}{\partial x \partial t} = 0$

GATE PREVIOUS YEAR QUESTIONS

15. Consider the linear differential equation $\frac{dy}{dx} = xy$. If $y = 2$ at $x = 0$, then the value of y at $x = 2$ is

[GATE 2016]

(a) e^{-2} (b) $2e^{-2}$ (c) e^2 (d) $2e^2$

16. A function $y(z)$ satisfies the ordinary differential equation $y'' + \frac{1}{z}y' - \frac{m^2}{z^2}y = 0$, where $m = 0, 1, 2, 3.......$

Consider the four statements P, Q, R, S as given below: **[GATE 2015]**

P: z^m and z^{-m} are linearly independent solutions for all values of m.

Q: z^m and z^{-m} are linearly independent solutions for all values of $m > 0$.

R: $\ln z$ and 1 are linearly independent solutions for $m = 0$.

S: z^m and $\ln z$ are linearly independent solutions for all values of m.

The correct option for the combination of valid statements is

(a) P, R and S only (b) P and R only (c) Q and R only (d) R and S only

17. The solution of the differential equation $\frac{d^2y}{dt^2} - y = 0$

subject to the boundary conditions $y(0) = 1$ and $y(\infty) = 0$, is **[GATE 2014]**

(a) $\cos t + \sin t$ (b) $\cosh t + \sinh t$ (c) $\cos t - \sin t$ (d) $\cosh t - \sinh t$

18. The solutions to the differential equation $\frac{dy}{dx} = -\frac{x}{y+1}$ are a family of **[GATE 2011]**

(a) Circles with different radii (b) Circles with different centres.

(c) Straight lines with different slopes (d) Straight lines with different intercepts on the y-axis

19. The solution of the differential equation for $y(t)$: $\frac{d^2y}{dt^2} - y = 2\cosh(t)$, subject to the initial conditions

$y(0) = 0$ and $\left.\frac{dy}{dt}\right|_{t=0} = 0$ is **[GATE 2010]**

(a) $\frac{1}{2}\cosh(t) + t\sinh(t)$ (b) $-\sinh(t) + t\cosh(t)$

(c) $t\cosh(t)$ (d) $t\sinh(t)$

20. Given the recurrence relation for the Legendre polynomials:

$$(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$$

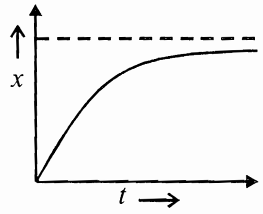
Which of the following integrals has a non-zero value? **[GATE 2010]**

(a) $\int_{-1}^{1} x^2 P_n(x) P_{n+1}(x)\,dx$ (b) $\int_{-1}^{1} x P_n(x) P_{n+2}(x)\,dx$

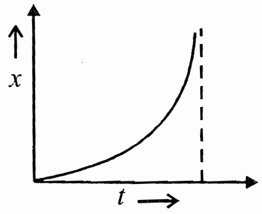
(c) $\int_{-1}^{1} x[P_n(x)]^2\,dx$ (d) $\int_{-1}^{1} x^2 P_n(x) P_{n+2}(x)\,dx$

21. Which one of the following curves gives the solution of the differential equation $k_1\frac{dx}{dt} + k_2 x = k_3$, where k_1, k_2 and k_3 are positive real constants with initial conditions $x = 0$ at $t = 0$? **[GATE 2009]**

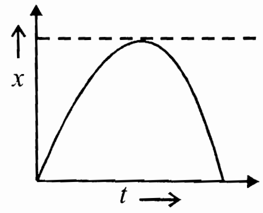
(a)



(b)



(c)



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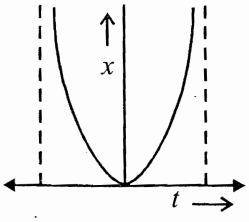


22. Consider the bessel equation:

$$\frac{d^2y}{dz^2}+\frac{1}{z}\frac{dy}{dz}+y=0\ \ (v=0)$$

Which of the following statements is correct? **[GATE 2008]**

(a) Equation has regular singular points at $z=0$ and $z=\infty$.

(b) Equation has 2 linearly independent solutions that are entire.

(c) Equation has an entire solution and a second linearly independent solution is singular at $z = 0$.

(d) Limit $z\to\infty$, taken along X- axis, exists for both the linearly independent solutions.

23. The points, where the series solution of the Legendre differential equation $\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+\frac{3}{2}\left(\frac{3}{2}+1\right)y=0$ will diverge, are located at: **[GATE 2007]**

(a) 0 and 1 (b) 0 and –1 (c) –1 and 1 (d) $\frac{3}{2}$ and $\frac{5}{2}$

24. Solution of the differential equation $x\frac{dy}{dx}+y=x^4$, with the boundary condition that $y=1$ at $x=1$, is **[GATE 2007]**

(a) $y=5x^4-4$ (b) $y=\frac{x^4}{5}+\frac{4x}{5}$ (c) $y=\frac{4x^4}{5}+\frac{1}{5x}$ (d) $y=\frac{x^4}{5}+\frac{4}{5x}$

Statement for Linked Answer Q. 25 and Q. 26:

For the differential equaiton $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$ **[GATE 2005]**

25. One of the solutions is:

(a) e^x (b) $In\ x$ (c) e^{-x^2} (d) e^{x^2}

26. The second linearly independent solution is:

(a) e^{-x} (b) xe^x (c) x^2e^x (d) x^2e^{-x}

27. Consider the differential equation $d^2x/dt^2 + 2\,dx/dt + x = 0$. At time $t = 0$, it is given that $x = 1$ and $dx/dt = 0$. At $t = 1$, the value of x is given by: **[GATE 2003]**

(a) $1/e$ (b) $2/e$ (c) 1 (d) $3/e$

28. The solution of the differential equation,

$$\left(1+x\right)\frac{d^2y(x)}{dx^2}+x\frac{dy(x)}{dx}-y(x)=0 \text{ is:}$$ **[GATE 2002]**

(a) $Ax^2 + B$ (b) $Ax + Be^{-x}$ (c) $Ax + Be^x$ (d) $Ax + Bx^2$

where A and B are constants.

29. The solution of the system of differential equations,

$$\frac{dy}{dx}=y-z\ and\ \frac{dz}{dx}=-4y+z$$

is given by (for A and B are arbitrary constants) **[GATE 2001]**

(a) $y(x) = Ae^{3x} + Be^{-x};\ z(x) = -2Ae^{3x} + 2Be^{-x}$ (b) $y(x) = Ae^{3x} + Be^{-x};\ z(x) = 2Ae^{3x} + 2B^{-x}$

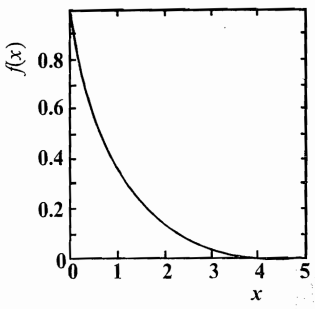
(c) $y(x) = Ae^{3x} + Be^{-x};\ z(x) = 2Ae^{3x} - 2Be^{-x}$ (d) $y(x) = Ae^{3x} + Be^{-x};\ z(x) = -2Ae^{3x} - 2Be^{-x}$

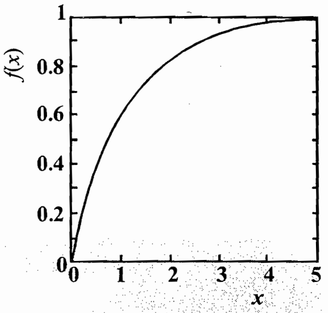
TIFR PREVIOUS YEAR QUESTIONS

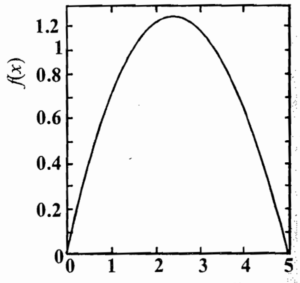
30. The differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$ has the complete solution, in terms of arbitrary constants A and B as **[TIFR 2013]**

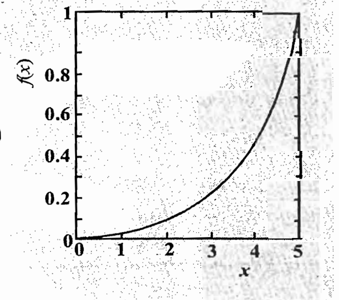
(a) $A\exp x+Bx\exp x$
(b) $A\exp x+B\exp(-x)$
(c) $A\exp x+Bx\exp(-x)$
(d) $x\{A\exp x+B\exp(-x)\}$

31. The solution of the integral equation $f(x)=x-\int_0^x dt\, f(t)$ has the graphical form **[TIFR 2014]**

(a)


(b)


(c)


(d)


2. Consider the differential equation

$$\frac{d^2y}{dx^2}=-4\left(y+\frac{dy}{dx}\right)$$

with the boundary conditions that $y(x)=0$ at $x=\frac{1}{5}$. When plotted as a function of x , for $x\geq 0$, we can say with certainity that the value of y **[TIFR 2015]**

(a) first increases, then decreases to zero.
(b) first decreases, then increases to zero.
(c) has an extremum in the range $0 < x < 1$.
(d) oscillates from positive to neagtive with amplitude decreasing to zero.

. The generating function for a set of polynomials in x is given by

$$f(x,t)=\left(1-2xt+t^2\right)^{-1}$$

The third polynomial (order x^2) in this set is **[TIFR 2015]**

(a) $2x^2+1$
(b) $4x^2+1$
(c) $2x^2-x$
(d) $4x^2-1$

. The function $y(x)$ satisfies the differential equation $x\frac{dy}{dx}=y(\ln y-\ln x+1)$ with the initial condition $y(1)=3$. What will be the value of $y(3)$? **[TIFR 2015]**

JEST PREVIOUS YEAR QUESTIONS

35. Consider the differential equation

$$\frac{dG(x)}{dx}+kG(x)=\delta(x)$$

where k is a constant. Which of the following statement is true? **[JEST 2013, 2015]**

(a) Both G(x) and G'(x) are continuous at $x = 0$

(b) G(x) is continuous at $x = 0$ but G'(x) is not.

(c) G(x) is discontinuous at $x = 0$

(d) The continuity properties of G(x) and G'(x) at $x = 0$ depend on the value of k.

36. What are the solutions to $f''(x)-2f'(x)+f(x)=0$? **[JEST 2014]**

(a) c_1e^x/x (b) c_1x+c_2/x (c) $c_1xe^x+c_2$ (d) $c_1e^x+c_2xe^x$

MISCELLANEOUS QUESTIONS

37. Given the equations $H'=-aG$ and $G'=bH, G$ can have oscillatory solutions

(a) $\forall\ a,b$ (b) for no choice of a, b

(c) If $a < 0$ and $b > 0$ (d) $a < 0$ and $b < 0$

38. Consider a forced harmonic oscillator which obeys the differential equation,

$$\frac{d^2y}{dt^2}+y=\sin t.$$

Which one of the following is the solution of the differential equation with initial condition y (0) = 0?

(a) $y(t)=6\sin t$ (b) $y(t)=12\sin t+\frac{t}{2}\cos t$

(c) $y(t)=12\sin t-\frac{t}{2}\cos t$ (d) $y(t)=6\cos t-\frac{t}{2}\cos t$

39. The solution of the differential equation

$$\frac{dy}{dx}+\frac{2}{3}\frac{y}{x}=0$$

with the condition y = 2 when x = 3 is given by

(a) $x^2y^3=72$ (b) $x^2y^3=108$ (c) $x^3y^2=108$ (d) $x^3y^2=72$

40. From the solution of the differential equation: $\frac{dy}{dx}=\frac{1}{1+x^2}$

what can you regarding the following series ?

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}+....$$

(a) The series is divergent (b) The series is absolutely convergent

(c) The series converges to $\frac{\pi}{4}$ (d) The series converges to 0

41. Consider the system of differential equations

$$\frac{dx}{dt}=-y, \qquad \frac{dy}{dt}=x$$

Plotting the various solutions in the x, y-plane one obtains

(a) Hyperbola (b) Parabola (c) Circles (d) Straight lines

42. If $f(x)=x^4-2=\sum_{n=0}^{\infty}a_nP_n(x)$, then a_3 is

(a) $\frac{1}{2}$ (b) 1 (c) -1 (d) 0

43. The value of $\frac{J_{1/2}(x)}{J_{-1/2}(x)}$ is:

(a) 0 (b) *tan x* (c) *cot x* (d) *tan h x.*

44. Legendre's differential equation:

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$$

has an ordinary point at $x=$

(a) -1 (b) 0 (c) 1 (d) ∞

45. For the differential equation: $z^2\frac{d^2y}{dz^2}+z\frac{dy}{dz}+y=0$

(a) $z=\infty$ is an ordinary point
(b) $z=0$ is a regular singular and $z=\infty$ is an irregular singular point
(c) $z=0$ and $z=\infty$ both are regular singular points
(d) $z=0$ is an irregular singular point

46. The Legendre polynomial $P_n(x)$, where $-1\le x\le +1$

(a) Is singular at $x=\pm 1$ (b) Satisfies $\int_{-1}^{1}dxP_n^2(x)=1$

(c) Satisfies $\int_{-1}^{1}dxP_n(x)=0$ for $n\ge 1$ (d) Is always an even function of x.

47. In order to obtain the solution of the initial value problem of the equation of motion, $\frac{d^2x}{dt^2}+5\frac{dx}{dt}+16x=0$.

How many initial conditions are required?

(a) 0 (b) 1 (c) 2 (c) 3

48. For the Bessel equation, $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2-n^2\right)y=0$, where n is an integer, the maximum number of linearly independent solutions, well-defined at $x=0$ is

(a) zero (b) one (c) two (d) three

49. The general solution of the linear differential equation, $\frac{d^4u}{dx^4}=0$ is, u(x) equal to

(a) 0 (b) c_3t+c_4 (c) $c_2t^2+c_3t+c_4$ (d) $c_1t^3+c_2t^2+c_3t+c_4$

50. The regular singular points of the associated Legendre differential equation in the finite domain are

(a) $1,-1$ (b) 0 (c) ∞ (d) 2

51. The Wronskian of the linearly independent solutions of the differential equation $\frac{d^2y}{dx^2}+4y=0$ is:

(a) Constant (b) 0 (c) ∞ (d) Undetermined

52. Given the Legendre polynomials $P_0(x)=1$, $P_1(x)=x$, and $P_2(x)\frac{3x^2-1}{2}$, then the polynomial $(3x^2+x-1)$ can be expressed as

(a) $P_2(x)-P_1(x)$ (b) $2P_2(x)+P_1(x)$

(c) $P_2(x)+P_1(x)$ (d) $2P_2(x)+P_1(x)+P_0(x)$

53. The solution of the differential equation $x\frac{dy}{dx}+y=1$, given the condition $y=0$ at $x=1$ is

(a) $y=(1-x^2)\ell n\, x$ (b) $y=\frac{x^2-1}{x}e^{-x}$ (c) $y=\frac{x-1}{x}$ (d) $y=\sinh(x)$

ANSWER KEY

1. (d) 2. (d) 3. (d) 4. (d) 5. (c) 6. (a) 7. (a)
8. (a) 9. (b) 10. (d) 11. (c) 12. (b) 13. (c) 14. (a)
15.(d) 16. (c) 17. (d) 18. (a) 19. (d) 20. (d) 21. (a)
22. (c) 23. (c) 24. (d) 25. (a) 26. (b) 27. (b) 28. (b)
29. (a) 30. (a) 31. (b) 32. (c) 33. (d) 34. (81) 35. (c)
36. (d) 37. (c) 38. (c) 39. (a) 40. (c) 41. (c) 42. (d)
43. (b) 44. (b) 45. (a) 46. (c) 47. (c) 48. (b) 49. (d)
50. (a) 51. (a) 52. (b) 53. (c)

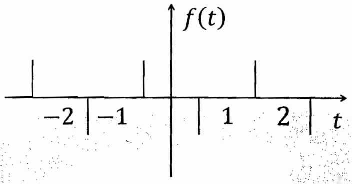
CSIR-UGC-NET/JRF DEC 2016

MATHEMATICAL PHYSICS

SECTION: FOURIER SERIES, FOURIER TRANSFORM, LAPLACE TRANSFORM

CSIR PREVIOUS YEAR QUESTIONS

1. Consider the periodic function $f(t)$ with time period T as shown in the fugure below:



[CSIR JUNE 2015]

The spikes are located at $t=\frac{1}{2}(2n-1)$, where $n=0,\pm1,\pm2....$, are Dirac delta functions of strength ±1 . The amplitudes a_n in the fourier expansion $f(t)=\sum_{n=-\infty}^{\infty} a_n e^{2\pi in/T}$ are given by

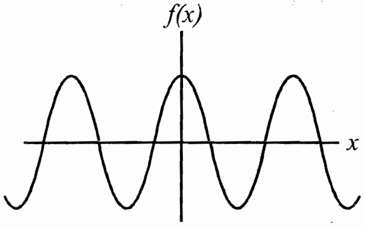
(a) $(-1)^n$ (b) $\frac{1}{n\pi}\sin\frac{n\pi}{2}$ (c) $i\sin\frac{n\pi}{2}$ (d) $n\pi$

2. Fourier Transform of the derivative of the dirac δ -function, namely $\delta'(x)$ is proportional to

[CSIR Dec 2013]

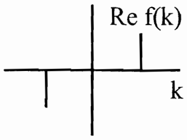
(a) 0 (b) 1 (c) $\sin k$ (d) ik

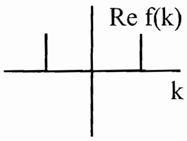
3. The graph of a real periodic function $f(x)$ for the range $[-\infty,\infty]$ is shown below



Which of the following graphs represents the real part of its Fourier transform?

[CSIR June 2014]

(a)


(b)


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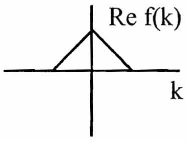
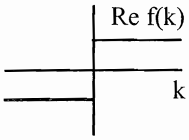


(d)


4. The Fourier transform of $f(x)$ is $\tilde{f}(k)=\int\limits_{-\infty}^{\infty} dx\, e^{ikx} f(x)$. If $f(x)=\alpha\,\delta(x)+\beta\,\delta'(x)+\gamma\,\delta''(x)$,

where $\delta(x)$ is the Dirac Delta function (and prime denotes derivative), what is $\tilde{f}(k)$?

[CSIR Dec 2015]

(a) $\alpha+i\beta k+i\gamma k^2$ (b) $\alpha+\beta k-\gamma k^2$ (c) $\alpha-i\beta k-\gamma k^2$ (d) $i\alpha+\beta k-i\gamma k^2$

5. A solution $y(x)$ satisfies the following differential equation:

$$\frac{d^2y}{dx^2}-\omega^2 y=-\delta(x-a)$$

where ω is positive. The Fourier transform $\tilde{f}(k)=\int\limits_{-\infty}^{\infty} dx\, e^{ikx}\, f(x)$ of f and the solution of the equation are, respectively

[CSIR Dec 2015]

(a) $\dfrac{e^{ika}}{k^2+\omega^2}$ and $\dfrac{1}{2\omega}\left[e^{-\omega|x-a|}+e^{\omega|x-a|}\right]$ (b) $\dfrac{e^{ika}}{k^2+\omega^2}$ and $\dfrac{1}{2\omega}e^{-\omega|x-a|}$

(c) $\dfrac{e^{ika}}{k^2-\omega^2}$ and $\dfrac{1}{2\omega}\left[e^{-i\omega|x-a|}+e^{i\omega|x-a|}\right]$ (d) $\dfrac{e^{ika}}{k^2-\omega^2}$ and $\dfrac{1}{2i\omega}\left[e^{-i\omega|x-a|}-e^{i\omega|x-a|}\right]$

6. What is the Fourier Transform $\int dx\, e^{ikx} f(x)$ of

$$f(x)=\delta(x)+\sum_{n=1}^{\infty}\frac{d^n}{dx^n}\delta(x)$$

where $\delta(x)$ is the Dirac delta function?

[CSIR June 2016]

(a) $\dfrac{1}{1-ik}$ (b) $\dfrac{1}{1+ik}$ (c) $\dfrac{1}{k+i}$ (d) $\dfrac{1}{k-i}$

7. The graph of the function $f(x)=\begin{Bmatrix}1 & \text{for } 2n\le x\le 2n+1\\ 0 & \text{for } 2n+1\le x\le 2n+2\end{Bmatrix}$ (where n = 0,1,2........). Its laplae transform $\tilde{f}(s)$ is

[CSIR Dec 2011]

(a) $\dfrac{1+e^{-s}}{s}$ (b) $\dfrac{1-e^{-s}}{s}$ (c) $\dfrac{1}{s\left(1+e^{-s}\right)}$ (d) $\dfrac{1}{s\left(1-e^{-s}\right)}$

8. The inverse Laplace tranform of $\dfrac{1}{s^2(s+1)}$ is

[CSIR June 2013]

(a) $\frac{1}{2}t^2e^{-t}$ (b) $\frac{1}{2}t^2+1-e^{-t}$ (c) $t-1+e^{-t}$ (d) $\frac{1}{2}t^2\left(1-e^{-t}\right)$

9. The Laplace transform of $6t^3+3\sin 4t$ is

[CSIR June 2015]

(a) $\dfrac{36}{s^4}+\dfrac{12}{s^2+16}$ (b) $\dfrac{36}{s^4}+\dfrac{12}{s^2-16}$ (c) $\dfrac{18}{s^4}+\dfrac{12}{s^2-16}$ (d) $\dfrac{36}{s^3}+\dfrac{12}{s^2+16}$

GATE PREVIOUS YEAR QUESTIONS

10. A periodic function $f(x) = x$ for $-\pi < x < \pi$ has the Fourier series representation

$$f(x) = \sum_{n=1}^{\infty}\left(-\frac{2}{n}\right)(-1)^n \sin nx$$

Using this, one finds the sum $\sum_{n=1}^{\infty}\frac{1}{n^2}$ to be **[GATE 2004]**

(a) $2\ln 2$ (b) $\frac{\pi^2}{3}$ (c) $\frac{\pi^2}{6}$ (d) $\pi \ln 2$

11. $f(x)$ symmetric periodic function of x i.e $f(x) = f(-x)$. Then, in general, the Fourier series of the function $f(x)$ will be of the form **[GATE 2013]**

(a) $f(x) = \sum_{n=1}^{\infty}(a_n \cos nkx + b_n \sin nkx)$ (b) $f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nkx)$

(c) $f(x) = \sum_{n=1}^{\infty}(b_n \sin nkx)$ (d) $f(x) = a_0 + \sum_{n=1}^{\infty}(b_n \sin nkx)$

12. A periodic function $f(x)$ of period 2π is defined in the interval $(-\pi < x < \pi)$ as:

$$f(x) = \begin{cases} -1 & \text{for} \quad -\pi < x < 0 \\ 1 & \text{for} \quad 0 < x < \pi \end{cases}$$

[GATE 2016]

The approximate Fourier series expansion for $f(x)$ is

(a) $f(x) = \frac{4}{\pi}\left[\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + ...\right]$

(b) $f(x) = \frac{4}{\pi}\left[\sin x - \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x - ....\right]$

(c) $f(x) = \frac{4}{\pi}\left[\cos x + \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x + ...\right]$

(d) $f(x) = \frac{4}{\pi}\left[\cos x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x - ....\right]$

13. The Heaviside function is defined as

$$H(t) = \begin{cases} +1 & for \quad t > 0 \\ -1 & for \quad t < 0 \end{cases}$$

and its fourier transform is given by $-2i/\omega$. The fourier transform of $\frac{1}{2}\left[H\left(t+\frac{1}{2}\right) - H\left(t-\frac{1}{2}\right)\right]$ is

[GATE 2015]

(a) $\frac{\sin\frac{\omega}{2}}{\frac{\omega}{2}}$ (b) $\frac{\cos\frac{\omega}{2}}{\frac{\omega}{2}}$ (c) $\sin\frac{\omega}{2}$ (d) 0

14. If the fourier transform $F\left[\delta(x-a)\right]=\exp(-i2\pi va)$, then $F^{-1}(\cos 2\pi va)$ will correspond to **[GATE 2008]**

(a) $\delta(x-a)-\delta(x+a)$ (b) a constant

(c) $\frac{1}{2}\left[\delta(x-a)+i\delta(x+a)\right]$ (d) $\frac{1}{2}\left[\delta(x-a)+\delta(x+a)\right]$

15. The k^{th} Fourier component of $f(x)=\delta(x)$ is **[GATE 2006]**

(a) 1 (b) zero (c) $(2\pi)^{-1/2}$ (d) $(2\pi)^{-3/2}$

16. The Fourier transform *F(k)* of a function *f(x)* is defined as:

$$F(k)=\int_{-\infty}^{\infty}dx\, f(x)\,\exp(ikx)$$

Then *F(k)* for $f(x)=exp(-x^2)$ is: [Given: $\int_{-\infty}^{\infty}\exp(-x^2)dx=\sqrt{\pi}$] **[GATE 2004]**

(a) $\pi\exp(-k)$ (b) $\sqrt{\pi}\,\exp\left(\frac{-k^2}{4}\right)$ (c) $\frac{\sqrt{\pi}}{2}\exp\left(\frac{-k^2}{2}\right)$ (d) $\sqrt{2\pi}\,\exp(-k^2)$

17. The Fourier transform of the function *f(x)* is $F(k)=\int e^{ikx}f(x)\,dx$. The Fourier transform of *df(x)/dx* is: **[GATE 2003]**

(a) $dF(k)/dk$ (b) $\int F(k)/dk$ (c) $-ikF(k)$ (d) $ikF(k)$

18. Fourier transform of which of the following functions does not exist? **[GATE 2002]**

(a) $e^{-|x|}$ (b) xe^{-x^2} (c) e^{x^2} (d) e^{-x^2}

19. Which of the following pairs of the given function $F(t)$ and its Laplace transform $f(s)$ is not correct? **[GATE 2013]**

(a) $F(t)=\delta(t),\ f(s)=1$ (singularity at 0) (b) $F(t)=1,\ f(s)=\frac{1}{s}(s>0)$

(c) $F(t)=\sin kt,\ f(s)=\frac{s}{s^2+k^2}(s>0)$ (d) $F(t)=te^{kt},\ f(s)=\frac{1}{(s-k)^2}(s>k,\ s>0)$

20. If $f(x)=\begin{cases}0 & for\ x<3\\ x-3 & for\ x\geq 3\end{cases}$ then, the laplace transform of $f(x)$ is **[GATE 2010]**

(a) $s^{-2}e^{3s}$ (b) s^2e^{-3s} (c) s^{-2} (d) $s^{-2}e^{-3s}$

21. If $\bar{f}(s)$ is the Laplace transform of *f(t)* the Laplace transform of *f(at)*, where *a* is a constnat, is **[GATE 2005]**

(a) $\frac{1}{a}\bar{f}(s)$ (b) $\frac{1}{a}\bar{f}(s/a)$ (c) $\bar{f}(s)$ (d) $\bar{f}(s/a)$

22. The Laplace transform of $f(t)=\sin \pi t$ is $F(s)=\frac{\pi}{(s^2+\pi^2)}$, $s>0$. Therefore, the Laplace transform of

$t\sin \pi t$ is: **[GATE 2004]**

(a) $\frac{\pi}{s^2\left(s^2+\pi^2\right)}$ (b) $\frac{2\pi}{s^2\left(s^2+\pi^2\right)^2}$ (c) $\frac{2\pi s}{\left(s^2+\pi^2\right)^2}$ (d) $\frac{2\pi}{\left(s^2+\pi^2\right)^2}$

TIFR PREVIOUS YEAR QUESTIONS

23. A function $f(x)$ is defined in the range $-1 \le x \le 1$ by

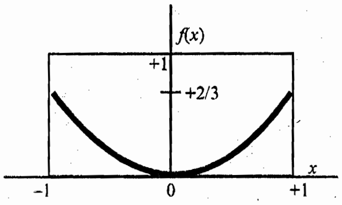
$$f(x)=\begin{cases}1-x & for\ \ x\ge 0\\ 1+x & for\ \ x<0\end{cases}$$

The first few terms in the Fourier series approximating this function are **[TIFR 2010]**

(a) $\frac{1}{2}+\frac{4}{\pi^2}\cos\pi x+\frac{4}{9\pi^2}\cos 3\pi x+\ldots$ (b) $\frac{1}{2}+\frac{4}{\pi^2}\sin\pi x+\frac{4}{9\pi^2}\sin 3\pi x+\ldots$

(c) $\frac{4}{\pi^2}\cos\pi x+\frac{4}{9\pi^2}\cos 3\pi x+\ldots$ (d) $\frac{1}{2}-\frac{4}{\pi^2}\cos\pi x+\frac{4}{9\pi^2}\cos 3\pi x-\ldots$

24. A student is asked to find a series approximation for the function $f(x)$ in the domain $-1\le x\le +1$, as indicated by the thick line in the figure below.



The student represents the function by a sum of three terms

$$f(x)\approx a_0+a_1\cos\frac{\pi x}{2}+a_2\sin\frac{\pi x}{2}$$ **[TIFR 2013]**

Which of the following would be the best choices for the coefficients a_0, a_1 and a_2?

(a) $a_0=1, a_1=-\frac{1}{3}, a_2=0$ (b) $a_0=\frac{2}{3}, a_1=-\frac{2}{3}, a_2=0$

(c) $a_0=\frac{2}{3}, a_1=0, a_2=-\frac{2}{3}$ (d) $a_0=-\frac{1}{3}, a_1=0, a_2=-1$

JEST PREVIOUS YEAR QUESTIONS

5. The output intensity I of radiation from a single mode of resonant cavity obeys

$$\frac{d}{dt}I=-\frac{\omega_0}{Q}I$$

where Q is the quality factor of the cavity and ω_0 is the resonant frequency. The form of the frequency spectrum of the output is **[JEST 2016]**

(a) Delta function (b) Gaussian (c) Lorentzian (d) Exponential

ANSWER KEY

1. (c)
2. (d)
3. (b)
4. (c)
5. (b)
6. (a)
7. (c)
8. (c)
9. (a)
10. (c)
11. (b)
12. (a)
13. (a)
14. (d)
15. (c)
16. (b)
17. (c)
18. (c)
19. (c)
20. (d)
21. (b)
22. (c)
23. (a)
24. (b)
25. (c)

CSIR-UGC-NET/JRF DEC 2016

MATHEMATICAL PHYSICS

SECTION: PROBABILITY

CSIR PREVIOUS YEAR QUESTIONS

1. An unbiased dice is thrown three times successively. The probability that the number of dots on the uppermost surface add up to 16 is **[CSIR Dec 2011]**

   (a) 1/16 (b) 1/36 (c) 1/108 (d) 1/216

2. A ball is picked at random from one of the two boxes that contain 2 black and 3 white and 3 black and 4 white balls respectively. What is the probability that it is white? **[CSIR June 2012]**

   (a) 34/70 (b) 41/70 (c) 36/70 (d) 29/70

3. In a series of five cricket matches, one of the captains calls 'heads' every time when the toss is taken. The probability that he will win 3 times and lose 2 times is **[CSIR Dec 2012]**

   (a) 1/8 (b) 5/8 (c) 3/16 (d) 5/16

4. Consider three particles A, B and C, each with attribute S that can take two values ± 1. Let $S_A = 1, S_B = 1$ and $S_C = -1$ at a given instant. In the next instant, each S value can change to -S with probability 1/3. The probability that $S_A + S_B + S_C$ remains unchanged is

   [CSIR June 2013]

   (a) 2/3 (b) 1/3 (c) 2/9 (d) 4/9

5. A loaded dice has the probabilities $\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}, \frac{6}{21}$ of turning up 1,2,3,4,5 and 6 respectively. If it is thrown twice, what is the probability that the sum of the numbers that turn up is even?

   [CSIR Dec 2013]

   (a) 144/441 (b) 225/441 (c) 221/441 (d) 220/441

6. A child makes a random walk on a square lattice of lattice constant 'a', taking a step in the north, east, south or west directions with probabilities 0.255, 0.255, 0.245, 0.245 respectively. After a large number of steps N, the expected position of the child w.r.t the strating point is at a distance **[CSIR Dec 2013]**

   (a) $\sqrt{2} \times 10^{-2} Na$ in the north-east direction (b) $\sqrt{2N} \times 10^{-2} a$ in the north-east direction

   (c) $2\sqrt{2} \times 10^{-2} Na$ in the south-east direction (d) 0

7. In one dimension, a random walker takes a step with equal probability to the left or right. What is the probability that the walker returns to the starting point after 4 steps? **[CSIR June 2014]**

   (a) 3/8 (b) 5/16 (c) 1/4 (d) 1/16

8. Let, $y = \frac{1}{2}(x_1 + x_2) - \mu$, where x_1 and x_2 are independent and identically distributed Gaussian random variables of mean μ and standard deviation σ . Then $\frac{\langle y^4 \rangle}{\sigma^4}$ is **[CSIR June 2014]**

(a) 1 (b) 3/4 (c) 1/2 (d) 1/4

9. Two independent random variables m and n, which can take the integer values 0, 1, 2, ..., ∞, follow the Poisson distribution, with distinct mean values μ and v respectively. Then

[CSIR Dec 2014]

(a) the probability distribution of the random variable $l = m + n$ is a binomial distribution.
(b) the probability distribution of the random variable $r = m - n$ is also a Poisson distribution.
(c) the variance of the random variable $l = m + n$ is equal to $\mu + v$.
(d) the mean value of the random variable $r = m - n$ is equal to 0.

10. A random walker takes a step of unit length in the positive direction with probability 2/3 and a step of unit length in the negative direction with probability 1/3. The mean displacement of the walker after n steps is

[CSIR Dec 2014]

(a) $n/3$ (b) $n/8$ (c) $2n/3$ (d) 0

11. Three real variables a, b and c are each randomly chosen from a uniform probability distribution in the interval [0, 1]. The probability that $a+b > 2c$ is **[CSIR June 2015]**

(a) 3/4 (b) 2/3 (c) 1/2 (d) 1/4

12. Consider a random walker on a square lattice. At each step the walker moves to a nearest neighbour site with equal probability for each of the four sites. The walker starts at the origin and takes 3 steps. The probability that during this walk no site is visited more than once is **[CSIR Dec 2015]**

(a) 12/27 (b) 27/64 (c) 3/8 (d) 9/16

13. Let X and Y be two independent random variables, each of which follow a normal distribution with same standard deviation σ , but with means μ and $-\mu$ respectively. Then the sum $X + Y$ follows a

[CSIR June 2016]

(a) distribution with two peaks at $\pm\mu$ and mean 0 and standard deviation $\sigma\sqrt{2}$
(b) normal distribution with mean 0 and standard deviation 2σ .
(c) distribution with two peaks at $\pm\mu$ and mean 0 and standard deviation 2σ .
(d) normal distribution with mean 0 and standard deviation $\sigma\sqrt{2}$.

14. A box of volume V containing N molecules of an ideal gas. is divided by wall with a hole into two conpartments. the volume of the smaller compartment $V/3$, the variance of the number of particles in it, is

[CSIR June 2016]

(a) $\frac{N}{3}$ (b) $\frac{2N}{9}$ (c) $\sqrt{N}$ (d) $\frac{\sqrt{N}}{3}$

TIFR PREVIOUS YEAR QUESTIONS

15. A 100 page book is known to have 200 printing errors distributed randomly through the pages. The probability that one of the pages will be found to be completely free of errors is closest to

[TIFR 2011]

(a) 67% (b) 50% (c) 25% (d) 13%

16. The probability function for a variable x which assumes only positive values is

$$f(x) = x\exp\left(-\frac{x}{\lambda}\right)$$

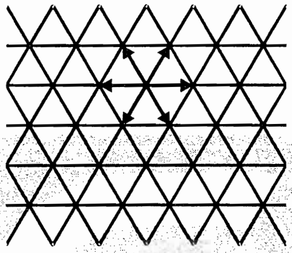
where $\lambda > 0$. The ratio $\langle x\rangle / \hat{x}$, where $\hat{x}$ is the most probable value and $\langle x\rangle$ is the mean value of the variable x, is **[TIFR 2014]**

(a) 2 (b) $\frac{1+\lambda}{1-\lambda}$ (c) $\frac{1}{\lambda}$ (d) 1

17. A random number generator outputs +1 or -1 with equal probability every time it is run. After it is run 6 times, what is the probability that the sum of the answers generated is zero? Assume that the individual runs are independent of each other. **[TIFR 2015]**

(a) 1/2 (b) 5/6 (c) 5/16 (d) 15/32

18. In a triangular lattice a particle moves from a lattice point to any of its 6 neighbouring points with equal probability, as shown in the figure on the right. **[TIFR 2016]**



The probability that the particle is back at its starting point after 3 moves is

(a) 5/18 (b) 1/6 (c) 1/18 (d) 1/36

JEST PREVIOUS YEAR QUESTIONS

19. An unbiased die is cast twice. The probability that the positive difference (bigger-smaller) between the two numbers is 2 is **[JEST 2012]**

(a) 1/9 (b) 2/9 (c) 1/6 (d) 1/3

20. A box contains 100 coins out of which 99 fair coins and 1 is a double-headed coin. Suppose you choose a coin at random and toss it 3 times. It turns out that the results of all 3 tosses are heads. What is the probability that the coin you have drawn is the double-headed one? **[JEST 2013]**

(a) 0.99 (b) 0.925 (c) 0.075 (d) 0.01

21. There are on average 20 buses per hour at a point, but at random times. The probability that there are no buses in five minutes is closest to **[JEST 2013]**

(a) 0.07 (b) 0.60 (c) 0.36 (d) 0.19

22. Two drunks start out together at the origin, each having equal probability of making a step simultaneously to the left or right along the x-axis. The probability that they meet after n steps is **[JEST 2013]**

(a) $\frac{1}{4^n}\frac{2n!}{n!^2}$ (b) $\frac{1}{2^n}\frac{2n!}{n!^2}$ (c) $\frac{1}{2^n}2n!$ (d) $\frac{1}{4^n}n!$

23. If the distribution function of x is $f(x) = xe^{-x/\lambda}$ over the interval $0 < x < \infty$, then mean value of x is

[JEST 2013]

(a) λ (b) 2λ (c) $\lambda/2$ (d) 0

24. If two ideal dice are rolled once, what is the probability of getting atleast one '6'? **[JEST 2015]**

(a) 11/36 (b) 1/36 (c) 10/36 (d) 5/36

25. You receive on average 5 emairs per day during a 365-days year. The number of days on average on which you do not receive any emails in that year are **[JEST 2016]**

(a) More than 5 (b) More than 2 (c) 1 (d) None of the above

26. The mean value of random variable x with probability density $p(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\left(x^2+\mu x\right)/2\sigma^2\right]$, is **[JEST 2016]**

(a) 0 (b) $\mu/2$ (c) $-\mu/2$ (d) σ

ANSWER KEY

1. (b) **2. (b)** **3. (d)** **4. (d)** **5. (b)** **6. (a)** **7. (a)**
8. (d) **9. (c)** **10. (a)** **11. (c)** **12. (d)** **13. (d)** **14. (b)**
15. (d) **16. (a)** **17. (c)** **18. (c)** **19. (b)** **20. (c)** **21. (d)**
22. (a) **23. (b)** **24. (a)** **25. (b)** **26. (c)**

CSIR-UGC-NET/JRF DEC 2016

MATHEMATICAL PHYSICS

SECTION: MISCELLANEOUS TOPICS

CSIR PREVIOUS YEAR QUESTIONS

. Which of the following matrices is an element of the group $SU(2)$? **[CSIR June 2011]**

(a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1+i}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \end{pmatrix}$ (c) $\begin{pmatrix} 2+i & i \\ 3 & 1+i \end{pmatrix}$ (d) $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$

. The character table of C_3, the group of symmetries of an equilateral triangle is given below

	$\chi^{(0)}$	$\chi^{(1)}$	$\chi^{(2)}$
$1\mathfrak{C}_1$	1	1	b
$3\mathfrak{C}_2$	1	a	c
$2\mathfrak{C}_3$	1	1	d

In the above $\mathfrak{C}_1, \mathfrak{C}_2, \mathfrak{C}_3$ denotes the three classes of C_{3v}, containing 1, 3 and 2 elements respectively, and $\chi^{(0)}, \chi^{(1)}$ and $\chi^{(2)}$ are the characters of the three irreducible representations $\Gamma^{(0)}, \Gamma^{(1)}$ and $\Gamma^{(2)}$ of C_{3v}.

(A) The entries a, b, c and d in this table are, respectively

(a) 2, 1, –1, 0 (b) –1, 2, 0, –1 (c) –1, 1, 0, –1 (d) –1, 1, 1, –1

(B) The reducible representation Γ of C_{3v} with character $\chi = (4,0,1)$ decomposes into its irreducible representations $\Gamma^{(0)}, \Gamma^{(1)}, \Gamma^{(2)}$ as **[CSIR June 2011]**

(a) $2\Gamma^{(0)} + 2\Gamma^{(1)}$ (b) $\Gamma^{(0)} + 3\Gamma^{(1)}$ (c) $\Gamma^{(0)} + \Gamma^{(1)} + \Gamma^{(2)}$ (d) $2\Gamma^{(2)}$

. The radioactive decay of a certain material satisfies Poisson statistics with a mean rate of λ per second, what should be the minimum duration of counting (in seconds) so that the relative error is less than 1% ? **[CSIR June 2012]**

(a) $100/\lambda$ (b) $10^4/\lambda^2$ (c) $10^4/\lambda$ (d) $1/\lambda$

. A bag contains many balls, each with a number painted on it. There are exactly n balls which have the number n (namely one ball with 1, two balls with 2, and so on until N balls with N on them). An experiment consists of choosing a ball at random, noting the number on it and returning it to the bag. If the experiment is repeated a large number of times, the average value of the number will tend to **[CSIR June 2012]**

(a) $\frac{2N+1}{3}$ (b) $\frac{N}{2}$ (c) $\frac{N+1}{2}$ (d) $\frac{N(N+1)}{2}$

5. The approximation $\cos\theta \approx 1$ is valid up to 3 decimal places as long as $|\theta|$ is less than: (take $180°/\pi \simeq 57.29°$). **[CSIR June 2013]**

(a) 1.28° (b) 1.81° (c) 3.28° (d) 4.01°

6. The solution of the partial differential equation

$$\frac{\partial^2}{\partial t^2}u(x,t)-\frac{\partial^2}{\partial x^2}u(x,t)=0$$

satisfying the boundary conditions u(0, t) = 0 = u(L, t) and initial conditions u(x, 0) = sin(πx/L) and

$\frac{\partial}{\partial t}u(x,t)|_{t=0}=\sin(2\pi x/L)$ is **[CSIR June 2013]**

(a) $\sin(\pi x/L)\cos(\pi t/L)+\frac{L}{2\pi}\sin(2\pi x/L)\cos(2\pi t/L)$

(b) $2\sin(\pi x/L)\cos(\pi t/L)-\sin(\pi x/L)\cos(2\pi t/L)$

(c) $\sin(\pi x/L)\cos(2\pi t/L)+\frac{L}{\pi}\sin(2\pi x/L)\sin(\pi t/L)$

(d) $\sin(\pi x/L)\cos(\pi t/L)+\frac{L}{2\pi}\sin(2\pi x/L)\sin(2\pi t/L)$

7. The expression **[CSIR Dec. 2013]**

$$\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}+\frac{\partial^2}{\partial x_4^2}\right)\frac{1}{(x_1^2+x_2^2+x_3^2+x_4^2)}$$ is proportional to

(a) $\delta(x_1+x_2+x_3+x_4)$ (b) $\delta(x_1)\delta(x_2)\delta(x_3)\delta(x_4)$

(c) $(x_1^2+x_2^2+x_3^2+x_4^2)^{-3/2}$ (d) $(x_1^2+x_2^2+x_3^2+x_4^2)^{-2}$

8. Let A and B be two vectors in three-dimensional Euclidean space. Under rotation, the tensor product $T_{ij}=A_iB_j$ **[CSIR Dec. 2013]**

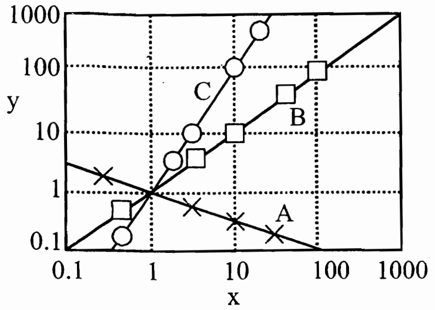
(a) reduces to a direct sum of three 3-dimensional representations

(b) is an irreducible 9-dimensional representation

(c) reduces to a direct sum of a 1-dimensional, a 3-dimensional and a 5-dimensional irreducible representations

(d) reduces to a direct sum of a 1-dimensional and an 8-dimensional irreducible representation

9. 69. Three sets of data *A*, *B* and *C* from an experiment, represented by ×, □ and ○, are plotted on a log-log scale. Each of these are fitted with straight lines as shown in the figure. **[CSIR Dec. 2013]**



The functional dependence *y*(*x*) for the sets A, B and C are, respectively

(a) $\sqrt{x}, x$ and x^2 (b) $-\frac{x}{2}, x$ and $2x$ (c) $\frac{1}{x^2}, x$ and x^2 (d) $\frac{1}{\sqrt{x}}, x$ and x^2

10. The following data is obtained in an expriment that measures the viscosity η as a function of molecular weight M for a set of polymers. **[CSIR June 2014]**

$M(Da)$	$\eta(kPa-s)$
990	0.28 ± 0.03
5032	30 ± 2
10191	250 ± 10
19825	2000 ± 200

The relation that best describes the dependence of η on M is

(a) $\eta \sim M^{4/9}$ (b) $\eta \sim M^{3/2}$ (c) $\eta \sim M^{2}$ (d) $\eta \sim M^{3}$

11. The integral $\int_0^1 \sqrt{x}dx$ is to be evaluated up to 3 decimal places using Simpson's 3-point rule. If the interval [0, 1] is divided into 4 equal parts, the correct result is **[CSIR June 2014]**

(a) 0.683 (b) 0.667 (c) 0.657 (d) 0.638

12. The function $\Phi(x,y,z,t)=\cos(z-vt)+\text{Re}(\sin(x+iy))$ satisfies the equation **[CSIR June 2014]**

(a) $\frac{1}{v^2}\frac{\partial^2\Phi}{\partial t^2}=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi$ (b) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial z^2}\right)\Phi=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Phi$

(c) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right)\Phi=\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\Phi$ (d) $\left(\frac{\partial^2}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\Phi=\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\Phi$

3. Let α and β be complex numbers. Which of the following sets of matrices forms a group under matrix multiplication? **[CSIR Dec. 2014]**

(a) $\begin{pmatrix}\alpha & \beta\\ 0 & 0\end{pmatrix}$ (b) $\begin{pmatrix}1 & \alpha\\ \beta & 1\end{pmatrix}$, where $\alpha\beta \neq 1$

(c) $\begin{pmatrix}\alpha & \alpha^*\\ \beta & \beta^*\end{pmatrix}$, where $\alpha\beta^*$ is real (d) $\begin{pmatrix}\alpha & \beta\\ -\beta^* & \alpha^*\end{pmatrix}$, where $|\alpha|^2+|\beta|^2=1$

4. A particle moves in two dimensions on the ellipse $x^2+4y^2=8$. At a particular instant it is at the point $(x, y) = (2, 1)$ and the x-component of its velocity is 6 (in suitable units). Then the y-component of its velocity is **[CSIR June 2015]**

(a) –3 (b) –2 (c) 1 (d) 4

5. The rank-2 tensor $x_i x_j$, where x_i are the Cartesian coordinates of the position vector in three dimensions, has 6 independent elements. Under rotation, these 6 elements decompose into irreducible sets (that is the elements of each set transform only into linear combinations of elements in that set) containing **[CSIR June 2015]**

(a) 4 and 2 elements (b) 5 and 1 elements

(c) 3, 2 and 1 elements (d) 4, 1 and 1 elements

5. Consider the differential equation $\frac{dy}{dx}=x^2-y$ with the initial condition $y=2$ at $x=0$. Let $y_{(1)}$ and $y_{(1/2)}$ be the solutions at $x=1$ obtained using Euler's forward algorithm with step size 1 and 1/2 respectively. The value of $(y_{(1)}-y_{(1/2)})/y_{(1/2)}$ is **[CSIR June 2015]**

(a) $-\frac{1}{2}$ (b) –1 (c) $\frac{1}{2}$ (d) 1

17. Let $f(x,t)$ be a solution of the wave equation $\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}$ in 1-dimension. If at $t=0$, $f(x,0) = e^{-x^2}$ and $\frac{\partial f}{\partial t}(x,0) = 0$ for all x, then $f(x,t)$ for all future times $t > 0$ is described by **[CSIR June 2015]**

(a) $e^{-(x^2 - v^2t^2)}$ (b) $e^{-(x-vt)^2}$

(c) $\frac{1}{4}e^{-(x-vt)^2} + \frac{3}{4}e^{-(x+vt)^2}$ (d) $\frac{1}{2}\left[e^{-(x-vt)^2} + e^{-(x+vt)^2}\right]$

18. In the scattering of some elementary particles, the scattering cross-section σ is found to depend on the total energy E and the fundamental constants h (Planck's constant) and c (the speed of light in vacuum). Using dimensional analysis, the dependence of σ on these quantities is given by **[CSIR Dec. 2015]**

(1) $\sqrt{\frac{hc}{E}}$ (2) $\frac{hc}{E^{3/2}}$ (3) $\left(\frac{hc}{E}\right)^2$ (4) $\frac{hc}{E}$

19. If $y = \frac{1}{\tan h(x)}$, then x is **[CSIR Dec. 2015]**

(a) $\ln\left(\frac{y+1}{y-1}\right)$ (b) $\ln\left(\frac{y-1}{y+1}\right)$ (c) $\ln\sqrt{\frac{y-1}{y+1}}$ (d) $\ln\sqrt{\frac{y+1}{y-1}}$

20. The value of the integral $\int_0^8 \frac{1}{x^2+5}dx$, evaluated using Simpson's $\frac{1}{3}$ rule with $h = 2$, is **[CSIR Dec. 2015]**

(a) 0.565 (b) 0.620 (c) 0.698 (d) 0.736

21. The Gauss hypergeometric function $F(a,b,c;z)$, defined by the Taylor series expansion around $z = 0$ as

$$F(a,b,c;z) = \sum_{n=0}^{\infty} \frac{a(a+1)\dots(a+n-1)\,b(b+1)\dots(b+n-1)}{c(c+1)\dots(c+n-1)\,n!} z^n$$

satisfies the equation relation **[CSIR June 2016]**

(a) $\frac{d}{dz}F(a,b,c;z) = \frac{c}{ab}F(a-1,b-1,c-1;z)$

(b) $\frac{d}{dz}F(a,b,c;z) = \frac{c}{ab}F(a+1,b+1,c+1;z)$

(c) $\frac{d}{dz}F(a,b,c;z) = \frac{ab}{c}F(a-1,b-1,c-1;z)$

(d) $\frac{d}{dz}F(a,b,c;z) = \frac{ab}{c}F(a+1,b+1,c+1;z)$

22. Using dimensional analysis, Planck defined a characteristic temperature T_P from powers of the gravitional constant G, Planck's constant h, Boltzmann constant k_B and the speed of light c in vacuum. The expression for T_P is proportional to **[CSIR June 2016]**

(a) $\sqrt{\frac{hc^5}{k_B^2 G}}$ (b) $\sqrt{\frac{hc^3}{k_B^2 G}}$ (c) $\sqrt{\frac{G}{hc^4 k_B^2}}$ (d) $\sqrt{\frac{hk_B^2}{Gc^3}}$

23. The integral equation **[CSIR June 2016]**

$$\phi(x,t)=\lambda\int dx'dt'\int\frac{d\omega\, dk}{(2\pi)^2}\frac{e^{-ik(x-x')+i\omega(t-t')}}{\omega^2-k^2-m^2+i\varepsilon}\phi^3(x',t')$$

is equivalent to the differential equation

(a) $\left(\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial x^2}-m^2+i\varepsilon\right)\phi(x,t)=-\frac{1}{6}\lambda\phi^3(x,t)$

(b) $\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}+m^2-i\varepsilon\right)\phi(x,t)=\lambda\phi^2(x,t)$

(c) $\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}+m^2-i\varepsilon\right)\phi(x,t)=-3\lambda\phi^2(x,t)$

(d) $\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}+m^2-i\varepsilon\right)\phi(x,t)=-\lambda\phi^3(x,t)$

24. A part of the group multiplication table for a six element group $G=\{e,a,b,c,d,f\}$ is shown below. (In the following e is the identity element of G). **[CSIR June 2016]**

	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	b	e	d		
b	b	e	x	f	y	z
c	c					
d	d					
f	f					

The entries x, y and z should be

(a) $x=a,\ y=d$ and $z=c$ (b) $x=c,\ y=a$ and $z=d$

(c) $x=c,\ y=d$ and $z=a$ (d) $x=a,\ y=c$ and $z=d$

25. In finding the roots of the polynomial $f(x)=3x^3-4x-5$ using the iterative Newton-Raphson method, the initial guess is taken to be $x=2$. In the next iteration its value is nearest to **[CSIR June 2016]**

(a) 1.671 (b) 1.656 (c) 1.559 (d) 1.551

TIFR PREVIOUS YEAR QUESTIONS

26. The infinite series

$$x+\frac{x^3}{3}+\frac{x^5}{5}+\frac{x^7}{7}+\ldots$$

where $-1<x<+1$, can be summed to the value **[TIFR 2011]**

(a) tan hx (b) $\ln\left(1-\frac{4}{\pi}\tan^{-1}x\right)$ (c) $\frac{1}{2}\ln\left[\frac{(1+x)}{(1-x)}\right]$ (d) $\frac{1}{2}\ln\left[\frac{(1-x)}{(1+x)}\right]$

27. The function $f(x)$ represents the nearest integer less than x, e.g. $f(3.14) = 3$. **[TIFR 2012]**
The derivative of this function (for arbitrary x) will be given in terms of the integers n as $f'(x) =$

(a) 0 (b) $\sum_n \delta(x-n)$ (c) $\sum_n |x-n|$ (d) $\sum_n f(x-n)$

28. Consider the integral $\displaystyle\int_{-p^2}^{+p^2} \frac{dx}{\sqrt{x^2-p^2}}$ **[TIFR 2012]**

where p is a constant. This integral has a real, nonsingular value if

(a) $p<-1$ (b) $p>1$ (c) $p=1$ (d) $p \to 0$
(e) $p \to \infty$

29. The value of the integral $\displaystyle\int_0^{\infty} dx\, x^9 \exp(-x^2)$ is **[TIFR 2013]**

(a) 20160 (b) 12 (c) 18 (d) 24

30. The integral $\displaystyle\int_{-\infty}^{\infty} dx\, \delta(x^2-\pi^2)\cos x$, evaluates to **[TIFR 2013]**

(a) -1 (b) 0 (c) $\dfrac{1}{\pi}$ (d) $-\dfrac{1}{\pi}$

31. In spherical polar coordinates $\vec{r} = (r,\theta,\varphi)$ the delta function $\delta(\vec{r}_1 - \vec{r}_2)$ can be written as
[TIFR 2014]

(a) $\delta(r_1-r_2)\delta(\theta_1-\theta_2)\delta(\varphi_1-\varphi_2)$ (b) $\dfrac{1}{r_1^2}\delta(r_1-r_2)\delta(\cos\theta_1-\cos\theta_2)\delta(\varphi_1-\varphi_2)$

(c) $\dfrac{1}{|\vec{r}_1-\vec{r}_2|^2}\delta(r_1-r_2)\delta(\cos\theta_1-\cos\theta_2)\delta(\varphi_1-\varphi_2)$ (d) $\dfrac{1}{r_1^2\cos\theta_1}\delta(r_1-r_2)\delta(\theta_1-\theta_2)\delta(\varphi_1-\varphi_2)$

32. The integral $\displaystyle\int_0^{\infty} \frac{dx}{x}\left[\exp\left(-\frac{x}{\sqrt{3}}\right) - \exp\left(-\frac{x}{\sqrt{2}}\right)\right]$ evaluates to **[TIFR 2016]**

(a) zero (b) 2.03×10^{-2} (c) 2.03×10^{-1} (d) 2.03

33. Given that infinite series $y(x) = 1 + 3x + 6x^2 + 10x^3 + \cdots + \dfrac{(n+1)(n+2)}{2}x^n + \cdots$, find the value of $y(x)$ for $x = 6/7$. **[TIFR 2016]**

JEST PREVIOUS YEAR QUESTIONS

34. If $[x]$ denotes the greatest integer not exceeding x, then $\int_0^{\infty}[x]e^{-x}dx$ **[JEST 2012]**

(a) $\dfrac{1}{e-1}$ (b) 1 (c) $\dfrac{e-1}{e}$ (d) $\dfrac{e}{e^2-1}$

35. As $x \to 1$, the infinite series $x - \frac{1}{3}x^3 + \frac{1}{5}x^4 - \frac{1}{7}x^7 + ...$ **[JEST 2012]**

(a) diverges
(b) converges to unity
(c) converges to $\frac{\pi}{4}$
(d) none of the above.

36. What is the value of the following series? **[JEST 2012]**

$$\left(1+\frac{1}{2!}+\frac{1}{4!}+.....\right)^2-\left(1+\frac{1}{3!}+\frac{1}{5!}+.....\right)^2$$

(a) 0
(b) e
(c) e^2
(d) 1

37. The Dirac delta function $\delta(x)$ satisfies the relation $\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0)$ for a well behaved function $f(x)$. If x has the dimension of momentum then **[JEST 2014]**

(a) $\delta(x)$ has the dimension of momentum
(b) $\delta(x)$ has the dimension of (mometum)^2
(c) $\delta(x)$ is dimensionless
(d) $\delta(x)$ has the dimension of (momentum)$^{-1}$.

38. The sum $\sum_{m=1}^{99} \frac{1}{\sqrt{m+1}+\sqrt{m}}$ is equal to **[JEST 2015]**

(a) 9
(b) $\sqrt{99}-1$
(c) $\frac{1}{(\sqrt{99}-1)}$
(d) 11

39. The sum of the infinite series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$ is **[JEST 2016]**

(a) 2π
(b) π
(c) $\frac{\pi}{2}$
(d) $\frac{\pi}{4}$

40. Given the condition $\nabla^2\Phi = 0$, the solution of the equation $\nabla^2\Psi = k\vec{\nabla}\Phi \cdot \vec{\nabla}\Phi$ is given by: **[JEST 2016]**

(a) $\Psi = k\Phi^2/2$
(b) $\Psi = k\Phi^2$
(c) $\Psi = k\Phi\,\ell n\,\Phi$
(d) $\Psi = k\Phi\,\ell n\,\Phi/2$

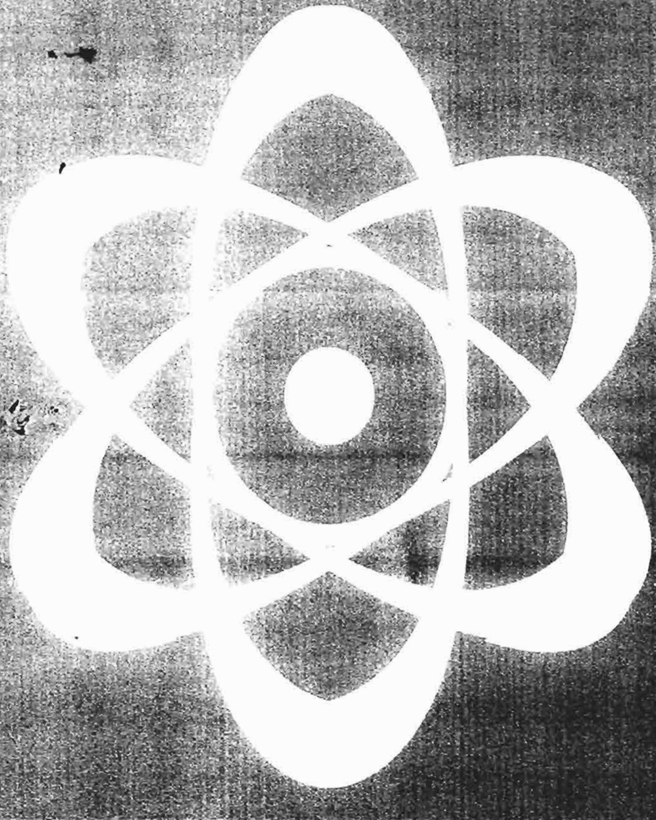
ANSWER KEY

1. (b) **2. (c)** **3. (c)** **4. (a)** **5. (b)** **6. (d)** **7. (b)**
8. (c) **9. (d)** **10. (d)** **11. (c)** **12. (a)** **13. (d)** **14. (a)**
15. (b) **16. (b)** **17. (d)** **18. (c)** **19. (d)** **20. (a)** **21. (d)**
22. (a) **23. (d)** **24. (d)** **25. (b)** **26. (c)** **27. (b)** **28. (d)**
29. (b) **30. (d)** **31. (b)** **32. (c)** **33. (343)** **34. (a)** **35. (c)**
36. (d) **37. (d)** **38. (a)** **39. (d)** **40. (a)**

QUANTUM MECHANICS

CSIR-UGC-NET Dec. 2016 | GATE 2017

NET | GATE | TIFR | JEST



Previous Years' Questions

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QUANTUM MECHANICS

Previous Years' Questions

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GATE Previous Years' Questions

1. The maximum kinetic energy of the photoelectrons in a photoelectric effect experiment depends on
(a) intensity of the incident light (b) frequency of the incident light **[GATE 1997]**
(c) polarization of the incident light (d) angle of incidence

2. The dispersion relation of a certain wave is **[GATE 1998]**

$$\omega = \sqrt{c^2k^2 + m^2}$$

where ω is the angular frequency, k is the wave vector, c is the velocity of light and m is a constant. The group velocity v of the wave has the following properties
(a) $v \to c$ as $k \to 0$ and $v \to c$ as $k \to \infty$ (b) $v \to c$ as $k \to 0$ and $v \to \infty$ as $k \to \infty$
(c) $v \to 0$ as $k \to 0$ and $v \to \infty$ as $k \to \infty$ (d) $v \to 0$ as $k \to 0$ and $v \to c$ as $k \to \infty$

3. An electron propagating along the x-axis passes through a slit of width $\Delta y = 1\, nm$. The uncertainty in the y-component of its velocity after passing through the slit is **[GATE 2002]**
(a) $7.332 \times 10^5\ m/\sec$ (b) $1.166 \times 10^5\ m/\sec$
(c) $3.436 \times 10^5\ m/\sec$ (d) $2.326 \times 10^5\ m/\sec$

4. For a free electron gas in a metal, the magnitudes of phase velocity (v_p) and group velocity (v_g) are such that **[GATE 2002]**
(a) $v_p = v_g$ (b) $v_p = \frac{1}{2} v_g$ (c) $v_p = \sqrt{2} v_g$ (d) $v_p = 2v_g$

5. A parallel beam of electrons of a given momentum pass through a screen S_1 containing a slit and produces a diffraction pattern on a screen S_2 placed behind it. The width of the central maximum observed on the screen S_2 can be increased by **[GATE 2003]**
(a) decreasing the distance between the screen S_1 and S_2.
(b) increasing the width of the slit in screen S_1.
(c) decreasing the momentum of the electrons.
(d) increasing the momentum of the electrons.

6. Which one of the following statements concerning the compton effect is not correct? **[GATE 2003]**
(a) The wavelength of the scattered photon is greater than or equal to the wavelength of the incident photon.
(b) The electron can acquire a kinetic energy equal to the energy of the incident photon.
(c) The energy of the incident photon equals to the kinetic energy of the electron plus the energy of the scattered photon.
(d) The kinetic energy acquired by the electron is the largest when the incident and scattered photons move in opposite direction.

7. Given a wave with the dispersion relation $\omega = ck + m$ for $k > 0$ and $m > 0$, which one of the following is true? **[GATE 2003]**
(a) the group velocity is greater than the phase velocity
(b) the group velocity is less than the phase velocity
(c) the group velocity and the phase velocity are equal
(d) there is no definite relation between the group velocity and the phase velocity

8. The de-Broglie wavelength for an electron of energy 150 eV is **[GATE 2005]**
(a) $10^{-8}\ m$ (b) $10^{-10}\ m$ (c) $10^{-12}\ m$ (d) $10^{-14}\ m$

9. Although mass-energy equivalence of special relativity allows conversion of a photon to an electron-positron pair, such a process cannot occur in free space because **[GATE 2005]**
(a) the mass is not conserved (b) the energy is not conserved
(c) the momentum is not conserved (d) the charge is not conserved

10. The de Broglie wavelength of particles of mass m with average momentum p at a temperature T in three dimensions is given by **[GATE 2009]**

(a) $\lambda = \dfrac{h}{\sqrt{2mk_BT}}$ (b) $\lambda = \dfrac{h}{\sqrt{3mk_BT}}$ (c) $\lambda = \dfrac{h}{\sqrt{2k_BT}}$ (d) $\lambda = \dfrac{h}{\sqrt{2m}}$

11. A particle is confined within a spherical region of radius one femtometer (10^{-15} m). Its momentum can be expected to be about

(a) $20\dfrac{keV}{c}$ (b) $200\dfrac{keV}{c}$ (c) $200\dfrac{MeV}{c}$ (d) $\dfrac{GeV}{c}$

12. A proton is confined to a cubic box, whose sides have length 10^{-12} m. What is the minimum kinetic energy of the proton? The mass of proton is 1.67×10^{-27} kg and Planck's constant is 6.63×10^{-34} Js. **[GATE 2013]**

(a) 1.1×10^{-17} J (b) 3.3×10^{-17} J (c) 9.9×10^{-17} J (d) 6.6×10^{-17} J

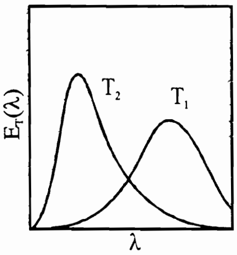
ANSWER KEY

1. (b) **2. (d)** **3. (b)** **4. (b)** **5. (c)** **6. (b)** **7. (b)**
8. (b) **9. (c)** **10. (b)** **11. (c)** **12. (c)**

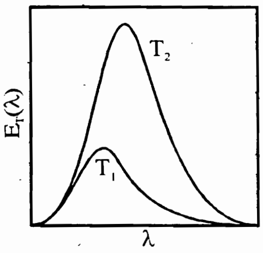
CSIR-UGC-NET Previous Years' Questions

1. Which of the graphs below gives the correct qualitative behavior of the energy density $E_T(\lambda)$ of blackbody radiation of wavelength λ at two temperatures T_1 and T_2 ($T_1 < T_2$)? **[CSIR June 2014]**

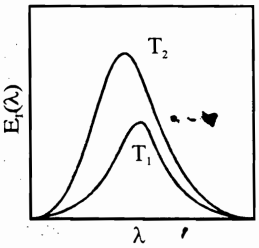
(a)



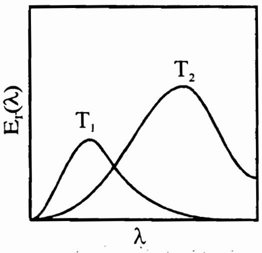
(b)



(c)



(d)



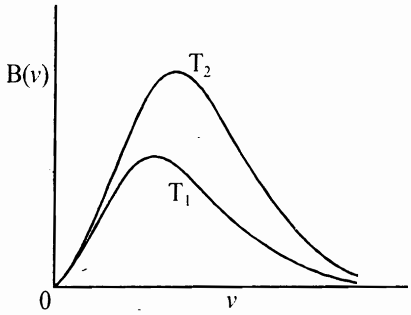
ANSWER KEY

1. (c)

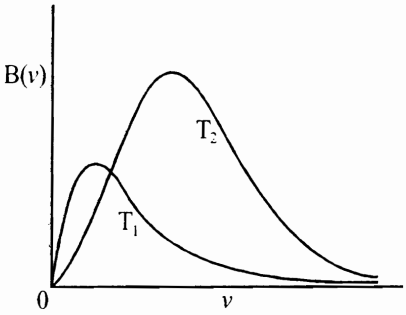
TIFR Previous Years' Questions

1. Two blackbodies radiate energy at temperature T_1 and T_2 $(T_1 > T_2)$. The energy emitted per unit time per unit solid angle per unit surface area of a blackbody in the frequency range ν to $\nu + d\nu$ is given by $B(\nu)d\nu$. Which one of the following graphs has the correct form ? **[TIFR 2015]**

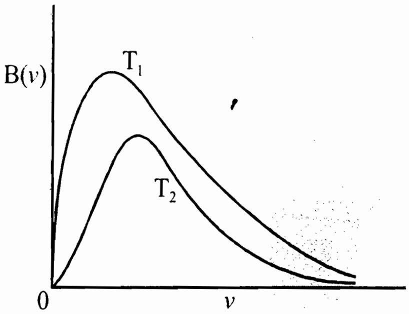
(a)



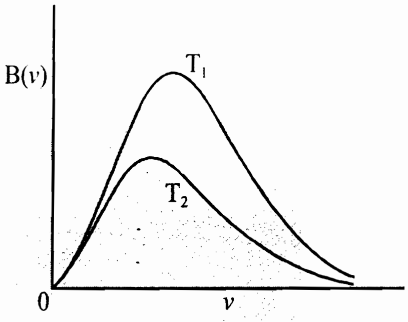
(b)



(c)



(d)



ANSWER KEY

1. (d)

Other Examinations Previous Years' Questions

1. The angular frequency ω of deep water waves varies as the inverse square rod of the wavelength λ, *i.e.*, $\omega(\lambda) \propto 1/\sqrt{\lambda}$. Which of the following is the relation between its group velocity v_g and phase velocity v_p?

   (a) $v_g = v_p/2$ (b) $v_g = v_p$ (c) $v_g = 2v_p$ (d) $v_g = \lambda v_p$

2. The intensity vs wavelength distribution of a blackbody is found to have its maximum at the wavelength $\lambda_{\max} = 0.2$ cm. Moreover, the total energy radiated J is measured to be 30 watts. If the temperature of the blackbody is now doubled, the new values of $\lambda_{\max}$ and J will be

   (a) 0.1 cm and 120 watts (b) 0.1 cm and 240 watts.
   (c) 0.1 cm and 480 watts (d) 0.4 cm an 60 watts

3. According to the uncertainty principle the kinetic energy of an electron confined to a spherical region of volume 10^{-33} m^3 is of the order of

   (a) 10^{-10} J (b) 10^{-12} J (c) 10^{-14} J (d) 10^{-16} J

4. In a Compton scattering process, a photon of wavelength λ is scattered off a charged particle of mass 'm', (initially at rest) by an angle θ. If the final wavelength of the photon is λ', then the difference $\lambda - \lambda'$

   (a) depends on θ, but not on λ (b) depends on λ, but not on m
   (c) depends on both λ and θ (d) depends on θ but not on m

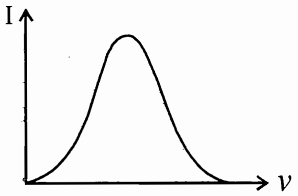
5. The kinetic energy of a proton and an α-particle (not under the influence of any force) are given to be equal. If we denote the de Broglie wavelengths of the proton by λ_p and that of the α-particle by λ_α, then

   (a) $\lambda_p = \lambda_\alpha$ (b) $\lambda_p = 4\lambda_\alpha$ (c) $\lambda_p = \frac{1}{2}\lambda_\alpha$ (d) $\lambda_p = 2\lambda_\alpha$

6. When a monochromatic point source of light is placed at a distance of 0.2 m from a photoelectric cell, the stopping potential V_s and the saturation current I_s are found to be 0.6 V and 18.0 mA, respectively. If the same source is now placed 0.6 m away from the photoelectric cell, one finds

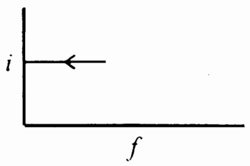
   (a) $V_S = 0.2$ V and $I_S = 6.0$ mA (b) $V_S = 0.6$ V and $I_S = 6.0$ mA
   (c) $V_S = 0.6$ V and $I_S = 2.0$ mA (d) $V_S = 0.2$ V and $I_S = 18.0$ mA

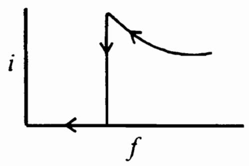
7. The graph in the figure below shows the intensity I as a function of frequency v of a perfect blackbody at a fixed temperature T;

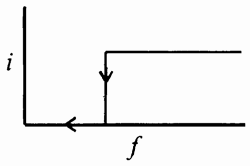


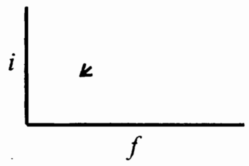
The corresponding graph at teperature 2T can be obtained by which of the following operations ?
For every point of the graph
(a) multiply the v-coordinate by 1 / 2 and the I-coordinate by 8
(b) multiply the v-coordinate by 2 and the I-coordinate by 8
(c) multiply the v-coordinate by 1 / 2 and the I-coordinate by 16
(d) multiply the v-coordinate by 2 and the I-coordinate by 16

8. In a photoelectric effect experiment, a stream of photons of frequency f and intensity (energy per unit time) is incident on the photocathode to produce a photocurrent i. If the frequency f is steadily reduced without any change of intensity, the plot of current vs frequency looks like

(a)


(b)


(c)


(d)


9. The energy of a 200 nm photon is:
(a) 0.01 eV (b) 100 eV (c) 6.2 eV (d) 1 eV

10. Phase velocity and group velocity for a wave will be identical
(a) If the phase velocity is independent of wavelength.
(b) If the phase velocity is dependent on wavelength.
(c) If the group velocity is dependent on wavelength.
(d) If the group velocity is dependent on refractive index of the medium.

11. An electron in an X-ray tube is accelerated through a voltage V and strikes a tungsten target. The shortest wavelength photon emitted will have
(a) $\lambda = \frac{\hbar c}{eV}$ (b) $\lambda = \frac{eV}{hc}$ (c) $\lambda = \frac{hc}{eV}$ (d) $\lambda = \frac{eV}{\hbar c}$

12. The dispersion relation for a certain type of excitation is given by $\omega = Ak^a$, where A and a > 0 are constants. For these excitations, the group velocity is less than the phase velocity when
(a) $a = 1$ (b) $a = \sqrt{2}$ (c) $a < 1$ (d) $a \geq 2$

13. A small object is viewed with a microscope, using light of wavelength λ to illuminate the object. Let α be the half-angle subtended by the objective of the microscope at the position of the object being examined. The uncertainty Δx in the position of the object is of the order of
(a) $\lambda / \sin\alpha$ (b) $\lambda / \cos\alpha$ (d) $\lambda \sin\alpha$ (d) $2\lambda \sin\alpha$

14. If a particle of mass $m = 5\times10^{-28}$ kg is confined to move in a space of size 10^{-15} m, the minimum value of its kinetic energy will approximately be
(a) $10^{-9}\ \text{kg m}^2/\text{s}^2$ (b) $6.6\times10^{-19}\ \text{kg m}^2/\text{s}^2$
(c) $10^{-12}\ \text{kg m}^2/\text{s}^2$ (d) $6.6\times10^{-15}\ \text{kg m}^2/\text{s}^2$

15. The relation between angular frequency ω and wave number k for given type of waves is $\omega^2 = \alpha k + \beta k^3$. The wave number k_0 for which the phase velocity equals the group velocity is:
(a) $3\sqrt{\frac{\alpha}{\beta}}$ (b) $\left(\frac{1}{3}\right)\sqrt{\frac{\alpha}{\beta}}$ (c) $\sqrt{\frac{\alpha}{\beta}}$ (d) $\left(\frac{1}{2}\right)\sqrt{\frac{\alpha}{\beta}}$

16. A particle of rest mass m_0 is moving uniformly in a straight line with relativistic velocity βc, where c is the velocity of light in vacuum and $0 < \beta < 1$. The phase velocity of the de-Broglie wave associated with the particle is:

(a) βc (b) $\frac{c}{\beta}$ (c) c (d) $\frac{c}{\beta^2}$

17. The speed of an electron, whose de-Broglie wavelength is equal to its compton wavelength, is (c is the speed of light)

(a) c (b) $c/\sqrt{2}$ (c) c/2 (d) c/3

ANSWER KEY

1. (a) 2. (c) 3. (d) 4. (a) 5. (d) 6. (c) 7. (d)
8. (b) 9. (c) 10. (a) 11. (c) 12. (c) 13. (a) 14. (c)
15. (c) 16. (b) 17. (b)

GATE Previous Years' Questions

1. The eigenfunction for the Hamiltonian of a particle of mass m in a one dimensional potential $V(x)$ is given by $\psi(x) = A\exp\left(-bx^2/2\right)$, where A and b are constants. It follows that **[GATE 1998]**

(a) $V(x) = \text{constant}$ (b) $V(x) \propto x^3$

(c) $V(x) \propto x^2$ (d) $V(x) \propto 1/x$

2. A particle of mass 'm' in one dimension is in a state described by **[GATE 1998]**

$$\psi(x,t) = A\exp\left(\frac{ipx - iEt}{\hbar}\right) + B\exp\left(\frac{-ipx - iEt}{\hbar}\right)$$

The probability current density is given by **Ans. (c)**

(a) $\frac{p}{m}$ (b) $\left(|A| - |B|\right)\frac{p}{m}$ (c) $\left(|A|^2 - |B|^2\right)\frac{p}{m}$ (d) $\left(|A|^2 + |B|^2\right)\frac{p}{m}$

3. Bound state eigenfunctions of an attractive finite range smooth potential behave for larger r as:

(a) $\exp(-r/r_0)$ where r_0 is a positive constant. (b) $\left(1/r^n\right)$ where n>0 **[GATE 2000]**

(c) constant (d) $\exp(ikr)/r$

4. A normalized state of a particle moving in a potential $V(x)$ is given by

$$\psi(x,t) = \sum_{n=0}^{\infty} C_n \phi_n(x) e^{-E_n t/\hbar}$$

where $\phi_n(x)$'s are the normalized eigenfunctions of the particle corresponding to the energies E_n's. Then **[GATE 2001]**

(a) $\sum_{n=0} |C_n|^2 = 1$

(b) The average energy of the particle in the state $\psi(x,t)$ is $\sum_{n=0}^{\infty} |C_n|^2 E_n$

(c) $\psi(x,t)$ is an eigenfunction of the Hamiltonian of the particle.

(d) $\psi(x,t)$ is an eigenfunction of the momentum operator.

5. Which of the following functions represents acceptable wave function of a particle in the range $-\infty \le x \le \infty$. **[GATE 2001]**

(a) $\varphi(x) = A\tan x, A > 0$ (b) $\varphi(x) = B\cos x$, B real

(c) $\varphi(x) = C\exp\left(-D/x^2\right), C > 0, D < 0$ (d) $\varphi(x) = E\,x\exp\left(-Fx^2\right), E, F > 0$

6. An electron in a time independent potential is in a state which is a superposition of the ground state ($E_0 = 11\, eV$) and the first excited state ($E_1 = 1\, eV$). The wave function of the electron will repeat itself with a period of **[GATE 2003]**

(a) 3.1×10^{-18} sec (b) 2.1×10^{-15} sec (c) 1.2×10^{-12} sec (d) 1.0×10^{-9} sec

7. A particle has the wave function $\psi(x,t) = A\left[\exp(i\omega t)\cos(kx)\right]$. Which one of the following is correct?

[GATE 2003]

(a) This is an eigen state of both energy and momentum.

(b) This is an eigen state of momentum and not energy.

(c) This is an eigen state of energy and not momentum.

(d) This is not an eigen state of energy or momentum.

Commom data for Q. 8 and Q.9

The normalized wave function ψ_1 and ψ_2 correspond to the ground state and the first excited state of a particle in a potential. You are given the information that the operator $\hat{A}$ acts on the wave functions as $\hat{A}\psi_1 = \psi_2$ and $\hat{A}\psi_2 = \psi_1$ **[GATE 2003]**

8. The expectation value of A for the state $\psi = (3\psi_1 + 4\psi_2)/5$ is:

(a) –0.32 (b) 0.0 (c) 0.75 (d) 0.96

9. Which of the following are eigenfunctions of $\hat{A}^2$?

(a) ψ_1 and ψ_2 (b) ψ_2 and not ψ_1 (c) ψ_1 and not ψ_2 (d) neither ψ_1 and ψ_2

10. The time independent Schrodinger equation of a system reprersents the conservation of the

[GATE 2004]

(a) total binding energy of the system. (b) total potential energy of the system.

(c) total kinetic energy of the system. (d) total energy of the system.

11. The wave function of a spin-less particle of mass m in a one-dimensional potential $V(x)$ is $\psi(x) = A\exp\left(-\alpha^2 x^2\right)$ corresponding to an eigenvalue $E_0 = \hbar^2\alpha^2/m$. The potential $V(x)$ is

[GATE 2004]

(a) $2E_0\left(1-\alpha^2 x^2\right)$ (b) $2E_0\left(1+\alpha^2 x^2\right)$ (c) $2E_0\alpha^2 x^2$ (d) $2E_0\left(1+2\alpha^2 x^2\right)$

12. A beam of monoenergetic particles having speed v is described by the wave funcion $\psi(x) = u(x)\exp(ikx)$, where $u(x)$ is a real function. This corresponds to a current density **[GATE 2004]**

(a) $u^2(x)v$ (b) v (c) zero (d) $u^2(x)$

13. A system in a normalized state $|\psi\rangle = c_1|\alpha_1\rangle + c_2|\alpha_2\rangle$, with $|\alpha_1\rangle$ and $|\alpha_2\rangle$ representing two different eigenstates of the system, requires that the constants c_1 and c_2 must satisfy the condition

[GATE 2005]

(a) $|c_1|.|c_2| = 1$ (b) $|c_1| + |c_2| = 1$ (c) $\left(|c_1| + |c_2|\right)^2 = 1$ (d) $|c_1|^2 + |c_2|^2 = 1$

14. The parities of the wave functions (i) $\cos kx$ and (ii) $\tanh kx$ are **[GATE 2005]**

(a) odd, odd (b) even, even (c) odd, even (d) even, odd

15. If the probability that x lies between x and x+dx is $p(x)dx = ae^{-ax}$, where $0 < x < \infty$, $a > 0$, then the probability that x lies between x_1 and $x_2\ (x_2 > x_1)$ is: **[GATE 2005]**

(a) $\left(e^{-ax_1} - e^{-ax_2}\right)$ (b) $a\left(e^{-ax_1} - e^{-ax_2}\right)$ (c) $e^{-ax_2}\left(e^{-ax_1} - e^{-ax_2}\right)$ (d) $e^{-ax_1}\left(e^{-ax_1} - e^{-ax_2}\right)$

16. The wavefunction of a particle in a one-dimensional potential at time $t = 0$ is

$$\psi(x,t=0) = \frac{1}{\sqrt{5}}\left[2\psi_0(x) - \psi_1(x)\right]$$

where $\psi_0(x)$ and $\psi_1(x)$ are the ground and the first excited states of the particle with corresponding energies E_0 and E_1. The wavefunction of the particle at a time 't' is: **[GATE 2006]**

(a) $\frac{1}{\sqrt{5}} e^{i(E_0E_i)t/2\hbar}\left[2\psi_0(x) - \psi_1(x)\right]$ (b) $\frac{1}{\sqrt{5}} e^{-iE_0t/\hbar}\left[2\psi_0(x) - \psi_1(x)\right]$

(c) $\frac{1}{\sqrt{5}} e^{-iE_0t/\hbar}\left[2\psi_0(x) - \psi_1(x)\right]$ (d) $\frac{1}{\sqrt{5}}\left[2\psi_0(x)e^{-iE_0t/\hbar} - \psi_1(x)e^{-iE_1t/\hbar}\right]$

17. A particle of mass m is represented by the wavefunction $\psi(x) = Ae^{ikx}$, where *k* is the wavevector and *A* is a constant. The magnitude of the probability current density of the particle is **[GATE 2006]**

(a) $|A|^2\frac{\hbar k}{m}$ (b) $|A|^2\frac{\hbar k}{2m}$ (c) $|A|^2\frac{(\hbar k)^2}{m}$ (d) $|A|^2\frac{(\hbar k)^2}{2m}$

18. The wave function of a particle, moving in a one-dimensional time-independent potential V(x), is given by $\psi(x) = e^{-iax+b}$, where a and **b** are constants. This means that the potential $V(x)$ is of the form **[GATE 2007]**

(a) $V(x) \propto x$ (b) $V(x) \propto x^2$ (c) $V(x) = 0$ (d) $V(x) \propto e^{-ax}$

19. For a particle of mass *m* in a one-dimensional harmonic oscillator potential of the form $V(x) = \frac{1}{2}m\omega^2x^2$, the first excited energy eigen state is $\psi(x) = xe^{-ax^2}$. The value of *a* is **[GATE 2007]**

(a) $\frac{m\omega}{4\hbar}$ (b) $\frac{m\omega}{3\hbar}$ (c) $\frac{m\omega}{2\hbar}$ (d) $\frac{2m\omega}{3\hbar}$

20. A particle is in the normalized state $|\psi\rangle$ which is superposition of the energy eigenstates $|E_0 = 10\text{ eV}\rangle$ and $|E_1 = 30\text{ eV}\rangle$. The average value of energy of the particle in the state $|\psi\rangle$ is 20 eV. The state $|\psi\rangle$ is given by **[GATE 2009]**

(a) $\frac{1}{2}|E_0 = 10\text{ eV}\rangle + \frac{\sqrt{3}}{4}|E_1 = 30\text{ eV}\rangle$ (b) $\frac{1}{\sqrt{3}}|E_0 = 10\text{ eV}\rangle + \sqrt{\frac{2}{3}}|E_1 = 30\text{ eV}\rangle$

(c) $\frac{1}{2}|E_0 = 10\text{ eV}\rangle - \frac{\sqrt{3}}{4}|E_1 = 30\text{ eV}\rangle$ (d) $\frac{1}{\sqrt{2}}|E_0 = 10\text{ eV}\rangle - \frac{1}{\sqrt{2}}|E_1 = 30\text{ eV}\rangle$

Common Data for Q.21 and Q.22.

A free particle of mass m moves along the x-direction. At $t = 0$, the normalized wave function of the particle is given by $\psi(x,0) = \frac{1}{(2\pi\alpha^2)^{1/4}} \exp\left[-\frac{x^2}{4\alpha^2} + i\alpha x\right]$, where α is a real constant **[GATE 2009]**

21. The expectation value of the momentum, in this state is

(a) $\hbar\alpha$ (b) $\hbar\sqrt{\alpha}$ (c) α (d) $\frac{\hbar}{\sqrt{\alpha}}$

22. The expectation value of the particle energy is

(a) $\frac{\hbar^2}{2m}\frac{1}{2\alpha^{3/2}}$ (b) $\frac{\hbar^2}{2m}\alpha^2$ (c) $\frac{\hbar^2}{2m}\frac{4\alpha^2+1}{4\alpha^{3/2}}$ (d) $\frac{\hbar^2}{8m\alpha^{3/2}}$ (e) $\frac{\hbar^2}{2m}\frac{4\alpha^4+1}{4\alpha^3}$

23. Which of the following is an allowed wavefunction for a particle in a bound state? N is a constant and $\alpha, \beta > 0$. **[GATE 2010]**

(a) $\psi = N\frac{e^{-\alpha r}}{r^3}$ (b) $\psi = N(1 - e^{-\alpha r})$

(c) $\psi = Ne^{-\alpha x}e^{-\beta(x^2+y^2+z^2)}$ (d) $\psi = \begin{cases} \text{non-zero constant} & \text{if } r < R \\ 0 & \text{if } r > R \end{cases}$

Common data for Q. 24 and Q. 25

The wavefunction of a particle moving in free space is given by, $\psi = e^{ikx} + 2e^{-ikx}$

24. The energy of the particle is. **[GATE 2012]**

(a) $\frac{5\hbar^2k^2}{2m}$ (b) $\frac{3\hbar^2k^2}{4m}$ (c) $\frac{\hbar^2k^2}{2m}$ (d) $\frac{\hbar^2k^2}{m}$

25. The probability current density for the real part of the wavefunction is

(a) 1 (b) $\frac{\hbar k}{m}$ (c) $\frac{\hbar k}{2m}$ (d) 0

26. Consider the wave function $Ae^{ikr}(r_0/r)$, where A is the normalization constant. For $r = 2r_0$, the magnitude of probability current density up to two decimal places, in units of $(A^2\hbar k/m)$, is _______. **[GATE 2013]**

27. The state of a system is given by **[GATE 2016]**

$$|\psi\rangle = |\phi_1\rangle + 2|\phi_2\rangle + 3|\phi_2\rangle$$

where, $|\phi_1\rangle, |\phi_2\rangle$ and $|\phi_2\rangle$ form an orthonormal set. The probability of finding the system in the state $|\phi_2\rangle$ is _____________.(Give your answer upto two decimal places)

ANSWER KEY

1. (c)	2. (c)	3. (a)	4. (a&b)	5. (d)	6. (b)	7. (c)
8. (d)	9. (a)	10. (d)	11. (c)	12. (a)	13. (d)	14. (d)
15. (a)	16. (d)	17. (a)	18. (c)	19. (c)	20. (d)	21. (a)
22. (e)	23. (c)	24. (c)	25. (d)	26. (0.25)	27. (0.27 to 0.29)	

CSIR-UGC-NET Previous Years' Questions

1. The wavefunction of a particle is given by $\psi = \left(\frac{1}{\sqrt{2}}\phi_0 + i\phi_1\right)$, where ϕ_0 and ϕ_1 are the normalized eigenfnctions with energies E_0 and E_1 corresponding to the ground state and first excited state, respectively. The expectation value of the Hamiltonian in the state ψ is: **[CSIR June -2011]**

(a) $\frac{E_0}{2} + E_1$ (b) $\frac{E_0}{2} - E_1$ (c) $\frac{E_0 - 2E_1}{3}$ (d) $\frac{E_0 + 2E_1}{3}$

2. The wavefunction of a particle at t = 0 is given by $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left[|u_1\rangle + |u_2\rangle\right]$, where $|u_1\rangle$ and $|u_2\rangle$ are the normalized eigenstates with eigenvalues E_1 and E_2 respectively, ($E_2 > E_1$). The shortest time after which $|\psi(t)\rangle$ will become orthogonal to $|\psi(0)\rangle$ is **[CSIR Dec 2011]**

(a) $-\frac{\hbar\pi}{(E_2 - E_1)}$ (b) $\frac{\hbar\pi}{(E_2 - E_1)}$ (c) $\frac{\sqrt{2}\hbar\pi}{(E_2 - E_1)}$ (d) $\frac{2\hbar\pi}{(E_2 - E_1)}$

3. If a particle is represented by the normalized wave function

$$\psi(x) = \begin{cases} \dfrac{\sqrt{15}\left(a^2 - x^2\right)}{4a^{5/2}} & \text{for } -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

the uncertainty Δp in its momentum is **[CSIR Dec 2012]**

(a) $\frac{2\hbar}{5a}$ (b) $\frac{5\hbar}{2a}$ (c) $\frac{\sqrt{10}\,\hbar}{a}$ (d) $\frac{\sqrt{5}\,\hbar}{\sqrt{2}\,a}$

4. The energies in the ground state and first excited state of a particle of mass $m = \frac{1}{2}$ in a potential V(x) are –4 and –1, respectively, (in units in which $\hbar = 1$). If the corresponding wavefunctions are related by $\psi_1(x) = \psi_0(x)\sinh x$, then the ground state eigenfunction is **[CSIR Dec 2012]**

(a) $\psi_0(x) = \sqrt{\text{sech}\, x}$ (b) $\psi_0(x) = \text{sech}\, x$ (c) $\psi_0(x) = \text{sech}^2 x$ (d) $\psi_0(x) = \text{sech}^3 x$

5. If $\psi(x) = A\exp(-x^4)$ is the eigenfunction of a one dimensional Hamiltonian with eigenvalue E = 0, the potential $V(x)$ (in units where $\hbar = 2m = 1$) is **[CSIR Dec 2013]**

(a) $12x^2$ (b) $16x^6$ (c) $16x^6 + 12x^2$ (d) $16x^6 - 12x^2$

6. Suppose the Hamiltonian of a conservative system in classical mechanics is $H = \omega xp$, where ω is a constant and x and p are the position and momentum respectively. The corresponding Hamiltonian in quantum mechanics, in the coordinate representation, is **[CSIR Dec 2014]**

(a) $-i\hbar\omega\left(x\frac{\partial}{\partial x} - \frac{1}{2}\right)$ (b) $-i\hbar\omega\left(x\frac{\partial}{\partial x} + \frac{1}{2}\right)$ (c) $-i\hbar\omega x\frac{\partial}{\partial x}$ (d) $-\frac{i\hbar\omega}{2}x\frac{\partial}{\partial x}$

7. Let ψ_1 and ψ_2 denote, the normalized eigenstates of a particle with energy eigenvalues E_1 and E_2 respectively, with $E_2 > E_1$. At time $t = 0$ the particle is prepared in a state $\Psi(t=0) = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$. The shortest time T at which $\Psi(t=T)$ will be orthogonal to $\Psi(t=0)$ is **[CSIR Dec 2014]**

(a) $\frac{2\hbar\pi}{(E_2 - E_1)}$ (b) $\frac{\hbar\pi}{(E_2 - E_1)}$ (c) $\frac{\hbar\pi}{2(E_2 - E_1)}$ (d) $\frac{\hbar\pi}{4(E_2 - E_1)}$

8. A Hermitian operator $\hat{O}$ has two normalised eigenstates $|1\rangle$ and $|2\rangle$ with eigenvalues 1 and 2, respectively. The two states $|u\rangle = \cos\theta|1\rangle + \sin\theta|2\rangle$ and $|v\rangle = \cos\phi|1\rangle + \sin\phi|2\rangle$ are such that $\langle v|\hat{O}|v\rangle = 7/4$ and $\langle u|v\rangle = 0$. Which of the following are possible values of θ and ϕ? **[CSIR Dec. 2015]**

(a) $\theta = -\frac{\pi}{6}$ and $\phi = \frac{\pi}{3}$ (b) $\theta = \frac{\pi}{6}$ and $\phi = \frac{\pi}{3}$

(c) $\theta = -\frac{\pi}{4}$ and $\phi = \frac{\pi}{4}$ (d) $\theta = \frac{\pi}{3}$ and $\phi = -\frac{\pi}{6}$

9. The eigenstates corresponding to eigenvalues E_1 and E_2 of a time-independent Hamiltonian are $|1\rangle$ and $|2\rangle$ respectively. If at $t = 0$, the system is in a state $|\psi(t=0)\rangle = \sin\theta|1\rangle + \cos\theta|2\rangle$ the value of $\langle\psi(t)|\psi(t)\rangle$ at time t will be **[CSIR June 2016]**

(a) 1 (b) $\frac{(E_1\sin^2\theta + E_2\cos^2\theta)}{\sqrt{E_1^2 + E_2^2}}$

(c) $e^{iE_1t/\hbar}\sin\theta + e^{iE_2t/\hbar}\cos\theta$ (d) $e^{-iE_1t/\hbar}\sin^2\theta + e^{-iE_2t/\hbar}\cos^2\theta$

ANSWER KEY

1. (d)	2. (b)	3. (d)	4. (c)	5. (d)	6. (b)	7. (b)
8. (a)	9. ( )					

JEST Previous Years' Questions

1. The ground state (apart from normalization) of a particle of unit mass moving in a one-dimensional potential $V(x)$ is $\exp(-x^2/2)\cosh(\sqrt{2}x)$. The potential $V(x)$, in suitable units so that $\hbar = 1$, is (up to an additive constant). **[JEST 2012]**

   (a) $\frac{x^2}{2}$ (b) $\frac{x^2}{2} - \sqrt{2}x\tanh(\sqrt{2}x)$

   (c) $\frac{x^2}{2} - \sqrt{2}x\tan(\sqrt{2}x)$ (d) $\frac{x^2}{2} - \sqrt{2}x\coth(\sqrt{2}x)$

2. The wave function of a free particle in one dimension is given by $\psi(x) = A\sin x + B\sin 3x$. Then $\psi(x)$ is an eigenstate of **[JEST 2012]**

   (a) the position operator (b) the Hamiltonian
   (c) the momentum operator (d) the parity operator

3. If the expectation value of the momentum is $\langle p \rangle$ for the wavefunction $\psi(x)$, then the expectation value of momentum for the wavefunction $e^{ikx/\hbar}\psi(x)$ is **[JEST 2013]**

   (a) k (b) $\langle p \rangle - k$ (c) $\langle p \rangle + k$ (d) $\langle p \rangle$

4. The Hamiltonian operator for a two-state system is given by

   $$H = a(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|)$$

   where a is a positive number with the dimension of energy. The energy eigenstates corresponding to the larger and smaller eigenvalues respectively are **[JEST 2014]**

   (a) $|1\rangle - (\sqrt{2}+1)|2\rangle, |1\rangle + (\sqrt{2}-1)|2\rangle$ (b) $|1\rangle + (\sqrt{2}-1)|2\rangle, |1\rangle - (\sqrt{2}+1)|2\rangle$

   (c) $|1\rangle + (\sqrt{2}-1)|2\rangle, (\sqrt{2}+1)|1\rangle - |2\rangle$ (d) $|1\rangle - (\sqrt{2}-1)|2\rangle, (\sqrt{2}+1)|1\rangle + |2\rangle$

5. Consider a three-state system with energies E, E, and E–3g (where g is a constant) and respective eigenstates

   $$|\psi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix}, |\psi_2\rangle = \frac{1}{\sqrt{6}}\begin{pmatrix}1\\1\\-2\end{pmatrix}, |\psi_3\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

   If the system is initially (at t = 0) in state

   $$|\psi_i\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

   What is the probability that at a later time 't' system will be in state

   $$|\psi_f\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

   [JEST 2014]

   (a) 0 (b) $\frac{4}{9}\sin^2\left(\frac{3gt}{2\hbar}\right)$ (c) $\frac{4}{9}\cos^2\left(\frac{3gt}{2\hbar}\right)$ (d) $\frac{4}{9}\sin^2\left(\frac{E-3gt}{2\hbar}\right)$

6. A ball bounces back off earth. You are asked to solve this quantum mechanically assuming the earth is an infinitely hard sphere. Consider surface of earth as the origin implying $V(0)=\infty$ and a linear potential eleswhere (i.e. $V(x)=-mgx\ for\ x>0$). Which of the following wave function is physically admissible for this problem (with $k>0$): **[JEST 2014]**

(a) $\psi(x)=\frac{e^{-kx}}{x}$ (b) $\psi(x)=xe^{-kx^2}$ (c) $\psi(x)=-Axe^{kx}$ (d) $\psi(x)=Ae^{-kx^2}$

7. Given that ψ_1 and ψ_2 are eigenstates of a Hamiltonian with eigenvalues E_1 and E_2 respectively, what is the energy uncertainty in the state $(\psi_1+\psi_2)$? **[JEST 2015]**

(a) $-\sqrt{E_1E_2}$ (b) $\frac{1}{2}|E_1-E_2|$ (c) $\frac{1}{2}|E_1+E_2|$ (d) $\frac{1}{\sqrt{2}}|E_2-E_1|$

8. If a Hamiltonian H is given as $H=|0\rangle\langle 0|-|1\rangle\langle 1|+i(|0\rangle\langle 1|-|1\rangle\langle 0|)$, where $|0\rangle$ and $|1\rangle$ are orthonormal states, the eigenvalues of H are **[JEST 2015]**

(a) ± 1 (b) $\pm i$ (c) $\pm\sqrt{2}$ (d) $\pm i\sqrt{2}$

ANSWER KEY

1. (b) **2. (d)** **3. (c)** **4. (b)** **5. (b)** **6. (b)** **7. (b)**

8. (c)

TIFR Previous Years' Questions

1. In a quantum mechanical system, an observable A is represented by an operator $\hat{A}$. If $|\psi\rangle$ is a state of the system, but not an eigenstate of $\hat{A}$, then the quantity

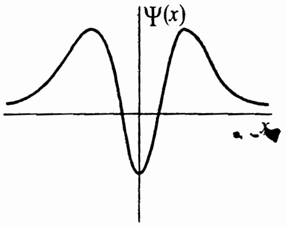
$$r = \langle\psi|\hat{A}|\psi\rangle^2 - \langle\psi|\hat{A}^2|\psi\rangle$$

satisfies the relation **[TIFR 2013]**

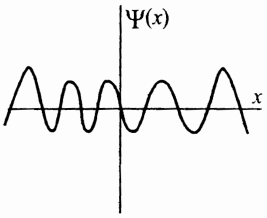
(a) $r<0$ (b) $r=0$ (c) $r>0$ (d) $r\geq 0$

2. A particle of energy E moves in one dimension under the influence of a potentials $V(x)$. If $E > V(x)$ for some range of x, which of the following graphs can represent a bound state wave function of the particle? **[TIFR 2013]**

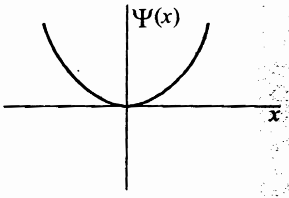
(a)



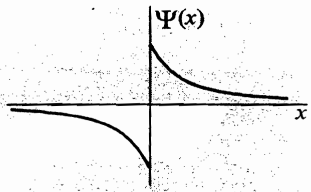
(b)



(c)



(d)



3. The state $|\psi\rangle$ of a quantum mechanical system, in a certain basis, is represented by the column vector

$$|\psi\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}.$$

The operator $\hat{A}$ corresponding to a dynamical variable A, is given, in the same basis, by the matrix

$$\hat{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

If, now, a measurement of the variable A is made on the system in the state $|\psi\rangle$, the probability that the result will be +1 is **[TIFR 2013]**

(a) $\frac{1}{\sqrt{2}}$ (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

4. A particle moving in a one dimension has the un-normalised wave function

$$\psi(x) = x\exp\left(-\frac{x^2}{\lambda^2}\right)$$

where λ is a real constant.The expectation value of its momentum is **[TIFR 2014]**

(a) $\frac{\hbar}{\lambda}\exp\left(-\frac{x^2}{\lambda^2}\right)$ (b) $\frac{\hbar^2}{\lambda^2}-2\frac{\hbar}{\lambda}$ (c) $\frac{\hbar}{\lambda}\exp(-1)$ (d) zero

5. The probability function for a variable x which assumes only positive values is **[TIFR 2014]**

$$f(x) = x\exp\left(-\frac{x}{\lambda}\right)$$

where $\lambda > 0$. The ratio $\langle x\rangle/\hat{x}$, where $\hat{x}$ is the most probable value and $\langle x\rangle$ is the mean value of the variable x, is

(a) 2 (b) $\frac{1+\lambda}{1-\lambda}$ (c) $\frac{1}{\lambda}$ (d) 1

6. A two state quantum system has two observables A and B. It is known that the observable A has eigenstates $|\alpha_1\rangle$ and $|\alpha_2\rangle$ with eigenvalues a_1 and a_2 respectively, while B has eigenstates $|\beta_1\rangle$ and $|\beta_2\rangle$ with eigenvalues b_1 and b_2 respectively, and that these eigenstates are related by **[TIFR 2015]**

$$|\beta_1\rangle = \frac{3}{5}|\alpha_1\rangle - \frac{4}{5}|\alpha_2\rangle \qquad |\beta_2\rangle = \frac{4}{5}|\alpha_1\rangle + \frac{3}{5}|\alpha_2\rangle$$

Suppose a measurement is made of the observable A and a value a_1 is obtained. If the observable B is now measured, the probability of obtaining the value b_1 will be

(a) 0.80 (b) 0.64 (c) 0.60 (d) 0.36

7. If x is a continuous variable which is uniformly distributed over the real line from $x = 0$ to $x \to \infty$ according to the distribution $f(x) = \exp(-4x)$ then the exception value of $\cos 4x$ is **[TIFR 2016]**

(a) zero (b) 1/2 (c) 1/4 (d) 1/16

8. A particle moving in one dimension is confined inside a rigid box located between $x = -a/2$ and $x = a/2$. If the particle is in its ground state $\psi(0) = \sqrt{2/a}\cos\frac{\pi x}{a}$ the quantum mechanical probability of its having a momentum p is given by **[TIFR 2016]**

(a) $\frac{8\hbar^4}{(\pi^2\hbar^2 - p^2a^2)^2}\cos^2\frac{pa}{2\hbar}$ (b) $\frac{\pi^2\hbar^4}{(\pi^2\hbar^2 - p^2a^2)^2}\sin^2\frac{pa}{2\hbar}$

(c) $\frac{2\hbar^4}{(\pi^2\hbar^2 + p^2a^2)^2}\cos^2\frac{pa}{2\hbar}$ (d) $\frac{16\hbar^4}{(\pi^2\hbar^2 - p^2a^2)^2}$

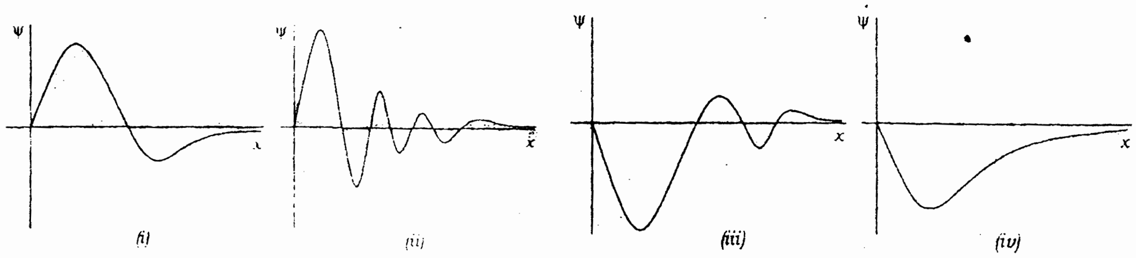
ANSWER KEY

1. (a) **2. (a)** **3. ( )** **4. (d)** **5. (a)** **6. (d)** **7. (b)**

8. (a)

Other Examinations Previous Years' Questions

1. A quantum particle in one dimension x moves under the influence of a potential that supports bound states. Plotted below are the wave functions ψ vs. x corresponding to four eigenstates of energy. Identify the wave functions corresponding to the ground state and the highest energy state among the those shown:



(i) (ii) (iii) (iv)

(a) (i) corresponds to the ground state and (iii) to the highest energy state
(b) (ii) corresponds to the ground state and (iv) to the highest energy state
(c) (iv) corresponds to the ground state and (ii) to the highest energy state.
(d) None of the ground state and (ii) corresponds to the highest energy state.

2. The time-dependent wave function of a particle of mass m moving in one dimension under the influence of a potential $V(x)$ is given to be

$$\psi(x,t)=\begin{cases}\alpha x e^{-\beta x} e^{i\gamma t/\hbar} & \text{for } x>0\\ 0 & \text{for } x<0\end{cases}$$

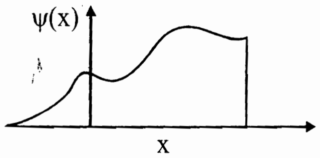
where α, β and γ are real numbers. For $x>0$, the potential V(x) is of the form $(\kappa_1$ and κ_2 are constants$)$

(a) $\kappa_1+\frac{\kappa_2}{x}$ (b) $\kappa_1+\frac{\kappa_2}{x^2}$ (c) $\kappa_1+\kappa_2 x$ (d) $\kappa_1+\kappa_2 x^2$

3. The wave function $\psi(\vec{r})$ of a particle moving in three-dimensional space has the physical dimensions of

(a) $(\text{Length})^{-3/2}$ (b) $(\text{Length})^{3/2}$ (c) $(\text{Length})^{-1}$ (d) (Length)

4. The plot of a particular function is as shown



(a) This is not acceptable as a bound state wave function because it is not zero at the origin.
(b) This is not acceptable as a bound state wave function because it is discontinuous.

(c) This is acceptable as a bound state wave function.

(d) This is not acceptable as a bound state wave function but acceptable as a scattering state wave function.

5. Let $\psi = c_1\phi_1 + c_2\phi_2 + c_3\phi_3$. $\phi_1 . \phi_2 . \phi_3$ being (orthogonal) eigen functions of H then.

(a) ψ is always an eigenfunction of H. (b) ψ can never be an eigen function of H

(c) ψ is an eigen function only if ϕ_1, ϕ_2, ϕ_3 are degenerate.

(d) ψ is an eigen function if any two of ϕ_1, ϕ_2, ϕ_3 are degenerate.

6. A is an operator which does not vary with time, and $\langle A \rangle$ the expection value of A in the state $|\psi\rangle$. Which of the following equation is correct in the Schrodinger representation.

(a) $\frac{d}{dt}\langle A \rangle = \frac{1}{i\hbar}\langle [A, H] \rangle$ (b) $\frac{d}{dt}\langle A \rangle = -\frac{1}{i\hbar}\langle [A, H] \rangle$

(c) $\frac{d}{dt}\langle A \rangle = -\frac{1}{i\hbar}\left\langle \frac{\text{[illegible]}}{H - A} \right\rangle$ (d) $\frac{d}{dt}\langle A \rangle = 0$

7. The Schordinger equation for a quantum system is a linear differential equation of the type

(a) First order in time and first order in space

(b) First order in space and second order in time.

(c) Second order in space and first order in time.

(d) Second order in space and second order in time.

8. Which of the following wave function is acceptable as the solution of the Schrodinger equation for all values of x?

(a) $\psi(x) = A\sec x$ (b) $\psi(x) = A\tan x$ (c) $\psi(x) = Ae^{x^2}$ (d) $\psi(x) = Ae^{-x^2}$

9. Which of the following wavefunctions can be bound-state solution of the one-dimensional Schrodinger equation for all values of x?

(a) $\psi = A\sec x$ (b) $\psi = A\,e^{x}$ (c) $\psi = A\,e^{x^2}$ (d) $\psi = A\,e^{-x^2}$

10. The dispersion relation for travelling waves admitted by the wave equation $\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial x^2} + \alpha\frac{\partial^4\psi}{\partial x^4} = 0$

(α : parameter) is:

(a) $\omega = \pm\sqrt{k^2 + \alpha k^4}$ (b) $\omega = \pm\sqrt{k + \alpha k^2}$

(c) $\omega = \pm\left(k^2 + \alpha k^4\right)$ (d) $\omega = \pm\left(k^2 + \alpha k^2\right)$

11. Given the Hamiltonian $\hat{H} = \hat{H}_0 + i\hat{T}$ where $\hat{H}_0$ and $\hat{T}$ are hermitian, which of the following statements are correct?

(a) $\hat{H}$ is a hermitian operator and the total probability is consrved.

(b) $\hat{H}$ is a hermitian and the total probability is not conserved.

(c) $\hat{H}$ is not a hermitian operator and the total probability is conserved.

(d) $\hat{H}$ is not a hermitian operator and the total probability is not conserved.

ANSWER KEY

1. (c) 2. (a) 3. (a) 4. (b) 5. (c) 6. (a) 7. (c)
8. (d) 9. (d) 10. (a) 11. (d)

GATE Previous Years' Questions

1. x and p are two operators which satisfy $[x,p]=i$. the operators X and P are defined as $X = x\cos\phi + p\sin\phi$ and $Y = -x\sin\phi + p\cos\phi$ for ϕ real. Then $[X, Y]$ equals **[GATE 2001]**

(a) 1 (b) –1 (c) i (d) –i

2. $\hat{A}$ and $\hat{B}$ are two quantum mechanical operators. If $[\hat{A},\hat{B}]$ stands for the commutator of $\hat{A}$ and $\hat{B}$, then $\left[[\hat{A},\hat{B}],[\hat{B},\hat{A}]\right]$ is equal to **[GATE 2002]**

(a) $\hat{A}\hat{B}\hat{A}\hat{B}-\hat{B}\hat{A}\hat{B}\hat{A}$ (b) $\hat{A}(\hat{A}\hat{B}-\hat{B}\hat{A})-\hat{B}(\hat{B}\hat{A}-\hat{A}\hat{B})$ (c) zero (d) $\left([\hat{A},\hat{B}]\right)^2$

3. The commutator $[x, P^2]$, where x and P are position and momentum operators respectively, is **[GATE 2003]**

(a) $2i\hbar P$ (b) $-i\hbar P$ (c) $2i\hbar\, x\, P$ (d) $-2i\hbar\, x\, P$

4. If $[x,p]=i\hbar$, the value of $[x^3,p]$ is **[GATE 2007]**

(a) $2i\hbar x^2$ (b) $-2i\hbar x^2$ (c) $3i\hbar x^2$ (d) $-3i\hbar x^2$

5. For a physical system, two obervables O_1 and O_2 are known to be compatible. Choose the correct implication from amongst those given below **[GATE 2008]**

(a) every eigenstate of O_1 must necessarily be an eigenstate O_2

(b) every non-degenerate eigenstate of O_1 must necessarily be an eigenstate of O_2

(c) when an observation of O_1 is carried out on an arbitrary state $|\psi\rangle$ of the physical system, a subsequent observation of O_2 leads to an unambiguous result.

(d) observation of O_1 and O_2, carried out on an arbitrary state $|\psi\rangle$ of the physical system, lead to the identical results irrespective of the order in which the observations are made.

6. $\hat{A}$ and $\hat{B}$ represents two physical characteristics of a quantum system. If $\hat{A}$ is Hermitian, then for the product $\hat{A}\hat{B}$ to be Hermitian, it is sufficient that **[GATE 2009]**

(a) $\hat{B}$ is Hermitian (b) $\hat{B}$ is anti-Hermitian

(c) $\hat{B}$ is Hermitian and $\hat{A}$ and $\hat{B}$ commute (d) $\hat{B}$ is Hermitian and $\hat{A}$ and $\hat{B}$ is anti-commute.

7. Which one of the functions given below represents the bound state eigen function of the operator $-\frac{d^2}{dx^2}$ in the region $0 \le x < \infty$, with the eigenvalue -4? **[GATE 2009]**

(a) $A_0 e^{2x}$ (b) $A_0 \cosh 2x$ (c) $A_0 e^{-2x}$ (d) $A_0 \sinh 2x$

8. Which of the following operators is Hermitian? **[GATE 2016]**

(a) $\frac{d}{dx}$ (b) $\frac{d^2}{dx^2}$ (c) $i\frac{d^2}{dx^2}$ (d) $\frac{d^3}{dx^3}$

9. If x and p are the x components of the position and the momentum operators of a particle respectively, the commutator $[x^2, p^2]$ is **[GATE 2016]**

(a) $i\hbar(xp-px)$ (b) $2i\hbar(xp-px)$ (c) $i\hbar(xp+px)$ (d) $2i\hbar(xp+px)$

10. For the parity operator P, which of the following statements is NOT true? **[GATE 2016]**

(a) $P^{\dagger} = P$ (b) $P^{2} = -P$ (c) $P^{2} = I$ (d) $P^{\dagger} = P^{-1}$

ANSWER KEY

1. (c) **2. (c)** **3. (a)** **4. (c)** **5. (a)** **6. (c)** **7. (c)**
8. (b) **9. (d)** **10. (b)**

CSIR-UGC-NET Previous Years' Questions

1. The commutator $\left[x^2, p^2\right]$ is: **[CSIR June 2012]**

(a) $2i\hbar\, xp$ (b) $2i\hbar(xp+px)$ (c) $2i\hbar px$ (d) $2i\hbar(xp-px)$

2. Which of the following is a self-adjoint operator in the spherical polar coordinate system (r,θ,φ)? **[CSIR June 2012]**

(a) $-\dfrac{i\hbar}{\sin^2\theta}\dfrac{\partial}{\partial\theta}$ (b) $-i\hbar\dfrac{\partial}{\partial\theta}$ (c) $-\dfrac{i\hbar}{\sin\theta}\dfrac{\partial}{\partial\theta}$ (d) $-i\hbar\sin\theta\dfrac{\partial}{\partial\theta}$

3. Given the usual canonical commutation relations, the commutator $[A,B]$ of $A=i(xp_y-yp_x)$ and $B=(yp_z+zp_y)$ is: **[CSIR Dec 2012]**

(a) $\hbar(xp_z-p_xz)$ (b) $-\hbar(xp_z-p_xz)$ (c) $\hbar(xp_z+p_xz)$ (d) $-\hbar(xp_z+p_xz)$

4. If the operators A and B satisfy the commutation relation [A, B] = I, where I is the identity operator, then **[CSIR June 2013]**

(a) $[e^A, B] = e^A$ (b) $[e^A, B] = [e^B, A]$
(c) $[e^A, B] = [e^{-B}, A]$ (d) $[e^A, B] = 1$

5. If A, B and C are non-zero Hermitian operators, which of the following relations must be false? **[CSIR Dec 2013]**

(a) [A, B] = C (b) AB + BA = C (c) ABA = C (d) A+ B = C

6. Let x and p denote, respectively, the coordinate and momentum operators satisfying the canonical commutation relation $[x,p]=i$ in natural units $(\hbar=1)$. Then the commutator $[x, pe^{-p}]$ is

[CSIR Dec 2014]

(a) $i(1-p)e^{-p}$ (b) $i(1-p^2)e^{-p}$ (c) $i(1-e^{-p})$ (d) ipe^{-p}

7. The waveform of a particle in one-dimension is denoted by $\psi(x)$ in the coordinate representation and by $\phi(p)=\int\psi(x)e^{-ipx/h}dx$ in the momentum representation. If the action of an operator $\hat{T}$ on $\psi(x)$ is given by $\hat{T}\psi(x)=\psi(x+a)$, where a is a constant, then $\hat{T}\phi(p)$ is given by **[CSIR June 2015]**

(a) $-\dfrac{i}{h}ap\phi(p)$ (b) $e^{-iap/h}\phi(p)$ (c) $e^{+iap/h}\phi(p)$ (d) $\left(1+\dfrac{i}{h}ap\right)\phi(p)$

8. A particle moves in one dimension in the potential $V=\dfrac{1}{2}k(t)x^2$, where $k(t)$ is a time dependent parameter.

Then $\dfrac{d}{dt}\langle V\rangle$, the rate of change of the expectation value $\langle V\rangle$ of the potential energy, is **[CSIR June 2015]**

(a) $\dfrac{1}{2}\dfrac{dk}{dt}\langle x^2\rangle+\dfrac{k}{2m}\langle xp+px\rangle$ (b) $\dfrac{1}{2}\dfrac{dk}{dt}\langle x^2\rangle+\dfrac{1}{2m}\langle p^2\rangle$

(c) $\dfrac{k}{2m}\langle xp+px\rangle$ (d) $\dfrac{1}{2}\dfrac{dk}{dt}\langle x^2\rangle$

9. Two different sets of orthogonal basis vectors $\left\{\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix}\right\}$ and $\left\{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix},\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\right\}$ are given for a two-dimensional real vector space. The matrix representation of a linear operator $\hat{A}$ in these bases are related by a unitary transformation. The unitary matrix may be chosen to be **[CSIR June 2015]**

(a) $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$ (b) $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}$

ANSWER KEY

1. (b) **2. (c)** **3. (c)** **4. (a)** **5. (a)** **6. (a)** **7. (c)**
8. (a) **9. (c)**

JEST Previous Years' Questions

1. The hermitian conjugate of the operator $(-\partial/\partial x)$ is **[JEST 2013]**

(a) $\partial/\partial x$ (b) $-\partial/\partial x$ (c) $i\partial/\partial x$ (d) $-i\partial/\partial x$

2. The operator $\left(\frac{d}{dx}-x\right)\left(\frac{d}{dx}+x\right)$ is equivalent to **[JEST 2013]**

(a) $\frac{d^2}{dx^2}-x^2$ (b) $\frac{d^2}{dx^2}-x^2+1$

(c) $\frac{d^2}{dx^2}-x\frac{d}{dx}+1$ (d) $\frac{d^2}{dx^2}-2x\frac{d}{dx}-x^2$

3. The operator A and B share all the eigenstates. Then the least possible value of the product of uncertainties $\Delta A\Delta B$ is **[JEST 2014]**

(a) $\hbar$ (b) 0 (c) $\hbar/2$ (d) Determinant (AB)

4. The adjoint of a differential operator $\frac{d}{dx}$ acting on a wavefunction $\psi(x)$ for a quantum mechanical system is:

(a) $\frac{d}{dx}$ (b) $-i\hbar\frac{d}{dx}$ (c) $-\frac{d}{dx}$ (d) $i\hbar\frac{d}{dx}$ **[JEST 2016]**

5. For operators P and Q, the commutator $[P,Q^{-1}]$ is: **[JEST 2016]**

(a) $Q^{-1}[P,Q]Q^{-1}$ (b) $-Q^{-1}[P,Q]Q^{-1}$ (c) $Q^{-1}[P,Q]Q$ (d) $-Q^{-1}[P,Q]Q^{-1}$

ANSWER KEY

1. (a) **2. (b)** **3. (b)** **4. (c)** **5. (b)**

TIFR Previous Years' Questions

1. Consider a quantum mechanical system with three linear operators $\hat{A}$, $\hat{B}$ and $\hat{C}$, which are related by

$$\hat{A}\hat{B}-\hat{C}=\hat{I}$$

where $\hat{I}$ is the unit operator. If $\hat{A}=\frac{d}{dx}$ and $\hat{B}=x$, then $\hat{C}$ must be **[TIFR 2013]**

(a) zero (b) $\frac{d}{dx}$ (c) $-x\frac{d}{dx}$ (d) $x\frac{d}{dx}$

ANSWER KEY

1. (d)

Other Examinations Previous Years' Questions

1. Two Hermitian operators A and B commute (on a finite dimensional linear vector space). This implies that
(a) every eigenvector of A is necessarily an eigenvector of B.
(b) a complete set of simultaneous eigenvector of A and B exist.
(c) every vector is necessarily an eigenvector of A and B.
(d) all eigenvectors of A are always orthogonal to those of B.

2. Let A and B be the Hermitian operators correspoinding to two physical observables of a system, such that $\left[A, B\right] \neq 0$. We may conclude in general that

(a) The uncertainty product is always $(\Delta A)(\Delta B) > \frac{h}{2}$

(b) The system can never be in a state in which $(\Delta A)(\Delta B) = 0$

(c) Neither A nor B can have any eigenstates.
(d) A and B may have one or more common eigenstate(s), but not a complete set of these.

3. Let $N = b^\dagger b$, where the operators b and $b^\dagger$. satisfy the relations $bb^\dagger + b^\dagger b = 1$ and $b^2 = 0 = (b^\dagger)^2$. The eigenvalues of N are
(a) Positive and negative integers (b) all positive integers
(c) ± 1 and 0 only (d) 0 and 1 only

4. The commutator $\left[\hat{x}, \left[\hat{x}, \hat{p}\right]\right]$ is
(a) $i\hbar\hat{x}$ (b) $-i\hbar\hat{x}$ (c) $2i\hbar\hat{x}$ (d) 0

5. An operator $\hat{F}$ is such that $\hat{F}\,\psi(x) = \psi(x+a)$. Here $\psi(x)$ is any wave function in the position representation and a is a constant. If $\hat{x}$ and $\hat{p}$ are position and momentum operators, then $\hat{F}$ is given by

(a) $\exp\left(\frac{-ia\hat{p}}{\hbar}\right)$ (b) $\exp\left(\frac{ia\hat{x}}{\hbar}\right)$ (c) $\exp\left(\frac{-ia\hat{x}}{\hbar}\right)$ (d) $\exp\left(\frac{ia\hat{p}}{\hbar}\right)$

6. If the momentum operator $\hat{p}$ is represented by multiplication by p on a wave function of a particle in one dimension, the position operator $\hat{x}$ is given by:

(a) $\hat{x} = i\hbar\frac{\partial}{\partial p}$ (b) $\hat{x} = -i\hbar\frac{\partial}{\partial p}$ (c) $\hat{x} = \frac{\partial}{\partial p}$ (d) $\hat{x} = x$ (multiplication by x)

7. For any operator $A, i(A^\dagger - A)$ is:
(a) Hermitian (b) Anti-Hermitian (c) Unitary (d) Orthogonal

8. The Hamiltonian of a three-level system is:

$$H = a\begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

The energy eigenvalues are
(a) $2a, a-a$ (b) $2a, a, 0$ (c) $2a, 0, 0$ (d) $2a, a, a$

ANSWER KEY

1. (b) 2. (b) 3. (b) 4. (d) 5. (d) 6. (a) 7. (a)

8. (a)

GATE Previous Years' Questions

1. In the first excited state of a one-dimensional harmoic oscillator with angular frequency ω, the energy eigenvalue is given by **[GATE 197]**

   (a) $\frac{1}{2}\hbar\omega$ (b) $\hbar\omega$ (c) $\frac{3}{2}\hbar\omega$ (d) $2\hbar\omega$

2. A particle of energy E is incident on a rectangular potential barrier of height V_0 and width b. **[GATE 1999]**

   (a) The reflection coefficient for $E<V_0$ tends to unity as b tends to infinity.
   (b) The reflection coefficient is zero for some specific values of $E>V_0$.
   (c) The transmission coefficient is always zero for $E<V_0$.
   (d) The transmission coefficient is zero for some specific values of $E>V_0$.

3. At t = 0, a one dimensional harmonic oscillator of frequency ω is in a state given by

$$\psi(x,0)=\frac{1}{2}u_0(x)+i\sqrt{3}/2\,u_1(x)$$

   where u_0 and u_1 are the first two normalized eigenfunctions. **[GATE 1998]**

   (a) The probability of finding the oscillator in state u_1 is $\frac{3}{4}$.

   (b) The probability of finding the oscillator in the state u_0 is $\frac{1}{2}$.

   (c) The average energy of the oscillator is $2\hbar\omega$.

   (d) The average energy of the oscillator is $(5\hbar\omega)$.

4. Let a particle move in a potential field given by **[GATE 2000]**

$$V(x)=\begin{cases}\frac{1}{2}m\omega^2x^2 & \text{for } x>0\\ \infty & \text{for } x\le 0\end{cases}$$

   The allowed energies of this particle are

   (a) $(n+1/2)\hbar\omega$ (b) $(2n+3/2)\hbar\omega$ (c) $(2n+1/2)\hbar\omega$ (d) $(n+5/2)\hbar\omega$

5. If the wave function of a particle trapped in space between x = 0 and x = L is given by $\psi(x)=A\sin\left(\frac{2\pi x}{L}\right)$, where A is a constant, for which value(s) of x will the probability of finding the particle be the maximum? **[GATE 2002]**

   (a) $\frac{L}{4}$ (b) $\frac{L}{2}$ (c) $\frac{L}{6}$ and $\frac{L}{3}$ (d) $\frac{L}{4}$ and $\frac{3L}{4}$

6. A free particle with energy E whose wave-function is a plane wave with wavelength λ enters a regionof constant potential V>0 where the wavelength of the particle is 2λ. The ratio (V/E) is: **[GATE 2003]**

   (a) 1/2 (b) 2/3 (c) 3/4 (d) 4/5

7. Which of the following is a valid normalized wave function for a particle in a one dimensional infinite potential well of width L centred at $x=0$? **[GATE 2003]**

   (a) $(2/L)\left[\cos(2\pi x/L)+\sin(2\pi x/L)\right]$ (b) $(2/L)^{1/2}\sin[n\pi x/L]$ for odd n

   (c) $(2/L)^{1/2}\cos[n\pi x/L]$ for odd n (d) $(2/L)\cos(\pi x/L)$

Common data for Q. 8 and Q.9

A particle is confined to the region $0 < x < L$ in one dimension. **[GATE 2004]**

8. If the particle is in the first excited state, then the probability of finding the particle is maximum at

(a) $x = \frac{L}{6}$ (b) $x = \frac{L}{2}$ (c) $x = \frac{L}{3}$ (d) $x = \frac{L}{4}$ and $\frac{3L}{4}$

9. If the particle is in the lowest energy state, then the probability of finding the particle in the region 0 to $\frac{L}{4}$ is:

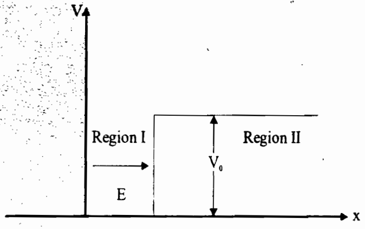
(a) $\frac{1}{4} - \frac{1}{(2\pi)}$ (b) $\frac{1}{4}$ (c) $\frac{1}{4} + \frac{1}{(2\pi)}$ (d) $\frac{1}{2}$ **[GATE 2004]**

10. A one-dimensional harmonic oscillator carrying a charge $-q$ is placed in a uniform electric field $\vec{E}$ along positive axis. The corresponding Hamiltonian operator is **[GATE 2005]**

(a) $\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 + qEx$ (b) $\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 - qEx$

(c) $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 + qEx$ (d) $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 - qEx$

11. A particle is incident with a constant energy E on a one-dimensional potential barrier as shown in the figure. **[GATE 2005]**



The wavefunctions in regions I and II are respectively
(a) decaying, oscillatory
(b) oscillatory, oscillatroy

(c) oscillatory, decaying
(d) decaying, decaying.

12. A one-dimensional harmonic oscillator is in the state

$$\psi(x) = \frac{1}{\sqrt{14}}\left[3\psi_0(x) - 2\psi_1(x) + \psi_2(x)\right]$$

Where $\psi_0(x), \psi_1(x)$ and $\psi_2(x)$ are the ground, first excited and second excited states, respectively. The probability of finding the oscillator in the ground state is: **[GATE 2006]**

(a) 0 (b) $\frac{3}{\sqrt{14}}$ (c) $\frac{9}{14}$ (d) 1

13. A free particle is moving in +x direction with a linear momentum p. The wavefunction of the particle normalised in a length L is: **[GATE 2006]**

(a) $\frac{1}{\sqrt{L}}\sin\frac{p}{h}x$ (b) $\frac{1}{\sqrt{L}}\cos\frac{p}{h}x$ (c) $\frac{1}{\sqrt{L}}e^{-i\frac{p}{h}x}$ (d) $\frac{1}{\sqrt{L}}e^{i\frac{p}{h}x}$

Common data for Q. 14, Q. 15 and Q. 16

A particle of mass m is confined in the ground state of a one-diemensional box, extending from $x = -2L$ to $x = +2L$. The wavefunction of the particle in this state is $\psi(x) = \psi_0 \cos\frac{\pi x}{4L}$. Where ψ_0 is a constant **[GATE 2007]**

14. The normalization factor ψ_0 of this wavelength is:

(a) $\sqrt{\frac{2}{L}}$ (b) $\sqrt{\frac{1}{4L}}$ (c) $\sqrt{\frac{1}{2L}}$ (d) $\sqrt{\frac{1}{L}}$

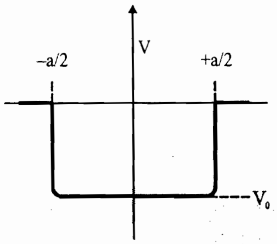
15. The energy eigenvalue corresponding to this state is:

(a) $\frac{\hbar^2\pi^2}{2mL^2}$ (b) $\frac{\hbar^2\pi^2}{4mL^2}$ (c) $\frac{\hbar^2\pi^2}{16mL^2}$ (d) $\frac{\hbar^2\pi^2}{32mL^2}$

16. The expectation value of p^2 (p is the momentum operator) in this state is:

(a) 0 (b) $\frac{\hbar^2\pi^2}{32\,L^2}$ (c) $\frac{\hbar^2\pi^2}{16\,L^2}$ (d) $\frac{\hbar^2\pi^2}{8\,L^2}$

17. There are only three bound states for a particle of mass m in a one-dimensional potential well of the form shown in the figure. The depth V_0 of the potential satisfies **[GATE 2007]**

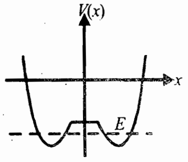


(a) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{9\pi^2\hbar^2}{2ma^2}$ (b) $\frac{\pi^2\hbar^2}{ma^2} < V_0 < \frac{2\pi^2\hbar^2}{ma^2}$

(c) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{8\pi^2\hbar^2}{ma^2}$ (d) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{50\pi^2\hbar^2}{ma^2}$

18. A particle with energy E is in a time-independent double well potential as shown in the figure. Which of the following statements about the particle is NOT correct? **[GATE 2007]**

(a) The particle will always be in a bound state.
(b) The probability of finding the particle in one well will be time-dependent.
(c) The particle will be confined to any one of the wells.
(d) The particle can tunnel from one well to the other, and back.



19. The energy levels of a particle of mass m in a potential of the form

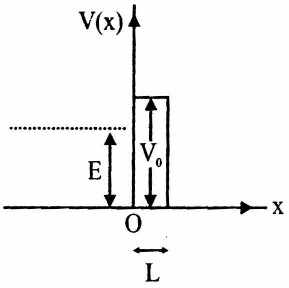
$$V(x) = \infty, \qquad x \le 0$$
$$= \frac{1}{2}m\omega^2x^2, \qquad x > 0$$

are given, in terms of quantum number $n = 0, 1, 2, 3..........$, by **[GATE 2007]**

(a) $\left(n+\frac{1}{2}\right)\hbar\omega$ (b) $\left(2n+\frac{1}{2}\right)\hbar\omega$ (c) $\left(2n+\frac{3}{2}\right)\hbar\omega$ (d) $\left(n+\frac{3}{2}\right)\hbar\omega$

Common data for Q. 20, Q. 21 and Q. 22

A beam of identical particles of mass m and energy E is incident from left on a potential barrier of width L (between $0 < x < L$) and height V_0 as shown in the figure $(E < V_0)$ **[GATE 2008]**



For $x > L$, there is tunneling with a transmission coefficient $T > 0$. Let A_0, A_R and A_T denote the amplitudes for the incident, reflected and the transmitted waves, respectively.

20. Throughout $0<x<L$, the wave -function
(a) Can be chosen to be real
(b) Is exponentially decaying.
(c) Is generally complex
(d) Is zero.

21. Let the probability current associated with the incident wave be S_0. Let R be the reflection coefficient. Then
(a) The probabiltiy current vanishes in the classically forbidden region.
(b) The probability current is TS_0 for $x > L$.
(c) For, $x<0$, the probability current is $S_0(I+R)$
(d) For $x>L$, the probability current is complex.

22. The ratio of the reflected to the incident amplitude $\frac{A_R}{A_0}$ is:

(a) $1-\frac{A_R}{A_0}$

(b) $\sqrt{1-T}$ in magnitude

(c) A real negative number

(d) $\sqrt{\left(1-\left|\frac{A_T}{A_0}\right|^2\right)\frac{E}{V_0-E}}$

23. A particle is placed in a one dimensional box of size L along the x-axis $(0<x<L)$. Which of the following is true? **[GATE 2008]**

(a) In the ground state, the probability of finding the particle in the interval $\left(\frac{L}{4},\frac{3L}{4}\right)$ is half.

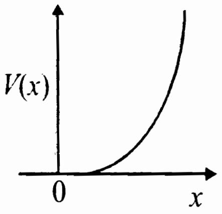
(b) In the first excited state, the probability of finding the particle in the interval $\left(\frac{L}{4},\frac{3L}{4}\right)$ is half. This also holds for states with $n = 4, 6, 8, \ldots\ldots$

(c) For an arbitory state $|\psi\rangle$, the probability of finding the particle in the left half of the well is half.

(d) In the ground state, the particle has a definite momentum.

24. A particle of mass m is confined in the potential,

$$V(x)=\begin{cases}\frac{1}{2}m\omega^2x^2 & \text{for } x>0,\\ \infty & \text{for } x\le 0\end{cases}$$



Let the wavefunction of the particle be given by

$$\psi(x)=-\frac{1}{\sqrt{5}}\psi_0+\frac{2}{\sqrt{5}}\psi_1,$$

where ψ_0 and ψ_1 are the eigenfunctions of the ground state and the first excited state respectively. The expectation value of the energy is: **[GATE 2010]**

(a) $\frac{31}{10}\hbar\omega$ (b) $\frac{25}{10}\hbar\omega$ (c) $\frac{13}{10}\hbar\omega$ (d) $\frac{11}{10}\hbar\omega$

Common data for Q.25 and Q. 26

In a one-dimensional harmonic oscillator, φ_0, φ_1 and φ_2 are respectively the ground, first and the second excited states. These three states are normalized and are orthogonal to one another. ψ_1 and ψ_2 are two states defined by $\psi_1=\varphi_0-2\varphi_1+3\varphi_2$; $\psi_2=\varphi_0-\varphi_1+\alpha\varphi_2$. Where α is a constant. **[GATE 2011]**

25. The value of α for which ψ_2 is orthogonal to ψ_1 is:

(a) 2 (b) 1 (c) −1 (d) −2

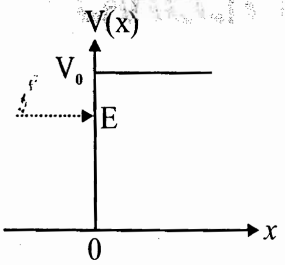
26. For the value of α determined in Q.37, the expectation value of energy of oscillator in the state ψ_2 is:

(a) $\hbar\omega$ (b) $3\hbar\omega/2$ (c) $3\hbar\omega$ (d) $9\hbar\omega/2$

27. An electron with energy E is incident from left on a potential barrier, given by

$$V(x)=0 \quad \text{for } x<0$$
$$=V_0 \quad \text{for } x>0$$

as shown in the figure **[GATE 2011]**

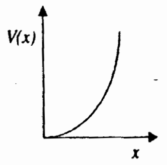


For $E<V_0$, the space part of the wavefunction for $x>0$ is of the form

(a) $e^{\alpha x}$ (b) $e^{-\alpha x}$ (c) $e^{i\alpha x}$ (d) $e^{-i\alpha x}$

where α is a real positive quantity.

28. A particle is constrained to move in a truncated harmonic potential well ($x > 0$) as shown in the figure. Which one of the following statements is CORRECT ? **[GATE 2012]**



(a) The parity of the first excited state is even (b) The parity of the ground state is even

(c) The ground state energy is $\frac{1}{2}\hbar\omega$ (d) The first excited state energy is $\frac{7}{2}\hbar\omega$

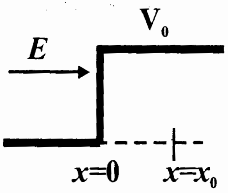
29. A one dimensional harmonic oscillator is in the superposition of number state, $|n\rangle$, given by

$$|\psi\rangle = \frac{1}{2}|2\rangle + \frac{\sqrt{3}}{2}|3\rangle$$

The average energy of the oscillator in the given state is _______ $\hbar\omega$. **[GATE 2014]**

30. Suppose a linear harmonic oscillator of frequency ω and mass m is in the state $|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\psi_0\rangle + e^{i\frac{\pi}{2}}|\psi_1\rangle\right]$ at $t = 0$ where $|\psi_0\rangle$ and $|\psi_1\rangle$ are the ground and the first excited states, respectively. The value of $\langle\psi|x|\psi\rangle$ in the units of $\sqrt{\frac{\hbar}{m\omega}}$ at $t = 0$ is_______ **[GATE 2015]**

31. A particle of mass m and energy E, moving in the positive x direction, is incident on a step potential at $x = 0$, as indicated in the figure. The height of the potential is V_0, where $V_0 > E$. At $x = x_0$, where $x_0 > 0$, the probability of the finding the electron is $\frac{1}{e}$ times the probability of finding it at $x = 0$. If $\alpha = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$, the value of x_0 is **[GATE 2016]**



(a) $\frac{2}{\alpha}$ (b) $\frac{1}{\alpha}$ (c) $\frac{1}{2\alpha}$ (d) $\frac{1}{4\alpha}$

ANSWER KEY

1. (c) 2. (a, b) 3. (a) 4. (b) 5. (d) 6. (c) 7. (c)
8. (d) 9. (a) 10. (b) 11. (c) 12. (c) 13. (d) 14. (c)
15. (d) 16. (b) 17. (a) 18. (b) 19. (c) 20. (b) 21. (b)
22. (b) 23. (c) 24. (a) 25. (c) 26. (b) 27. (b) 28. (d)
29. (3.25) 30. (0) 31. (c)

CSIR-UGC-NET Previous Years' Questions

1. Consider a particle in a one dimensional potential that satisfies $V(x)=V(-x)$. Let $|\psi_0\rangle$ and $|\psi_1\rangle$ denote the ground and the first excited states, respectively, and let $|\psi\rangle=\alpha_0|\psi_0\rangle+\alpha_1|\psi_1\rangle$ be a normalized state with α_0 and α_1 being real constants. The expectation value $\langle x\rangle$ of the position operator x in the state $|\psi\rangle$ is given by **[CSIR Dec 2011]**

(a) $\alpha_0^2\langle\psi_0|x|\psi_0\rangle+\alpha_1^2\langle\psi_1|x|\psi_1\rangle$ (b) $\alpha_0\alpha_1\left[\langle\psi_0|x|\psi_1\rangle+\langle\psi_1|x|\psi_0\rangle\right]$

(c) $\alpha_0^2+\alpha_1^2$ (d) $2\alpha_0\alpha_1$

2. Let $|0\rangle$ and $|1\rangle$ denote the normalized eigenstates corresponding to the ground and first excited state of a one dimensional harmonic oscillator. The uncertainty Δp in the state $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$, is: **[CSIR Dec 2011]**

(a) $\Delta p=\sqrt{\hbar m\omega}/2$ (b) $\Delta p=\sqrt{\hbar m\omega/2}$ (c) $\Delta p=\sqrt{\hbar m\omega}$ (d) $\Delta p=\sqrt{2\hbar m\omega}$

3. A particle in one-dimension is in the potental

$$V(x)=\begin{cases}\infty & \text{if } x<0\\ -V_0 & \text{if } 0\le x\le \ell\\ 0 & \text{if } x>\ell\end{cases}$$

If there is at least one bound state, the minimum depth of the potential is: **[CSIR June 2012]**

(a) $\frac{\hbar^2\pi^2}{8m\ell^2}$ (b) $\frac{\hbar^2\pi^2}{2m\ell^2}$ (c) $\frac{2\hbar^2\pi^2}{m\ell^2}$ (d) $\frac{\hbar^2\pi^2}{m\ell^2}$

4. Let $|0\rangle$ and $|1\rangle$ denote the normalized eigenstates corresponding to the ground and the first excited states of a one-dimensional harmonic oscillator. The uncertainty Δx in the state $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ is: **[CSIR Dec 2012]**

(a) $\Delta x=\sqrt{\hbar/2m\omega}$ (b) $\Delta x=\sqrt{\hbar/m\omega}$ (c) $\Delta x=\sqrt{2\hbar/m\omega}$ (d) $\Delta x=\sqrt{\hbar/4m\omega}$

5. Consider the normalized state $|\psi\rangle$ of a particle in a one-dimensional harmonic oscillator: $|\psi\rangle=b_1|0\rangle+b_2|1\rangle$ where $|0\rangle$ and $|1\rangle$ denote the ground and first excited states respectively, and b_1 and b_2 are real constants. The expectation value of the displacement x in the state $|\psi\rangle$ will be a minimum when **[CSIR June 2013]**

(a) $b_2=0,\ (b_1=1)$ (b) $b_2=\frac{1}{\sqrt{2}}b_1$ (c) $b_2=\frac{1}{2}b_1$ (d) $b_2=-b_1$

6. A particle is in the ground state of an infinite square well potential given by,

$$V(x)=\begin{cases}0 & \text{for } -a\le x\le a\\ \infty & \text{otherwise}\end{cases}$$

The probability to find the particle in the interval between $-\frac{a}{2}$ and $\frac{a}{2}$ is

[CSIR Dec 2013]

(a) $\frac{1}{2}$ (b) $\frac{1}{2}+\frac{1}{\pi}$ (c) $\frac{1}{2}-\frac{1}{\pi}$ (d) $\frac{1}{\pi}$

7. A particle in the infinite square well

$$V(x)=\begin{cases}0 & 0<x<a\\ \infty & \text{otherwise}\end{cases}$$

is prepared in a state with the wavefunction

$$\psi(x)=\begin{cases}A\sin^3\left(\frac{\pi x}{a}\right) & 0<x<a\\ 0 & \text{otherwise}\end{cases}$$

The expectation value of the energy of the particle is **[CSIR June 2014]**

(a) $\frac{5\hbar^2\pi^2}{2ma^2}$ (b) $\frac{9\hbar^2\pi^2}{2ma^2}$ (c) $\frac{9\hbar^2\pi^2}{10ma^2}$ (d) $\frac{\hbar^2\pi^2}{2ma^2}$

8. Let $|\psi\rangle=c_0|0\rangle+c_1|1\rangle$ (where c_0 and c_1 are constants with $c_0^2+c_1^2=1$) be a linear combination of the wavefunctions of the ground and first excited states of the one-dimensional harmonic oscillator. For what value of c_0 is the expectation value $\langle x\rangle$ a maximum? **[CSIR Dec 2014]**

(a) $\langle x\rangle=\sqrt{\frac{\hbar}{m\omega}},\ c_0=\frac{1}{\sqrt{2}}$ (b) $\langle x\rangle=\sqrt{\frac{\hbar}{2m\omega}},\ c_0=\frac{1}{2}$

(c) $\langle x\rangle=\sqrt{\frac{\hbar}{2m\omega}},\ c_0=\frac{1}{\sqrt{2}}$ (d) $\langle x\rangle=\sqrt{\frac{\hbar}{m\omega}},\ c_0=\frac{1}{2}$

9. The ground state energy of a particle of mass m in the potential $V(x)=V_0\cosh\left(\frac{x}{L}\right)$, where L and V_0 are constants (with $V_0\gg\frac{\hbar^2}{2mL^2}$) is approximately **[CSIR Dec. 2015]**

(a) $V_0+\frac{\hbar}{L}\sqrt{\frac{2V_0}{m}}$ (b) $V_0+\frac{\hbar}{L}\sqrt{\frac{V_0}{m}}$

(c) $V_0+\frac{\hbar}{4L}\sqrt{\frac{V_0}{m}}$ (d) $V_0+\frac{\hbar}{2L}\sqrt{\frac{V_0}{m}}$

10. The state of a particle of mass m in a one-dimensional rigid box in the interval 0 to L is given by the normalised wavefunction **[CSIR June 2016]**

$$\psi(x)=\sqrt{\frac{2}{L}}\left(\frac{3}{5}\sin\left(\frac{2\pi x}{L}\right)+\frac{4}{5}\sin\left(\frac{4\pi x}{L}\right)\right).$$

If its energy is measured, the possible outcomes and the average value of energy are, respectively.

(a) $\frac{h^2}{2mL^2},\ \frac{2h^2}{mL^2}$ and $\frac{73}{50}\frac{h^2}{mL^2}$

(b) $\frac{h^2}{8mL^2},\ \frac{h^2}{2mL^2}$ and $\frac{19}{40}\frac{h^2}{mL^2}$

(c) $\frac{h^2}{2mL^2},\ \frac{2h^2}{mL^2}$ and $\frac{19}{10}\frac{h^2}{mL^2}$

(d) $\frac{h^2}{8mL^2},\ \frac{2h^2}{mL^2}$ and $\frac{73}{200}\frac{h^2}{mL^2}$

11. A particle of mass m moves in one-dimension under the influence of the potential $V(x)=-\alpha\delta(x)$, where α is a positive constant. The uncertainty in the product $(\Delta x)(\Delta p)$ in its ground state is

(a) $2\hbar$ (b) $\frac{\hbar}{2}$ (c) $\frac{\hbar}{\sqrt{2}}$ (d) $\sqrt{2}\hbar$ **[CSIR June 2016]**

ANSWER KEY

1. (b) **2. (c)** **3. (a)** **4. (a)** **5. (d)** **6. (b)**

7. (c) **8. (c)** **9. (d)** **10. ( )** **11. ( )**

JEST Previous Years' Questions

1. A quantum mechanical particle in a harmonic oscillator potential has the initial wavefunction $\psi_0(x)+\psi_1(x)$, where ψ_0 and ψ_1 are the real wavefunctions in the ground and first excited states of the harmonic oscillator Hamiltonian. For convenience we take $m=\hbar=\omega=1$ for the oscillator. What is the probability density of finding the particle at x at time $t=\pi$? **[JEST 2013]**

(a) $(\psi_1(x)-\psi_0(x))^2$ (b) $(\psi_1(x))^2-(\psi_0(x))^2$

(c) $(\psi_1(x)+\psi_0(x))^2$ (d) $(\psi_1(x))^2+(\psi_0(x))^2$

2. A particle of mass m is contained in a one-dimensional infinite well extending from x = –L/2 to x = L/2. The particle is in its ground state given by $\psi_0(x)=\sqrt{2/L}\cos(\pi x/L)$. The walls of the box are moved suddenly to form a box extending from x = –L to x = L. What is the probability that the particle will be in the ground state after this sudden expansion? **[JEST 2013]**

(a) $(8/3\pi)^2$ (b) 0 (c) $(16/3\pi)^2$ (d) $(4/3\pi)^2$

3. The lowest quantum mechanical energy of a particle confined in a one-dimensional box of size L is 2 eV. The energy of the quantum mechanical ground state for a system of three non-interacting spin-1/2 particles is **[JEST 2014]**

(a) 6 eV (b) 10 eV (c) 12 eV (d) 16 eV

4. Consider a square well of depth $-V_0$ and width a with V_0a fixed. Let $V_0\to\infty$ and $a\to 0$. This potential well has **[JEST 2014]**

(a) No bound states (b) 1 bound state (c) 2 bound states (d) Infinitely many bound states

5. Consider a harmonic oscillator in the state $|\psi\rangle = e^{-\frac{|\alpha|^2}{2}}e^{\alpha a^\dagger}|0\rangle$, where $|0\rangle$ is the ground state, $a^\dagger$ is the raising operator and α is a complex number. What is the probability that the harmonic oscillator is in the n-th eigenstate $|n\rangle$? **[JEST 2015]**

(a) $e^{-|\alpha|^2}\frac{|\alpha|^{2n}}{n!}$ (b) $e^{-\frac{|\alpha|^2}{2}}\frac{|\alpha|^n}{\sqrt{n!}}$ (c) $e^{-|\alpha|^2}\frac{|\alpha|^n}{n!}$ (d) $e^{-\frac{|\alpha|^2}{2}}\frac{|\alpha|^{2n}}{n!}$

6. A particle of mass m moves in 1-dimensional potential $V(x)$, which vanishes at infinity. The exact ground state eigenfunction is $\psi(x)=A\,\text{sech}(\lambda x)$ where A and λ are constant. The ground state energy eigenvalue of this system is, **[JEST 2015]**

(a) $E=\frac{\hbar^2\lambda^2}{m}$ (b) $E=-\frac{\hbar^2\lambda^2}{m}$ (c) $E=-\frac{\hbar^2\lambda^2}{2m}$ (d) $E=\frac{\hbar^2\lambda^2}{2m}$

7. An electron confined within a thin layer of semiconductor may be treated as a free particle inside an infinitely deep one-dimensional potential well. If the difference in energies between the first and the second energy levels is δE, then the thickness of the layer is: **[JEST 2016]**

(a) $\sqrt{\frac{3\hbar^2\pi^2}{2m\delta E}}$ (b) $\sqrt{\frac{2\hbar^2\pi^2}{3m\delta E}}$ (c) $\sqrt{\frac{\hbar^2\pi^2}{2m\delta E}}$ (d) $\sqrt{\frac{\hbar^2\pi^2}{m\delta E}}$

ANSWER KEY

1. (a) 2. (a) 3. (c) 4. (b) 5. (a) 6. (c) 7. (a)

TIFR Previous Years' Questions

1. A particle $_1$ is confined in a one-dimensional infinite potential well with walls at $x = \pm 1$. Another Particle P_2 is confined in a one-dimensional infinite potential well with walls at $x = 0, 1$. Comparing the two particles, one can conclude that **[TIFR 2010]**

(a) the no. of nodes in the nth excited state of P_1 is twice that of P_2

(b) the no. of nodes in the nth excited state of P_1 is half that of P_2

(c) the energy of the nth level of P_1 is the same as that of P_2

(d) the energy of the nth level of P_1 is one quarter of that of P_2

2. The wave function Ψ of a quantum mechanical system described by a Hamiltonian $\hat{H}$ can be written as a linear combination of Φ_1 and Φ_2 which are the eigenfunctions of $\hat{H}$ with eigenvalues E_1 and E_2 respectively At $t = 0$, the system is prepared is the state $\Psi_0 = \frac{4}{5}\Phi_1 + \frac{3}{5}\Phi_2$ and then allowed to evolve with time. The wavefunction at time $T = \frac{\frac{1}{2}h}{(E_1 - E_2)}$ will be (accurate to within a phase). **[TIFR 2010]**

(a) $\frac{4}{5}\Phi_1 + \frac{3}{5}\Phi_2$ (b) Φ_1 (c) $\frac{4}{5}\Phi_1 - \frac{3}{5}\Phi_2$ (d) Φ_2

(e) $\frac{3}{5}\Phi_1 + \frac{4}{5}\Phi_2$ (f) $\frac{3}{5}\Phi_1 - \frac{4}{5}\Phi_2$

3. A particle of mass m is placed in the ground state of a one-dimensional harmonic oscillator potential of the form

$$V(x) = \frac{1}{2}kx^2$$

where the stiffness constant k can be varied externally. The ground state wavefunction has the form $\psi(x) \propto \exp(-ax^2\sqrt{k})$ where a is a constant. If, suddenly, the parameter k is changed to 4k, the probability that the particle will remain in the ground state of the new potential is **[TIFR 2011]**

(a) 0.47 (b) 0.06 (c) 0.53 (d) 0.67

(e) 0.33 (f) 0.94

4. A particle in a one-dimensional potential has the wavefunction $\psi(x) = \frac{1}{\sqrt{a}}\exp\left(\frac{-|x|}{a}\right)$ where a is a constant. It follows that for a positive constant V_0, the potential $V(x)$ is **[TIFR 2012]**

(a) $V_0 x^2$ (b) $V_0|x|$ (c) $-V_0\delta(x)$ (d) $-V_0/|x|$

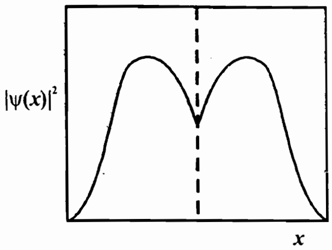
5. A harmonic oscillator has the wave function, $\psi(x,t) = \frac{1}{5}[3\varphi_0(x,t) - 2\sqrt{2}\varphi_1(x,t) + 2\sqrt{2}\varphi_2(x,t)]$

where $\varphi_n(x,t)$ is the eigenfunction belonging to the n-th energy eigenvalue $\left(n + \frac{1}{2}\right)\hbar\omega$. The expectation value $\langle E\rangle$ of energy for the state $\psi(x,t)$ is **[TIFR 2013]**

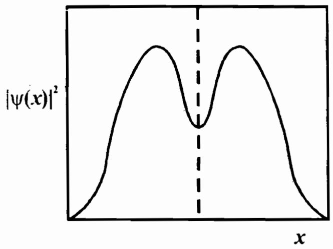
(a) $1.58\hbar\omega$ (b) $0.46\hbar\omega$ (c) $\hbar\omega$ (d) $1.46\hbar\omega$

6. A particle is confined to a one-dimensional box of length L. If a vanishingly thin but strongly repulsive partition is introduced in the exact centre of the box, and the particle is allowed to come to its ground state, then the probability density for finding the particle will appear as **[TIFR 2014]**

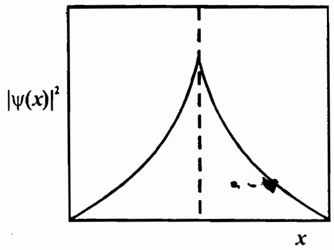
(a)



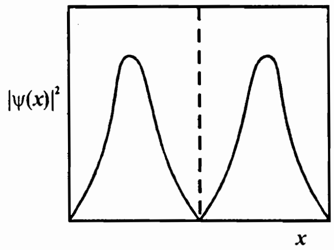
(b)



(c)



(d)



7. A particle is moving in one dimension under a potential $V(x)$ such that, for large positive values of x, $V(x) \approx kx^{\beta}$, where $k > 0$ and $\beta \geq 1$. If the wavefunction in this region has the form $\psi(x) \sim \exp(-x^{\lambda})$, which of the following is true? **[TIFR 2015]**

(a) $\lambda = \frac{\beta}{2} + 1$ (b) $\lambda = \beta$ (c) $\lambda = 2\beta - 2$ (d) $\lambda = \frac{\beta^2}{2}$

8. A one-dimensional box contains a particle whose ground state energy is ϵ. It is observed that a small disturbance causes the particle to emit a photon of energy $h\nu = 8\epsilon$, after which it is stable. Just before emission, a possible state of the particle in terms of the energy eigenstates $\{\psi_1, \psi_2, ...\}$ would be **[TIFR 2015]**

(a) $\frac{\psi_1 - \psi_2}{\sqrt{2}}$ (b) $\frac{\psi_2 + 2\psi_3}{\sqrt{5}}$ (c) $\frac{-4\psi_4 + 5\psi_5}{\sqrt{41}}$ (d) $\frac{\sqrt{2}\psi_1 - 3\psi_2 + 5\psi_5}{6}$

9. 1000 neutral spinless particles are confined in a one-dimensional box of length 100 nm. At a given instant of time, if 100 of these particle have energy $4\epsilon_0$ and the remaining 900 have energy $225\epsilon_0$, then the number of particles in the left half of the box will be approximately **[TIFR 2015]**

(a) 625 (b) 500 (c) 441 (d) 100

10. A one-dimensional quantum harmonic oscillator is in its ground state **[TIFR 2015]**

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar}$$

Two experiments, [A] and [B], are performed on the system. In [A], the frequency ω of the oscillator is suddenly doubled, while in [B] the frequency ω is suddenly halved. If p_A and p_B denote the probability in each case that the system is found in its new ground state immediately after the frequency change, which of the following is true?

(a) $p_A = \sqrt{2} p_B$ (b) $p_A = 2p_B$ (c) $2p_A = p_B$ (d) $p_A = p_B$

11. A one-dimensional harmonic oscillator of mass m and natural frequency ω is in the quantum state

$$|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \frac{i}{\sqrt{3}}|1\rangle + \frac{i}{\sqrt{3}}|2\rangle$$ **[TIFR 2016]**

at time $t = 0$, where $|n\rangle$ denotes the eigenstate with eigenvalue $\left(n + \frac{1}{2}\right)\hbar\omega$. It follows that the exception value $\langle x\rangle$ of the position operator $\hat{x}$ is

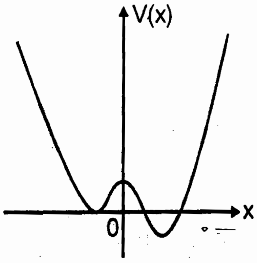
(a) $x(0)\left[\cos\omega t + \frac{1}{\sqrt{3}}\sin\omega t\right]$

(b) $x(0)[\cos\omega t - \sin\omega t]$

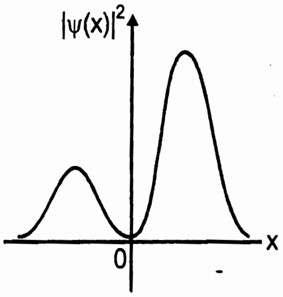
(c) $x(0)\left[\cos\omega t - \frac{1}{2}\sin\omega t\right]$

(d) $x(0)\left[\cos\omega t + \frac{1}{\sqrt{2}}\sin\omega t\right]$

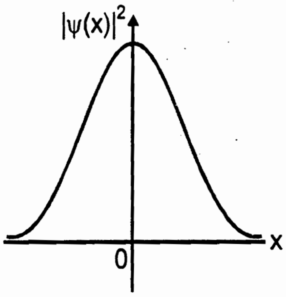
12. A particle is confined inside a one-dimensional potential well $V(x)$, as shown on the right. One of the possible probability distributions $|\psi(x)|^2$ for an energy eigenstate for this particle is **[TIFR 2016]**



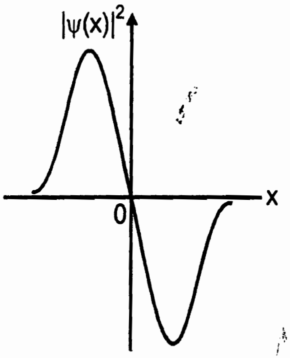
(a)



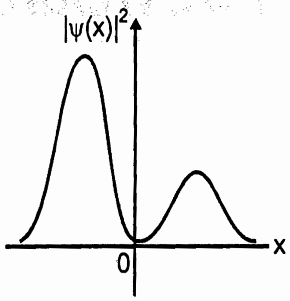
(b)



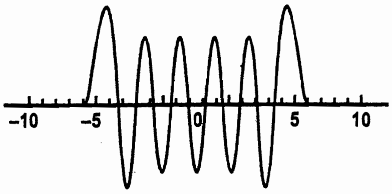
(c)



(d)

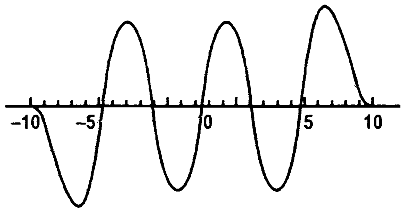


13. Two harmonic oscillators A and B are in excited eigenstates with the same excitation energy E, as measured from their respective ground state energies. The natural frequency of A is twice that of B.

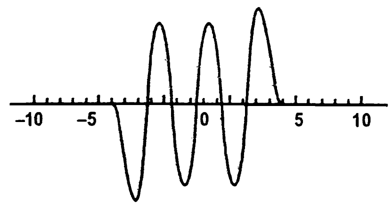


If the wavefunction of B is as sketched in the above picture, which of the following would best represent the wavefunction of A ? **[TIFR 2016]**

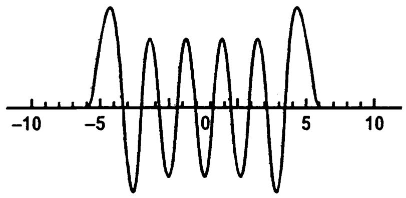
(a)



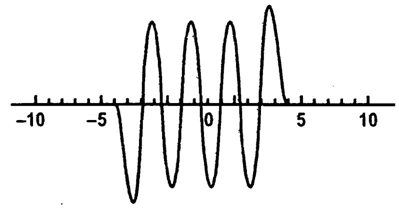
(b)



(c)



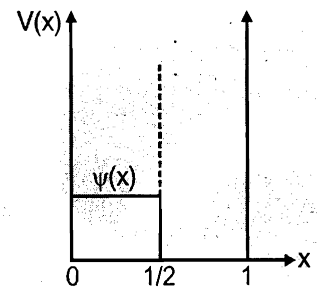
(d)



14. A particle is confined in a one-dimensional box of unit length, i.e. $L = 1$, i.e. in a potential

$$V(x) = \begin{cases} 0 & \text{if} \quad 0 < x < 1 \\ \infty & \quad \text{otherwise} \end{cases}$$

The energy eigenvalues of this particle are denoted $E_0, E_1, E_2, E_3, \ldots$



In a particular experiment, the wavefunction of this particle, at $t = 0$, is given by

$$\psi(x) = \begin{cases} \sqrt{2} & \text{if} \quad 0 < x < 1/2 \\ 0 & \quad \text{otherwise} \end{cases}$$

If, simultaneouly, i.e. at $t = 0$, a measurement of the energy of the particle is made, find $100p_3$, where p_3 is the probability that the measurement will yield the energy E_3. **[TIFR 2016]**

ANSWER KEY

1. (d)	**2. (c)**	**3. (f)**	**4. (c)**	**5. (d)**	**6. (a)**	**7. (a)**
8. (b)	**9. (b)**	**10. (d)**	**11. (d)**	**12. (a)**	**13. (b)**	**14. (0)**

Other Examinations Previous Years' Questions

1. A particle in one dimension is in the ground state (lowest energy quantum state) of the potential well given by

$$V(x) = \begin{cases} 0 & \text{for} \, |x| < \dfrac{L}{2} \\ \infty & \text{otherwise} \end{cases}$$

Let P_+ be the probability that the particle is found to move along the positive x-direction and p be the magnitude of the momentum for that state of motion. Then

(a) $P_+ = 0$ and $p = 0$ (b) $P_+ = \frac{1}{2}$ and $p = \frac{\pi}{2L}$ (c) $P_+ = \frac{1}{2}$ and $p = \frac{\pi}{L}$ (d) $P_+ = 1$ and $p = \frac{\pi}{L}$

2. For the ground state of a particle moving freely in a one-dimensional box $0 \le x \le L$ with rigid reflecting end-points, the uncertainty product $(\Delta x)(\Delta p)$ is:

(a) $h/2$ (b) $\sqrt{2}\,h$ (c) 0.09 h (d) $h/\sqrt{3}$

3. The ground state energy of a particle in an infinite square-well potential of width L is E. If the width of the wall is reduced to L/2, then the ground state energy becomes

(a) 2E (b) E/2 (c) 4E (d) E/4

4. Consider the operators a and a^+, satisfying the commutation relation $\left[a, a^\dagger\right] = I$, the unit operator. There are no normalisable eigenstates ~~of the operator~~

(a) $a^\dagger a$ (b) a (c) $a^\dagger$ (d) $\left(a^\dagger a\right)^2$

5. The first excited state of a one dimensional harmonic oscillator has eigenfunction $\phi(x) = Nxe^{-\alpha^2 x^2/2}$ where N is:

(a) $\sqrt{2\alpha^3/\sqrt{\pi}}$ (b) $2\alpha^3/\sqrt{\pi}$ (c) $\sqrt{\alpha^2/\pi}$ (d) α/π

6. The expectation value of the momentum in the ground state of one - dimensional harmonic oscillator of mass m and frequency ω is:

(a) $\dfrac{1}{\sqrt{2m\hbar\omega}}$ (b) $\sqrt{2m\hbar\omega}$ (c) 0 (d) ∞

7. The unnormalized ground state wave function of a partial confined to a one dimensional box of length L is:

(a) $A\sin\left(\dfrac{2\pi x}{L}\right)$ (b) $A\sin\left(\dfrac{\pi x}{L}\right)$ (c) $A\sin\left(\dfrac{L}{\pi x}\right)$ (d) $A\sin\left(\dfrac{L}{2\pi x}\right)$

8. A stream of particles of mass m and energy E encounter a potential step of height W (<E). In the limit W approaching E, the fraction of particles reflected from the barrrier will

(a) Appraoch zero (b) Appraoch unity (c) Diverge to infinity (d) Approach half.

9. Consider one dimensional attractive potential of arbitrary shape. Which of the following statement is true?

(a) Only one bound state exists.
(b) It has at least one bound state.
(c) It may have bound state only if the depth of the potential is greater than some critical depth.
(d) It has no bound state.

10. For a one-dimensional harmonic oscillator, let $\psi_n(x)$ denote the eigenstate of the n^{th} quantum state. Which one of the statements is correct?

(a) Parity of $\psi_n(x)$ is even when n is even. (b) Parity of $\psi_n(x)$ is odd when n is even.

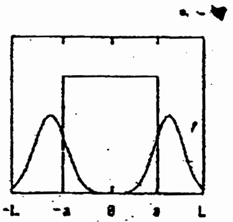
(c) Parity of $\psi_n(x)$ is even where n is odd (d) $\psi_n(x)$ has mixed parity.

11. The schematic form of the ground state wave-function in the one-dimensional potential.

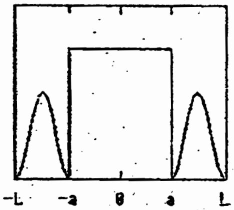
$$V(x)=\begin{cases} V_0, & \text{for } a \ge |x| \\ 0, & \text{for } a < |x| < L \\ \infty, & \text{for } |x| \ge L \end{cases}$$

where V_0, a positive constant, is given by

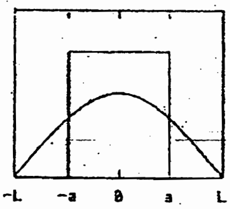
(a)



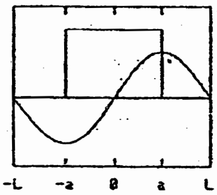
(b)



(c)



(d)



12. A particle of mass m in one dimension moves in a potential $V(x)=\frac{k}{2a}\left(1-e^{-ax^2}\right)$, where k and α are positive constants. If the particle executes small oscillations around x = 0, its quantum mechanical zero-point energy is

(a) $\frac{1}{2}\sqrt{\frac{\hbar^2 k}{m}}$ (b) $\frac{1}{2}\sqrt{\frac{\hbar^2 k}{m\alpha}}$ (c) $\frac{1}{2}\sqrt{\frac{\hbar^2 m}{k}}$ (d) $\frac{1}{2}\sqrt{\frac{\hbar^2 m\alpha}{k}}$

13. A particle is confined to a one dimensional harmonic oscillator potential in the region $0 < x < \infty$. At $x = 0$ there is n infinite barrier. The energy levels of the particle are separated by

(a) $\frac{1}{2}\hbar\omega$ (b) $\hbar\omega$ (c) $\frac{3}{2}\hbar\omega$ (d) $2\hbar\omega$.

14. A function of the form $\left(x^2-2\right)e^{-ax^2}$ can represent the state of a one dimensional quantum mechanical system in

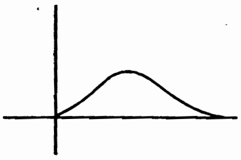
(a) its ground state (b) its first excited state

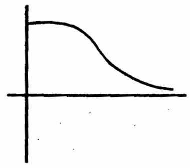
(c) its second exited state (d) a superposition of first and second excited state

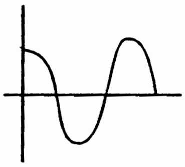
15. A system is in the state $\psi = \frac{1}{2}\left(\psi_1 + \sqrt{3}\,\psi_2\right)$ where ψ_1 and ψ_2 denote energy eigenstates corresponding to the eigenvalues E_1 and E_2 respectively. When a measurement of energy is made, the probability of obtaining the value E_1 is

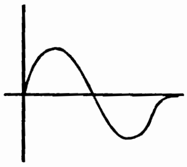
(a) $\sqrt{3}/2$ (b) $3/4$ (c) $1/2$ (d) $1/4$

16. Consider a particle in a one dimensional potential $V(x) = \begin{cases} x^2 \text{ for } x > 0 \\ \infty \text{ for } x \le 0 \end{cases}$. The schematic form of the ground state wave function is as follows"

(a) 

(b) 

(c) 

(d) 

17. $\psi_0(x), \psi_1(x)$ and $\psi_2(x)$ are the simple harmonic oscillator wave functions corresponding to energies $\frac{1}{2}\hbar\omega, \frac{3}{2}\hbar\omega$ and $\frac{5}{2}\hbar\omega$ respectively. At $t = 0$ a particle is in the state

$$\psi(x) = \frac{i}{3}\psi_0(x) - \frac{\sqrt{5}}{3}\psi_1(x) + \frac{1}{\sqrt{3}}\psi_2(x).$$

If an energy measurement is made the most probable outcome is

(a) $\frac{5}{2}\hbar\omega$ (b) $\frac{1}{2}\hbar\omega$ (c) $\frac{9}{2}\hbar\omega$ (d) $\frac{3}{2}\hbar\omega$

18. One-dimensional barrier is given by the potential $V(x) = \begin{cases} U \text{ for } |x| < 1 \\ 0 \text{ for } |x| > 1 \end{cases}$ where U is a positive constant. Particles arriving from the left with an energy $E < U$ impinge on the barrier. The general form of the wave function in the region $|x| < 1$ is

(a) $a\,e^{kx} + b\,e^{-kx}$ (b) $a\,e^{-ikx} + b\,e^{-kx}$ (c) $a\,e^{kx} + b\,e^{-ikx}$ (d) $a\,e^{ikx} + b\,e^{-ikx}$

19. Let a particle move in a potential field given by

$$V(x) = \begin{Bmatrix} \left(\frac{1}{2}\right) m\omega^2 x^2 & \text{for } x > 0 \\ \infty & \text{for } x \le 0 \end{Bmatrix}$$

The allowed energies of this particle are

(a) $(n + 1/2)\hbar\omega$ (b) $(2n + 3/2)\hbar\omega$ (c) $(2n + 1/2)\hbar\omega$ (d) $(n + 5/2)\hbar\omega$

where 'n' is a non-negative odd integer.

20. The ground state energy of a particle in the one-dimensional potential

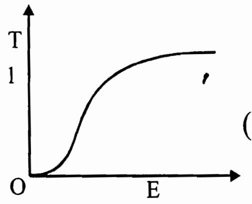
$$V(x)=\begin{cases}\frac{1}{2}m\omega^2x^2 & \text{for } x>0\\ \infty & \text{for } x<0\end{cases}\quad \text{is equal to}$$

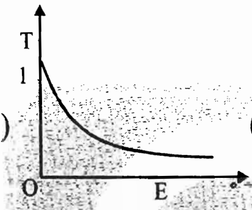
(a) $\hbar\omega$ (b) $\frac{3}{2}\hbar\omega$ (c) $2\hbar\omega$ (d) $\frac{5}{2}\hbar\omega$

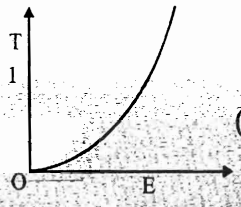
21. The particle in a box has a ground state wavefunction given by $\phi(x)=\frac{1}{\sqrt{a}}\cos\left(\frac{\pi x}{2a}\right)$. The expectation value of x is:

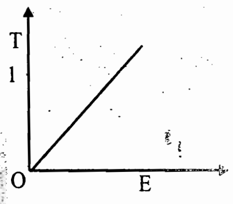
(a) 0 (b) $\frac{a^2}{3}$ (c) $\frac{a^2}{\pi^2}$ (d) 1

22. The variation of transmission probability with energy E, of the particles hitting a potential barrier of finite height, is represented as

(a)


(b)


(c)


(d)


23. Consider a particle trapped inside an infinite potential well. If an observer makes a measurement to detect the particle just outside the well then

(a) He will always find the particle (b) He will never find the particle

(c) He will find the particle half the timer

(d) He will find the particle only if the measurement is performed for sufficiently long time.

24. The matter wave corresponding to a free particle moving in one dimension satisfies the time-dependent Schrodinger wave equation $i\hbar\frac{\delta\psi(x,t)}{\delta t}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$

If $\hbar k$ is the momentum, then the wavepacket formed out of the matter waves moves with a group velocity V_g equal to

(a) $\frac{\hbar}{2m}k^2$ (b) $\frac{\hbar^2}{2m}k^2$ (c) $\frac{\hbar^2}{2m}k$ (d) $\frac{\hbar k}{m}$

25. A particle limited to the x-axis has the wave function $\psi=\sqrt{3}\,x$ between $x=0$ and $x=1$; $\psi=0$ elsewhere.

The expectation value $\langle x\rangle$ of the particle's position is:

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{3}{4}$

ANSWER KEY

1. (c) 2. (c) 3. (c) 4. (b&c) 5. (a) 6. (c) 7. (b)
8. (b) 9. (b) 10. (a) 11. (a) 12. (a) 13. (d) 14. (c)
15. (d) 16. (a) 17. (d) 18. (a) 19. (a) 20. (b) 21. (a)
22. (a) 23. (b) 24. (d) 25. (d)

GATE Previous Years' Questions

1. A system is known to be in a state described by the wave function **[GATE 1998]**

$$\psi(\theta,\phi)=\frac{1}{\sqrt{30}}\left(5Y_4^0+Y_6^0-2Y_6^3\right)$$

where $Y_\ell^m(\theta,\phi)$ are the spherical harmonics. The probability of finding the system in a state with $m=0$ is

(a) zero (b) 2/15 (c) 1/4 (d) 13/15

2. In a Stern-Gerlach experiment the atomic beam whose angular momentum state is to be determined, must travel through **[GATE 2000]**

(a) homogeneous radio frequency magnetic field
(b) homogeneous static magnetic field
(c) inhomogeneous static magnetic field
(d) inhomogeneous radio frequency magnetic field

3. A spin ½ particle with g>0, is subjected to a magnetic field $\vec{H}=H_0\hat{z}$. If the quantization axis is along $+\hat{z}$, then the minimum energy eigenstate is given by, **[GATE 2000]**

(a) $|\downarrow\rangle$ (b) $\frac{1}{\sqrt{2}}|\uparrow\rangle-|\downarrow\rangle$ (c) $|\uparrow\rangle$ (d) $\frac{1}{\sqrt{2}}|\uparrow\rangle+|\downarrow\rangle$

4. If a spin 1 particle is in the state $|m=0\rangle$ with respect to a quantization axis $\hat{n}$, which of the following is correct? **[GATE 2000]**

(a) $\langle\vec{S}\rangle=0$ (b) $\langle\vec{S}\rangle=\hat{n}$ (c) $\langle\vec{S}\rangle=\sqrt{2}\hat{n}$ (d) $\langle\vec{S}\rangle=-\hat{n}$

5. The Hamiltonian of a particle is given by $H=\frac{p^2}{2m}+V(|\vec{r}|)+\phi(|\vec{r}|)\vec{L}\cdot\vec{S}$, where $\vec{S}$ is the spin, $V(|\vec{r}|)$ and $\phi(|\vec{r}|)$ are potential functions and $\vec{L}=\vec{r}\times\vec{p}$ is the angular momentum. The Hamiltonian does NOT commute with **[GATE 2004]**

(a) $\vec{L}+\vec{S}$ (b) $\vec{S}^2$ (c) L_z (d) $\vec{L}^2$

6. Let $\vec{L}=(L_x,L_y,L_z)$ denote the orbital angular momentum operators of a particle and let $L_+=L_x+iL_y$ and $L_-=L_x-iL_y$. The particle is in an eigenstate of L^2 and L_z eigenvalues $\ell(\ell+1)\hbar^2$ and $\hbar\ell$ respectively. The expectation value of $L_+\,L_-$ in this state is: **[GATE 2001]**

(a) $\ell\hbar^2$ (b) $2\ell\hbar^2$ (c) 0 (d) $\ell\hbar$

7. An electron is in a state spin wave function $\phi_s=\begin{pmatrix}\sqrt{3}/2\\ 1/2\end{pmatrix}$ in the S_z representation. What is the probability of finding the z-componenet of its spin along the $-\hat{z}$ direction? **[GATE 2002]**

(a) 0.75 (b) 0.50 (c) 0.35 (d) 0.25

8. In a Stern-Gerlach experiment, the magnetic field is in +z direction. A particle comes out of this experiment in $|+\hat{Z}\uparrow\rangle$ state. Which of the following statements is true? **[GATE 2002]**

(a) The particle has a definite value of the y-component of the spin angular momentum.
(b) The particle has a definite value of the square of the spin angular momentum.
(c) The particle has a definite value of x-components of spin angular momentum.
(d) The particle has definite values of x- and y-components of spin angular momentum.

9. A spin half particle is in the state $S_z=\hbar/2$. The expectation values of S_x, S_x^2, S_y, S_y^2 are given by **[GATE 2003]**

(a) $0,0,\hbar^2/4,\hbar^2/4$ (b) $0,\hbar^2/4,\hbar^2/4,0$ (c) $0,\hbar^2/4,0,\hbar^2/4$ (d) $\hbar^2/4,\hbar^2/4,0,0$

10. The commutator, $\left[L_z, Y_{\ell m}(\theta,\phi)\right]$ where L_z is the *z* component of the orbital angular momentum and $Y_{\ell m}(\theta,\phi)$ is a spherical harmonic, is: **[GATE 2005]**

(a) $\ell(\ell+1)\hbar$ (b) $-m\hbar$ (c) $m\hbar$ (d) $+\ell\hbar$

11. For a spin 1/2 particle, the expectation value of $S_xS_yS_z$. where S_x, S_y and S_z are spin operators, is **[GATE 2005]**

(a) $\dfrac{i\hbar^3}{8}$ (b) $-\dfrac{i\hbar^3}{8}$ (c) $\dfrac{i\hbar^3}{16}$ (d) $-\dfrac{i\hbar^3}{16}$

12. The commutator $\left[L_x, y\right]$, where L_x is the x-component of the angular momentum operator and y is the y-component of the position operator, is equal to: **[GATE 2006]**

(a) 0 (b) $i\hbar x$ (c) $i\hbar y$ (d) $i\hbar z$

13. Which one of the following relations is true for Pauli matrices? **[GATE 2006]**

(a) $\sigma_x\sigma_y=\sigma_y\sigma_x$ (b) $\sigma_x\sigma_y=\sigma_z$ (c) $\sigma_x\sigma_y=i\sigma_z$ (d) $\sigma_x\sigma_y=-\sigma_y\sigma_x$

14. Three operators X, Y and Z satisfy the commutation relations

$$[X,Y]=i\hbar Z, [Y,Z]=i\hbar X \text{ and } [Z,X]=i\hbar Y$$

The set of all possible eigenvalues of the operator Z, in units of $\hbar$, is **[GATE 2007]**

(a) $\{0,\pm1,\pm2,\pm3,\ldots\ldots.\}$ (b) $\left\{\frac{1}{2},1,\frac{3}{2},2,\frac{5}{2},\ldots\ldots.\right\}$

(c) $\left\{0,\pm\frac{1}{2},\pm1,\pm\frac{3}{2},\pm2,\pm\frac{3}{2},\ldots\ldots.\right\}$ (d) $\left\{-\frac{1}{2},+\frac{1}{2}\right\}$

15. Let $|\psi_0\rangle$ denote the ground state of the hydrogen atom. Choose the correct statement from those given below: **[GATE 2008]**

(a) $\left[L_x,L_y\right]|\psi_0\rangle=0$ (b) $J^2|\psi_0\rangle=0$ (c) $\vec{L}\cdot\vec{S}|\psi_0\rangle\neq0$ (d) $\left[S_x,S_y\right]|\psi_0\rangle=0$

16. $\sigma_i\,(i=1,2,3)$ represents the Pauli spin matrices. Which one of the following is NOT true? **[GATE 2009]**

(a) $\sigma_i\sigma_j+\sigma_j\sigma_i=2\delta_{ij}$ (b) $\mathrm{Tr}(\sigma_i)=0$

(c) The eigenvalues of σ_i are ±1 (d) $\det(\sigma_i)=1$

17. The spin function of a free particle, in the basis in which S_z is diagonal, can be written as $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ with eigenvalues $+\frac{\hbar}{2}$ and $\frac{\hbar}{2}$, respectively. In the given basis, the normalized eigenfunction of S_y with eigenvalue $-\frac{\hbar}{2}$

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix}0\\i\end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix}i\\0\end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix}i\\1\end{pmatrix}$ **[GATE 2009]**

18. For a spin *s* particle, in the eigen basis of S^2, S_z the expectation value $\langle sm | S_x^2 | sm \rangle$ is

[GATE 2010]

(a) $\frac{\left[s(s+1)-m^2\right]\hbar^2}{2}$ (b) $\left[s(s+1)-2m^2\right]\hbar^2$

(c) $\left[s(s+1)-m^2\right]\hbar^2$ (d) $m^2\hbar^2$

19. If L_x, L_y and L_z are the x, y and z components of angular momentum operator L, the commutator $\left[L_x L_y, L_z\right]$ is equal to **[GATE 2011]**

(a) $i\hbar\left(L_x^2+L_y^2\right)$ (b) $i\hbar\left(L_x^2-L_y^2\right)$ (c) $2i\hbar L_z$ (d) zero

20. Which one of the following commutation relations is not correct? **[GATE 2013]**

(a) $\left[L^2, L_z\right]=0$ (b) $\left[L_x, L_y\right]=i\hbar L_z$ (c) $\left[L_z, L_+\right]=\hbar L_+$ (d) $\left[L_z, L_-\right]=\hbar L_-$

21. A spin-half particle is in a linear superposition $0.8|\uparrow\rangle+0.6|\downarrow\rangle$ of its spin-up and spin-down states. If $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigen states of σ_z then what is the expectation value, up to one decimal place, of the operator $10\sigma_z+5\sigma_x$? .... **[GATE 2013]**

22. If $\vec{L}$ is the orbital angular momentum and $\vec{S}$ is the spin angular momentum, then $\vec{L}\cdot\vec{S}$ does not commute with **[GATE 2014]**

(a) S_z (b) L^2 (c) S^2 (d) $\left(\vec{L}+\vec{S}\right)^2$

23. ψ_1 and ψ_2 are two orthogonal states of a spin ½ system. it is given that

$$\psi_1=\frac{1}{\sqrt{3}}\begin{pmatrix}1\\0\end{pmatrix}+\sqrt{\frac{2}{3}}\begin{pmatrix}0\\1\end{pmatrix}$$ **[GATE 2014]**

where $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ represents the spin-up and spin-down states, respectively. When the system is in the state ψ_2, its probability to be in the spin-up state is ________________

24. If L_+ and L_- are the angular momentum ladder operators, then the expectation value of $\left(L_+L_-+L_-L_+\right)$ in the state $|l=1, m=1\rangle$ of an atom is __________ $\hbar^2$. **[GATE 2014]**

25. Let $\vec{L}$ and $\vec{p}$ be the angular and linear momentum operators respectively, for a particle. The commutator $\left[L_x, p_y\right]$ gives **[GATE 2015]**

(a) $-i\hbar p_x$ (b) 0 (c) $i\hbar p_x$ (d) $i\hbar p_z$

26. An operator for a spin-1/2 particle is given by $\hat{A}=\lambda\vec{\sigma}.\vec{B}$, where $\vec{B}=\frac{B}{\sqrt{2}}(\hat{x}+\hat{y})$, $\vec{\sigma}$ denotes Pauli spin matrices and λ is a constant. The eigenvalue of $\hat{A}$ are **[GATE 2015]**

(a) $\pm\frac{\lambda B}{\sqrt{2}}$ (b) $\pm\lambda B$ (c) $0,\lambda B$ (d) $0,-\lambda B$

27. σ_x,σ_y and σ_z are the Pauli matrices. The expression $2\sigma_x\sigma_y+\sigma_y\sigma_x$ is equal to **[GATE 2016]**

(a) $-3i\sigma_z$ (b) $-i\sigma_z$ (c) $i\sigma_z$ (d) $3i\sigma_z$

28. Let $|\ell,m\rangle$ be the simultaneous eigenstates of L^2 and L_z. Here $\vec{L}$ is the angular momentum operator with Cartesian components (L_x,L_y,L_z), ℓ is the angular momentum quantum number and m is the azimuthal quantum number. The value of $\langle 1,0|(L_x+iL_y)|1,-1\rangle$ is **[GATE 2016]**

(a) 0 (b) $\hbar$ (c) $\sqrt{2}\,\hbar$ (d) $\sqrt{3}\,\hbar$

ANSWER KEY

1. (d) **2. (c)** **3. (c)** **4. (a)** **5. (c)** **6. (b)** **7. (d)**
8. (b) **9. (c)** **10. (c)** **11. (a)** **12. (d)** **13. (c)** **14. (c)**
15. ( ) **16. (d)** **17. (a)** **18. (a)** **19. (b)** **20. (d)** **21. (7.6)**
22. (a) **23. (0.67)** **24. (2)** **25. (d)** **26. (b)** **27. (c)** **28. (c)**

CSIR-UGC-NET Previous Years' Questions

1. The Hamiltonian of an electron in a constant magnetic field $\vec{B}$ is given by $H = \mu\vec{\sigma}.\vec{B}$ where μ is a positive constant and $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ denotes the Pauli matrices. Let $\omega = \mu B/\hbar$ and I be the 2×2 unit matrix. Then the operator $e^{iHt/\hbar}$ simplifies to **[CSIR June 2011]**

(a) $I\cos\frac{\omega t}{2} + \frac{i\vec{\sigma}.\vec{B}}{B}\sin\frac{\omega t}{2}$ (b) $I\cos\omega t + \frac{i\vec{\sigma}.\vec{B}}{B}\sin\omega t$

(c) $I\sin\omega t + \frac{i\vec{\sigma}.\vec{B}}{B}\cos\omega t$ (d) $I\sin 2\omega t + \frac{i\vec{\sigma}.\vec{B}}{B}\cos 2\omega t$

2. The component along an arbitrary direction $\hat{n}$, with direction cosines (n_x, n_y, n_z), of the spin of a spin-½ particle is measured. The result is: **[CSIR June 2012]**

(a) 0 (b) $\pm\frac{\hbar}{2}n_z$ (c) $\pm\frac{\hbar}{2}(n_x, n_y, n_z)$ (d) $\pm\frac{\hbar}{2}$

3. In a basis in which the z-component S_z of the spin is diagonal, an electron is in a spin state

$$\psi = \begin{pmatrix} (1+i)/\sqrt{6} \\ \sqrt{2/3} \end{pmatrix}$$

The probabilities that a measurement of S_z will yield the values $\hbar/2$ and $-\hbar/2$ are, respectively **[CSIR June 2013]**

(a) 1/2 and 1/2 (b) 2/3 and 1/3 (c) 1/4 and 3/4 (d) 1/3 and 2/3

4. A spin $-\frac{1}{2}$ particle is in the state $\chi = \frac{1}{\sqrt{11}}\begin{pmatrix} 1+i \\ 3 \end{pmatrix}$ in the eigenbasis of S^2 and S_z. If we measure S_z the probabilities of getting $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$, respectively, are **[CSIR Dec 2013]**

(a) $\frac{1}{2}$ and $\frac{1}{2}$ (b) $\frac{2}{11}$ and $\frac{9}{11}$ (c) 0 and 1 (d) $\frac{1}{11}$ and $\frac{3}{11}$

5. Let ψ_{nlm} denote the eigenfunctions of a Hamiltonian for a spherically symmetric potential $V(r)$. The expectation value of L_z in the state

$\psi = \frac{1}{6}\left[\psi_{200} + \sqrt{5}\,\psi_{210} + \sqrt{10}\,\psi_{21-1} + \sqrt{20}\,\psi_{211}\right]$ is **[CSIR Dec 2013]**

(a) $-\frac{5}{18}\hbar$ (b) $\frac{5}{6}\hbar$ (c) $\hbar$ (d) $\frac{5}{18}\hbar$

6. The expectation value of the x-component of the orbital angular momentum L_x in the state $\psi = \frac{1}{5}\left[3\psi_{2,1,-1} + \sqrt{5}\psi_{2,1,0} - \sqrt{11}\psi_{2,1,+1}\right]$ (where ψ_{nlm} are the eigenfunctions in usual notation), is

[CSIR Dec 2013]

(a) $-\frac{\hbar\sqrt{10}}{25}(\sqrt{11}-3)$ (b) 0 (c) $\frac{\hbar\sqrt{10}}{25}(\sqrt{11}+3)$ (d) $\hbar\sqrt{2}$

7. A particle is prepared in a simultaneous eigenstate of L^2 and L_z. If $\ell(\ell+1)\hbar^2$ and $m\hbar$ are respectively the eigenvalues of L^2 and L_z, then the expectation value $\langle L_x^2 \rangle$ of the particle in this state satisfies

[CSIR Dec 2013]

(a) $\langle L_x^2 \rangle = 0$

(b) $0 \le \langle L_x^2 \rangle \le \ell^2\hbar^2$

(c) $0 \le \langle L_x^2 \rangle \le \frac{\ell(\ell+1)\hbar^2}{3}$

(d) $\frac{\ell\hbar^2}{2} \le \langle L_x^2 \rangle \le \frac{\ell(\ell+1)\hbar^2}{2}$

8. Consider the normalized wavefunction $\phi = a_1\psi_{11} + a_2\psi_{10} + a_3\psi_{1-1}$ where ψ_{lm} is a simultaneous normalized eigenfunction of the angular momentum operators L^2 and L_z, with eigenvalues $l(l+1)\hbar^2$ and $m\hbar$ respectively. If ϕ is an eigenfunction of the operator L_x with eigenvalue $\hbar$, then

[CSIR Dec 2014]

(a) $a_1 = -a_3 = \frac{1}{2}, a_2 = \frac{1}{\sqrt{2}}$

(b) $a_1 = a_3 = \frac{1}{2}, a_2 = \frac{1}{\sqrt{2}}$

(c) $a_1 = a_3 = \frac{1}{2}, a_2 = -\frac{1}{\sqrt{2}}$

(d) $a_1 = a_2 = a_3 = \frac{1}{\sqrt{3}}$

9. Let $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$, where $\sigma_1, \sigma_2, \sigma_3$ are the Pauli matrices. If $\vec{a}$ and $\vec{b}$ are two arbitrary constant vectors in three dimensions, the commutator $[\vec{a}\cdot\vec{\sigma}, \vec{b}\cdot\vec{\sigma}]$ is equal to (in the following I is the identity matrix)

[CSIR Dec 2014]

(a) $(\vec{a}\cdot\vec{b})(\sigma_1+\sigma_2+\sigma_3)$

(b) $2i(\vec{a}\times\vec{b})\cdot\vec{\sigma}$

(c) $(\vec{a}\cdot\vec{b})I$

(d) $|\vec{a}||\vec{b}|I$

10. If L_i are the components of the angular momentum operator $\vec{L}$, then the operator $\sum i = 1,2,3\left[\left[\vec{L}, L_i\right], L_i\right]$ equals

[CSIR June 2015]

(a) $\vec{L}$ (b) $2\vec{L}$ (c) $3\vec{L}$ (d) $-\vec{L}$

11. Let ψ_{nlm} denote the eigenstates of a hydrogen atom in the usual notation. The state **[CSIR Dec. 2015]**

$\frac{1}{5}[2\psi_{200} - 3\psi_{211} + \sqrt{7}\psi_{210} - \sqrt{5}\psi_{21-1}]$ is an eigenstate of

(a) L^2, but not of the Hamiltonian or L_z

(b) the Hamiltonian, but not of L^2 or L_z

(c) the Hamiltonian, L^2 and L_z

(d) L^2 and L_z, but not of the Hamiltonian

12. The Hamiltonian for a spin-$\frac{1}{2}$ particle at rest is given by $H = E_0(\sigma_z + \alpha\sigma_x)$, where σ_x and σ_z are Pauli spin matrices and E_0 and α are constants. The eigenvalues of this Hamiltonian are **[CSIR Dec. 2015]**

(a) $\pm E_0\sqrt{1+\alpha^2}$ (b) $\pm E_0\sqrt{1-\alpha^2}$

(c) E_0 (doubly degenerate) (d) $E_0\left(1 \pm \frac{1}{2}\alpha^2\right)$

13. The product of the uncertainties $(\Delta L_x)(\Delta L_y)$ for a particle in the state $a|1,1\rangle + b|1,-1\rangle$ (where $|l,m\rangle$ denotes an eigenstate of L^2 and L_z) will be a minimum for **[CSIR Dec. 2015]**

(a) $a = \pm ib$ (b) $a = 0$ and $b = 1$

(c) $a = \frac{\sqrt{3}}{2}$ and $b = \frac{1}{2}$ (d) $a = \pm b$

14. If $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$ are the components of the angular momentum operator in three dimensions, the commutator $\left[\hat{L}_x, \hat{L}_x\hat{L}_y\hat{L}_z\right]$ may be simplified to **[CSIR June 2016]**

(a) $i\hbar L_x\left(\hat{L}_z^2 - \hat{L}_y^2\right)$ (b) $i\hbar\hat{L}_z\hat{L}_y\hat{L}_x$ (c) $i\hbar L_x\left(2\hat{L}_z^2 - \hat{L}_y^2\right)$ (d) 0

ANSWER KEY

1. (b)	2. (d)	3. (d)	4. (b)	5. (d)	6. (a)	7. (d)
8. (b)	9. (b)	10. (b)	11. (b)	12. (a)	13. (d)	14. ( )

JEST Previous Years' Questions

1. Define $\sigma_x = \left(f^\dagger + f\right)$ and $\sigma_y = -i\left(f^\dagger - f\right)$, where the σ' are Pauli spin matrices and $f, f^\dagger$ obey anticommutation relations $\{f,f\}=0, \{f, f^\dagger = 1\}$. Then σ_z is given by **[JEST 2012]**

   (a) $f^\dagger f - 1$ (b) $2f^\dagger f - 1$ (c) $2f^\dagger f + 1$ (d) $f^\dagger f$

2. Consider a spin-1/2 particle in the presence of homogeneous magnetic field of magnitude B along z-axis which is prepared initially in a state $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$ at time t = 0. At what time 't' will the particles be in the state $-|\psi\rangle$ (μ_B is Bohr magneton)? **[JEST 2012]**

   (a) $t = \frac{\pi\hbar}{\mu_B B}$ (b) $t = \frac{2\pi\hbar}{\mu_B B}$ (c) $t = \frac{\pi\hbar}{2\mu_B B}$ (d) Never

3. The quantum state $\sin x|\uparrow\rangle + \exp(i\phi)\cos x|\downarrow\rangle$, where $\langle\uparrow||\downarrow\rangle = 0$ and x, ϕ are, real, is orthogonal to: **[JEST 2012]**

   (a) $\sin x|\uparrow\rangle$ (b) $\cos x|\uparrow\rangle + \exp(i\phi)\sin x|\downarrow\rangle$

   (c) $-\cos x|\uparrow\rangle + \exp(i\phi)\sin x|\downarrow\rangle$ (d) $-\exp(i\phi)\cos x|\uparrow\rangle + \sin x|\downarrow\rangle$

4. If J_x, J_y, J_z are angular momentum operators, the eigenvalues of the operator $\left(J_x + J_y\right)/\hbar$ are **[JEST 2013]**

   (a) real and discrete with rational spacing (b) real and discrete with irrational spacing

   (c) real and continuous (d) not all real

5. Consider the state $\begin{pmatrix} 1/2 \\ 1/2 \\ 1/\sqrt{2} \end{pmatrix}$ corresponding to the angular momentum $l = 1$ in the L_z basis of states with m $= +1, 0, -1$. If L_z^2 is measured in this state yielding a result 1, what is the state after the measurement? **[JEST 2013]**

   (a) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1/\sqrt{3} \\ 0 \\ \sqrt{2/3} \end{pmatrix}$ (c) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$

6. What are the eigenvalues of the operator $H = \vec{\sigma}.\vec{\alpha}$, where $\vec{\sigma}$ are the three Pauli matrices and $\vec{\alpha}$ is a vector? **[JEST 2013]**

   (a) $\alpha_x + \alpha_y$ and α_z (b) $\alpha_x + \alpha_y$ and $i\alpha_z$ (c) $\pm\left(\alpha_x + \alpha_y + \alpha_z\right)$ (d) $\pm|\vec{\alpha}|$

7. Consider an eigenstate of $\vec{L}^2$ and L_z operator denoted by $|\ell, m\rangle$. Let $A = \hat{n}.\vec{L}$ denote an operator, where $\hat{n}$ is a unit vector parametrized in terms of two angles as $(n_x, n_y, n_z) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. The width ΔA in $|\ell, m\rangle$ state is: **[JEST 2014]**

(a) $\sqrt{\dfrac{\ell(\ell+1)-m^2}{2}}\hbar\cos\theta$ (b) $\sqrt{\dfrac{\ell(\ell+1)-m^2}{2}}\hbar\sin\theta$

(c) $\sqrt{\ell(\ell+1)-m^2}\hbar\sin\theta$ (d) $\sqrt{\ell(\ell+1)-m^2}\hbar\cos\theta$

8. Suppose a spin $\frac{1}{2}$ particle is in the state, $|\psi\rangle = \frac{1}{\sqrt{6}}\begin{bmatrix}1+i\\2\end{bmatrix}$. If S_x (x component of the spin angular momentum operator) is measured what is the probability of getting $+\frac{\hbar}{2}$? **[JEST 2014]**

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{5}{6}$ (d) $\frac{1}{6}$

9. A spin-1 particle is in a state $|\psi\rangle$ described by the column matrix $(1/\sqrt{10})\{2, \sqrt{2}, 2i\}$ in the S_z basis. What is the probability that a measurement of operator S_z will yield the result $\hbar$ for the state $S_x|\psi\rangle$?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$ **[JEST 2016]**

10. A spin-$\frac{1}{2}$ particle in a uniform external magnetic field has energy eigenstates $|1\rangle$ and $|2\rangle$. The system is prepared in ket-state $(|1\rangle + |2\rangle)/\sqrt{2}$ at time $t = 0$. It evolves to the state described by the ket $(|1\rangle - |2\rangle)/\sqrt{2}$ in time T. The minimum energy difference between two levels is: **[JEST 2016]**

(a) $\frac{h}{6T}$ (b) $\frac{h}{4T}$ (c) $\frac{h}{2T}$ (d) $\frac{h}{T}$

11. If $Y_{xy} = \frac{1}{\sqrt{2}}(Y_{2,2} - Y_{2,-2})$, where $Y_{l,m}$ are spherical harmonics, then which of the following is true? **[JEST 2016]**

(a) Y_{xy} is an eigenfunction of both L^2 and L_z (b) Y_{xy} is an eigenfunction of L^2 but not L_z

(c) Y_{xy} is an eigenfunction of L_z but not L^2 (d) Y_{xy} is not an eigenfunction of either L^2 or L_z

12. A spin-$\frac{1}{2}$ particle is in a state $(|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$, where $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of S_z operator. The exceptation value of the spin angular momentum measured along x-direction is: **[JEST 2016]**

(a) $\hbar$ (b) $-\hbar$ (c) 0 (d) $\frac{\hbar}{2}$

13. If the direction with respect to a right-handed cartesian coordinate system of the ket vector $\left|z,+\right\rangle$ is $(0,0,1)$, then the direction of the ket vector obtained by application of rotations: $\exp(-i\sigma_z\pi/2)\exp(i\sigma_y\pi/4)$,on the ket $\left|z,+\right\rangle$ is (σ_y, σ_z are the Pauli matrices): **[JEST 2016]**

(a) $(0,1,0)$ (b) $(1,0,0)$ (c) $(1,1,0)/\sqrt{2}$ (d) $(1,1,1)/\sqrt{3}$

ANSWER KEY

1. (b) **2. (a)** **3. (c)** **4. (a)** **5. (b)** **6. (c)** **7. (b)**

8. (c) **9. (d)** **10. (c)** **11. (b)** **12. (d)** **13. (b)**

TIFR Previous Years' Questions

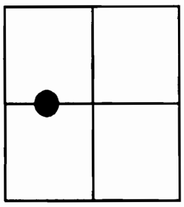
1. Consider the Hamiltonian

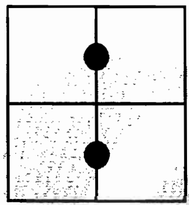
$$H = f\vec{\sigma}.\vec{x}$$

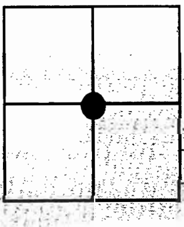
here $\vec{x}$ is the position vector, f is a constant and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$, where $\sigma_x, \sigma_y, \sigma_z$ are the three Pauli matrices. The energy eigenvalues are **[TIFR 2014]**

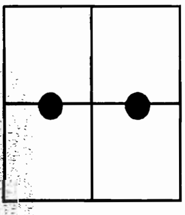
(a) $f\left(\sqrt{x^2+y^2} \pm z\right)$ (b) $f(x \pm iy)$ (c) $\pm f\sqrt{x^2+y^2+z^2}$ (d) $\pm f(x+y+z)$

2. In a Stern-Gerlach experiment with spin-½ particles, the beam is found to form two spots on the screen, one directly above the other. The experimenter now makes a hole in the screen at the position of the upper spot. The particles that go through this hole are then passed through another Stern-Gerlach apparatus but with its magnets rotated by 90 degrees counter-clockwise about the axis of the beam direction. Which of the following shows what happens on the second screen? **[TIFR 2015]**

(a)


(b)


(c)


(d)


3. A rigid rotator has the wave function **[TIFR 2015]**

$$\psi(\theta,\varphi) = N\left[2i\,Y_{1,0}(\theta,\varphi) + (2+i)Y_{2,-1}(\theta,\varphi) + 3i\,Y_{1,1}(\theta,\varphi)\right]$$

where $Y_{l,m}(\theta,\varphi)$ are the spherical harmonics, and N is a normalization constant. If $\vec{L}$ is the orbital angular momentum operator, and $L_\pm = L_x \pm iL_y$ the expectation value of L_+L_- is

(a) $\frac{21\hbar^2}{9}$ (b) $\frac{23\hbar^2}{9}$ (c) $\frac{25\hbar^2}{9}$ (d) 0

ANSWER KEY

1. (c) **2. (d)** **3. (b)**

Other Examinations Previous Years' Questions

1. The eigenstates of $\hat{S}_z$ of a quantum particle with spin $\frac{1}{2}$ are $|\uparrow\rangle = \left|s=\frac{1}{2}, s_z=\frac{1}{2}\right|$ and $|\downarrow\rangle = \left|s=\frac{1}{2}, s_z=-\frac{1}{2}\right|$.

Which of the following statements about eigenstates of hte operator $\hat{S}_z$ is true?

(a) $|\uparrow\rangle$ is an eigenstate of $\hat{S}_x$ (b) $|\downarrow\rangle$ is an eigenstate of $\hat{S}_x$

(c) $|\uparrow\rangle + |\downarrow\rangle$ is an eigenstate of $\hat{S}_x$

(d) No linear combination of $|\uparrow\rangle$ and $|\downarrow\rangle$ can be an eigenstate of $\hat{S}_x$.

2. J_x, J_y, J_z are the operators corresponding to the components of the angular momentum $\vec{J}$ and ψ_{jm} is the eigenfunction of J^2 and J_z with eigenvalues.

$$J^2\psi_{jm} = j(j+1)\hbar^2\psi_{jm}, \qquad J_z\psi_{jm} = m\hbar\psi_{jm}$$

Then the eigenvalue relation for $J_+ = J_x + iJ_y$ is:

(a) $J_+\psi_{jm} = \sqrt{j(j+1)+m(m+1)}\,\hbar\psi_{jm}$ (b) $J_+\psi_{jm} = \sqrt{j(j+1)-m(m+1)}\,\hbar\psi_{jm+1}$

(c) $J_+\psi_{jm} = \sqrt{(j-m)}\,\hbar\psi_{jm}$ (d) $J_+\psi_{jm} = \left[\sqrt{j(j+1)\hbar - m(m+1)\hbar}\right]\psi_{jm+1}$

3. P is a beam of atoms with spin quantum number $\frac{1}{2}$ and zero orbital angular momentum, with all angular momenta being along x axis. Q is a similar beam which is unpolarized. These beams can be differentiated experimentally by passing P and Q separately through Stern-Gerlach apperatus with

(a) Magnetic field along X axis (b) Magnetic field along Y axis.

(c) Magnetic field along Z axis (d) Magnetic field along YZ plane.

4. The expression $\exp i(\vec{\sigma}.\vec{n})\theta$ can be rewritten as

(a) $(\vec{\sigma}.\vec{n})\cos\theta + i\sin\theta$ (b) $\sin\theta + i(\vec{\sigma}.\vec{n})\cos\theta$

(c) $\cos\theta - i(\vec{\sigma}.\vec{n})\cos\theta$ (d) $\cos\theta + i(\vec{\sigma}.\vec{n})\sin\theta$

5. The eigenvalues of the pauli spin matrix σ_x are

(a) i, –i (b) –i, i (c) 1, –1 (d) 1, 1

6. Pauli's spin operators $\sigma_x, \sigma_y, \sigma_z$ are represented by 2×2 matrices. These are

(a) All anti-hermitian and traceless (b) All orthogonal and traceless

(c) All Hermitian and traceless (d) Only σ_z is hermitian and traceless

7. The orbital angular momentum eignstate $\psi_{\ell,m}(r,\theta,\phi)$ with $l=0, m=0$ (in standard notation) is an eigensta

of the following operators

(a) L_x, L_y, L_z and L^2 (b) L_x and L^2 only (c) L_y and L^2 only (d) L_z and L^2 only

8. The Dirac Hamiltonian for the relativistic Hydrogen atom has a coupling $\vec{L}\cdot\vec{S}$ between the orbital angular momentum and the spin of the electron. Let s_z, l_z and j_z denote the z-components of the spin, orbital and total angular momenta respectively. For the electron in an eigenstate of energy

(a) j_z and s_z (but not l_z) can be measured precisely

(b) j_z (but not s_z and l_z) can be measured precisely

(c) j_z and l_z (but not s_z) can be measured precisely

(d) l_z (but not l_z and s_z) can be measured precisely

9. Which of the following describes the same physical state of a spin-1/2 particle as $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix}$?

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 0\end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -1\end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix}i\\ -1\end{pmatrix}$

10. If $J_+|j,m\rangle = 0$ where 'J_+' denotes the angular momentum raising operator, $J_+ = J_x + iJ_y$ the value of m is

(a) -j+1 (b) $-j$ (c) $j-1$ (d) j

11. The wavefunction of a particle is of the form $\psi(r,\theta,\phi) = u(r,\theta)\cos(\phi)$. If the z-component of the angular momentum L_z is measured, then the possible outcome is $+\hbar$ with probability

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

12. The total spin-angular momentum of a system of three free electrons is:

(a) $S_{tot} = \frac{1}{2}$ and $S_{tot} = \frac{3}{2}$ (b) $S_{tot} = \frac{1}{2}$ only

(c) $S_{tot} = \frac{3}{2}$ only (d) $S_{tot} = 0$ $S_{tot} = 1$

Statement for Linked Answer type Q. 13 and Q. 14

An electron is placed in an uniform magnetic field $\vec{H}$ that points in the $\hat{x}$ direction. The Hamiltonian of the system is $H = -K\vec{S}\cdot\vec{H}$ where $k > 0$ is a constant and $\vec{S}$ is the electron spin operator.

13. The energy of the ground state of the Hamiltonian is:

(a) $-KS\hbar$ (b) $\frac{1}{2}KH\hbar$ (c) $-\frac{1}{2}KH\hbar$ (d) 0

14. The expectation value of S_z in the ground state of this Hamiltonian is:

(a) $-\hbar/2$ (b) 0 (c) $\hbar/2$ (d) $\hbar$

ANSWER KEY

1. (c) 2. (b) 3. (d) 4. (d) 5. (c) 6. (c) 7. (a)

8. (b) 9. (d) 10. (d) 11. (c) 12. (a) 13. (c) 14. (b)

GATE Previous Years' Questions

1. The Hamiltonian of the electron in the hydrogen atom is **[GATE 1991]**

$$H = p^2/(2m) - e^2/(4\pi\varepsilon_0 r)$$

where the symbols have their usual meanings. Let $|\psi\rangle$ denote the ground state of H, with $\langle\psi|\psi\rangle = 1$

(a) $|\psi\rangle$ is an eigen state of $p^2/(2m)$ (b) $\langle\psi|r|\psi\rangle$ is zero

(c) $\langle\psi|(1/r)|\psi\rangle$ is infinite (d) $\langle\psi|H|\psi\rangle < 0$

2. A particle is moving in a Coulomb potential. An operator A commutes with the Hamiltonian of the system. The observable corresponding to A is **[GATE 1995]**
(a) position (b) linear momentum (c) kinetic energy (d) angular momentum

3. The ground state wave function ψ of hydrogen atom in spherical polar co-ordinates has no angular dependence but only radial dependence. Then ψ is an eigen function of **[GATE 1996]**

(a) L_x, L_y and L_z simultaneously (b) L_z but not of L_x and L_y

(c) none of L_x, L_y and L_z (d) p_z

4. Degeneracy of the first excited state of an isolated hydrogen atom is: **[GATE 1997]**
(a) 2 (b) 4 (c) 6 (d) 8

5. Given that binding energy of hydrogen atom in the ground state is 13.6 eV, the binding energy for n = 2 state of the positronium atom is **[GATE 1998]**
(a) 13.6 eV (b) 6.8 eV (c) 3.4 eV (d) 1.7 eV

6. The Bohr model gives the value for the ionisation potential of Li^{2+} ion as **[GATE 2000]**
(a) 13.6 eV (b) 27.2 eV (c) 40.8 eV (d) 122.4 eV **Ans. (d)**

7. The spinless particle moves in a central potential $V(r)$. Which of the following statement is TRUE? **[GATE 2001]**
(a) The kinetic energy and the potential energy of the particle cannot have simultaneous sharp values.
(b) The total energy and the potential energy of the particle can have simultaneous sharp values.
(c) The total energy and the square of the orbital angular momentum about the origin cannot have simultaneous sharp values.
(d) The total energy of the particle can have only discrete eigenvalues.

8. A quantum particle of mass m is confined to a square region in xy-plane whose vertices are given by (0, 0), (L, 0), (L, L) and (0, L). Which of the following represents an admissible wave function of the particle (for ℓ, m, n positive integers)? **[GATE 2001]**

(a) $\frac{2}{L}\sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{m\pi y}{L}\right)$ (b) $\frac{2}{L}\sin\left(\frac{\ell\pi x}{L}\right)\cos\left(\frac{n\pi y}{L}\right)$

(c) $\frac{2}{L}\sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi y}{L}\right)$ (d) $\frac{2}{L}\cos\left(\frac{n\pi x}{L}\right)\sin\left(\frac{\ell\pi y}{L}\right)$

Common data for Q. 9 and Q.10
A particle is located in a three dimensional cubic well of width L with impenetrable walls **[GATE 2003]**

9. The sum of he energies of the third and the fourth levels is:

(a) $10\pi^2\hbar^2/mL^2$ (b) $10\pi^2\hbar^2/3\,mL^2$ (c) $11\pi^2\hbar^2/2mL^2$ (d) $15\pi^2\hbar^2/2mL^2$

10. The degeneracy of the fourth level is given by
(a) 1 (b) 2 (c) 3 (d) 4

11. In a hydrogen atom, the accidental or Coulomb degenaracy for the n = 4 state is **[GATE 2004]**
(a) 4 (b) 16 (c) 18 (d) 32

Linked Answer for Q.12 and Q.13

The one-electron states for non-interacting electrons confined in a cubic box of side a are $\varepsilon_0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 < \varepsilon_4$ etc. **[GATE 2004]**

12. The energy of the lowest state is:

(a) zero (b) $\frac{\hbar^2\pi^2}{2ma^2}$ (c) $\frac{\hbar^2\pi^2}{ma^2}$ (d) $\frac{3\hbar^2\pi^2}{2ma^2}$

13. The degeneracy (including spin) of the level ε_3 is:

(a) 2 (b) 4 (c) 6 (d) 8

14. The expectation value of the z coordinate (z), in the ground state of the hydrogen atom (wavefunction: $\psi_{100}(r) = Ae^{-r/a_0}$, where A is the normalization constant and a_0 is the Bohr radius), is **[GATE 2005]**

(a) a_0 (b) $\frac{a_0}{2}$ (c) $\frac{a_0}{4}$ (d) 0

15. The degeneracy of the n = 2 level for a three dimensional isotropic oscillator is **[GATE 2005]**

(a) 4 (b) 6 (c) 8 (d) 10

16. A particle is moving in a spherically symmetric potential $V(r) = \alpha r^2$, where α is a postive constant. In a stationary state, the expectation value of the kinetic energy of the particle is **[GATE 2006]**

(a) $\langle T\rangle = \langle V\rangle$ (b) $\langle T\rangle = 2\langle V\rangle$ (c) $\langle T\rangle = 3\langle V\rangle$ (d) $\langle T\rangle = 4\langle V\rangle$

17. An atomic state of hydrogen is represented by the following wave function:

$$\psi(r,\theta,\phi) = \frac{1}{\sqrt{2}}\left(\frac{1}{a_0}\right)^{3/2}\left(1-\frac{r}{2a_0}\right)e^{-r/2a_0}\cos\theta$$

where a_0 is a constant. The quantum number of the state are **[GATE 2007]**

(a) $\ell = 0, m = 0, n = 1$ (b) $\ell = 1, m = 1, n = 2$ (c) $\ell = 1, m = 0, n = 2$ (d) $\ell = 2, m = 0, n = 3$

18. The radial wave function of the electrons in the state of n = 1 and l = 0 in a hydrogen atom is $R_{10} = \frac{2}{a_0^{3/2}}\exp\left(-\frac{r}{a_0}\right)$, a_0 is the Bohr radius. The most probable value of r for an electron is:

[GATE 2008]

(a) a_0 (b) $2a_0$ (c) $4a_0$ (d) $8a_0$

19. Consider the combined system of proton and electron in the hydrogen atom in its (electronic) ground state. Let I denote the quantum number associated with the total angular momentum and let $\langle m\rangle$ denote the magnitude of the expectation value of the net magnetic moment in the state. Which of the following pairs represents a possible state of the system (μ_B is Bohr magneton)? **[GATE 2008]**

(a) $I = 0, \langle m\rangle = 0$ (b) $I = \frac{1}{2}, \langle m\rangle = 1\mu_B$ (c) $I = 1, \langle m\rangle \approx 1\mu_B$ (d) $I = 0, \langle m\rangle = 2\mu_B$

20. The three principal moments of inertia of a methanol (CH_3OH) molecule have the property $I_x = I_y = I$ and $I_z \neq I$. The rotational energy eigenvalues are **[GATE 2010]**

(a) $\frac{\hbar^2}{2I}\ell(\ell+1) + \frac{\hbar^2 m_\ell^2}{2}\left(\frac{1}{I_z} - \frac{1}{I}\right)$ (b) $\frac{\hbar^2}{2I}\ell(\ell+1)$

(c) $\frac{\hbar^2 m_\ell^2}{2}\left(\frac{1}{I_z}-\frac{1}{I}\right)$ (d) $\frac{\hbar^2}{2I}\ell(\ell+1)+\frac{\hbar^2 m_\ell^2}{2}\left(\frac{1}{I_z}+\frac{1}{I}\right)$

21. The normalized ground state wavefunction of a hydrogen atom is given by $\psi(r)=\frac{1}{\sqrt{4\pi}}\frac{2}{a^{3/2}}e^{-r/a}$, where a is the Bohr radius and r is the distance of the electron from the nucleus, located at the origin. The expectation value $\left\langle\frac{1}{r^2}\right\rangle$ is: **[GATE 2011]**

(a) $\frac{8\pi}{a^2}$ (b) $\frac{4\pi}{a^2}$ (c) $\frac{4}{a^2}$ (d) $\frac{2}{a^2}$

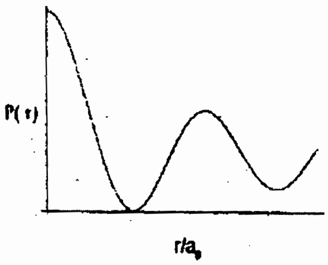
22. A particle of mass m is confined in a two dimensional square well potential of dimension a. This potential $V(x,y)$ is given by

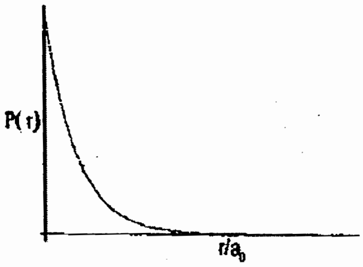
$$V(x,y)=0 \text{ for } 0<x<a \text{ and } 0<y<a$$
$$=\infty \text{ else where}$$

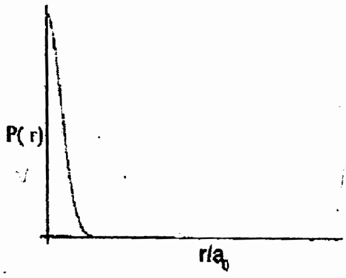
The energy of the first excited state for this particle is given by, **[GATE 2012]**

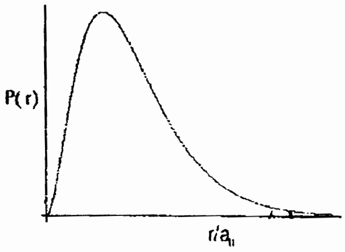
(a) $\frac{\pi^2\hbar^2}{ma^2}$ (b) $\frac{2\pi^2\hbar^2}{ma^2}$ (c) $\frac{5\pi^2\hbar^2}{2ma^2}$ (d) $\frac{4\pi^2\hbar^2}{ma^2}$

23. The ground state wavefunction for the hydrogen atom is given by $\Psi_{100}=\frac{1}{\sqrt{4\pi}}\left(\frac{1}{a_0}\right)^{3/2}e^{-r/a_0}$, where a_0 is the Bohr radius. The plot of the radial probability density, P(r) for the hydrogen atom in the ground state is **[GATE 2012]**

(a)


(b)


(c)


(d)


24. An electron in the ground state of the hydrogen atom has the wave function

$$\psi(\vec{r}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-(r/a_0)}$$

where a_0 is constant. The expectation value of the operator $\hat{Q} = z^2 - r^2$, where $z = r\cos\theta$ is (Hint: $\int_0^\infty e^{-ar} r^n dr = \frac{\Gamma(n)}{\alpha^{n+1}} = \frac{(n-1)!}{\alpha^{n+1}}$) **[GATE 2014]**

(a) $-\frac{a_0^2}{2}$ (b) $-a_0^2$ (c) $-\frac{3a_0^2}{2}$ (d) $-2a_0^2$

25. A particle of mass 'm' is subjected to a potential, **[GATE 2014]**

$$V(x,y) = \frac{1}{2}m\omega^2\left(x^2+y^2\right), \quad -\infty \le x \le \infty, -\infty \le y \le \infty$$

The state with energy $4\hbar\omega$ is g-fold degenerate. The value of g is ________

26. A hydrogen atom is in the state

$$\psi = \sqrt{\frac{8}{21}}\psi_{200} - \sqrt{\frac{3}{7}}\psi_{310} + \sqrt{\frac{4}{21}}\psi_{321}$$ **[GATE 2014]**

where n, ℓ, m in $\psi_{n\ell m}$ denote the principal, orbital and magnetic quantum numbers, respectively. If $\vec{L}$ is the angular momentum operator, the average value of L^2 is ____________ $\hbar^2$.

27. The moment of inertia of a rigid diatomic molecule A is 6 times that of another rigid diatomic molecule B. If the rotational energies of the two molecules are equal, then the corresponding values of the rotational quantum numbers J_A and J_B are **[GATE 2014]**

(a) $J_A = 2, J_B = 1$ (b) $J_A = 3, J_B = 1$ (c) $J_A = 5, J_B = 0$ (d) $J_A = 6, J_B = 1$

ANSWER KEY

1. (d) 2. (d) 3. (a) 4. (d) 5. (d) 6. (d) 7. (a)

8. (c) 9. (a) 10. (c) 11. (b) 12. (d) 13. (c) 14. (d)

15. (b) 16. (a) 17. (c) 18. (a) 19. (b) 20. (a) 21. (d)

22. (c) 23. (d) 24. (d) 25. (4) 26. (2) 27. (b)

CSIR-UGC-NET Previous Years' Questions

1. The energy levels of the non-relativistic electron in a hydrogen atom (i.e. in a Coulomb potential $V(r) \propto -1/r$ are given by $E_{n\ell m} \propto -1/n^2$, where n is the principal quantum number. and the corresponding wave functions are given by $\psi_{n\ell m}$, where ℓ is the orbital angular momentum quantum number and m is the magnetic quantum number. The spin of the electron is not considered. Which of the following is a correct statement? **[CSIR June 2011]**

   (a) There are exactly $(2\ell+1)$ different wave functions $\psi_{n\ell m}$, for each $E_{n\ell m}$.

   (b) There are $\ell(\ell+1)$ different wave functions $\psi_{n\ell m}$, for each $E_{n\ell m}$.

   (c) $E_{n\ell m}$ does not depend on ℓ and m for the Coulomb potential.

   (d) There is a unique wave function $\psi_{n\ell m}$ and $E_{n\ell m}$.

2. If an electron is in the ground state of the hydrogen atom, the probability that its distance from the proton is more than one Bohr radius is approximately **[CSIR June 2011]**

   (a) 0.68 (b) 0.48 (c) 0.28 (d) 0.91

3. Given that the ground state energy of the hydrogen atom is -13.6 eV, the ground state energy of positronium (which is a bound state of an electron and a positron) is **[CSIR Dec 2011]**

   (a) 6.8 eV (b) -6.8 eV (c) -13.6 eV (d) -27.2 eV

4. The energy of the first excited quantum state of a particle in the two-dimensional potential $V(x,y) = \frac{1}{2}m\omega^2\left(x^2+4y^2\right)$ is: **[CSIR Dec 2011]**

   (a) $2\hbar\omega$ (b) $3\hbar\omega$ (c) $\frac{3}{2}\hbar\omega$ (d) $\frac{5}{2}\hbar\omega$

5. A particle of mass m is in a cubic box of size a. The potential inside the box $(0 \le x < a, 0 \le y < a, 0 \le z < a)$ is zero and infinite outside. If the particle is in an eigenstate of energy $E = \frac{14\pi^2 h^2}{2ma^2}$, its wave function is: **[CSIR June 2012]**

   (a) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{3\pi x}{a} \sin\frac{5\pi y}{a} \sin\frac{6\pi z}{a}$ (b) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{7\pi x}{a} \sin\frac{4\pi y}{a} \sin\frac{3\pi z}{a}$

   (c) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{4\pi x}{a} \sin\frac{8\pi y}{a} \sin\frac{2\pi z}{a}$ (d) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{\pi x}{a} \sin\frac{2\pi y}{a} \sin\frac{3\pi z}{a}$

6. Let $\Psi_{n\ell m}$ denote the eigenfunctions of a Hamiltonian for a spherically symmetric potential V(r). The wavefunction $\Psi = \frac{1}{4}\left[\Psi_{210} + \sqrt{5}\Psi_{21-1} + \sqrt{10}\Psi_{213}\right]$ is an eigenfunction of **[CSIR June 2012]**

   (a) H, L^2 and L_z (b) H and L_z (c) H and L^2 (d) L^2 and L_z

7. The wave function of a state of the hydrogen atom is given by

$$\psi = \psi_{200} + 2\psi_{211} + 3\psi_{210} + \sqrt{2}\,\psi_{21-1}$$

where $\psi_{n\ell m}$ denotes the normalized eigen function of the state with quantum numbers n, l and m in the usual notation. The expectation value of L_z in the state ψ is: **[CSIR Dec 2012]**

(a) $\frac{15\hbar}{16}$ (b) $\frac{11\hbar}{16}$ (c) $\frac{3\hbar}{8}$ (d) $\frac{\hbar}{8}$

8. The un-normalized wavefunction of a particle in a spherically symmetric potential is given by $\psi(\vec{r}) = zf(r)$ where f(r) is a function of the radial variable r. The eigenvalue of the operator $\vec{L}^2$ (namely the square of the orbital angular momentum) is **[CSIR June 2013]**

(a) $\hbar^2/4$ (b) $\hbar^2/2$ (c) $\hbar^2$ (d) $2\hbar^2$

9. If ψ_{nlm} denotes the eigenfunction of the Hamiltonian with a potential V = V(r) then the expectation value of the operator $L_x^2 + L_y^2$ in the state $\psi = \frac{1}{5}[3\psi_{211} + \psi_{210} - \sqrt{15}\psi_{21-1}]$ is **[CSIR June 2013]**

(a) $39\hbar^2/25$ (b) $13\hbar^2/25$ (c) $2\hbar^2$ (d) $26\hbar^2/25$

10. A particle of mass m in three dimensions is in the potential

$$V(r) = \begin{cases} 0 & r < a \\ \infty & r \geq a \end{cases}$$

Its ground state energy is **[CSIR June 2014]**

(a) $\frac{\pi^2\hbar^2}{2ma^2}$ (b) $\frac{\pi^2\hbar^2}{ma^2}$ (c) $\frac{3\pi^2\hbar^2}{2ma^2}$ (d) $\frac{9\pi^2\hbar^2}{2ma^2}$

11. Given that $\hat{p}_r = -i\hbar\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)$, the uncertainty Δp_r in the ground state

$$\psi_0(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

of the hydrogen atom is **[CSIR June 2014]**

(a) $\frac{\hbar}{a_0}$ (b) $\frac{\sqrt{2}\hbar}{a_0}$ (c) $\frac{\hbar}{2a_0}$ (d) $\frac{2\hbar}{a_0}$

12. A particle of mass m in the potential $V(x,y) = \frac{1}{2}m\omega^2(4x^2 + y^2)$, is in an eigenstate of energy $E = \frac{5}{2}\hbar\omega$. The corresponding un-normalized eigenfunction is **[CSIR June 2014]**

(a) $y\exp\left[-\frac{m\omega}{2\hbar}(2x^2 + y^2)\right]$ (b) $x\exp\left[-\frac{m\omega}{2\hbar}(2x^2 + y^2)\right]$

(c) $y\exp\left[-\frac{m\omega}{2\hbar}(x^2 + y^2)\right]$ (d) $xy\exp\left[-\frac{m\omega}{2\hbar}(x^2 + y^2)\right]$

13. An electron is in the ground state of a hydrogen atom. The probability that it is within the Bohr radius is approximately equal to **[CSIR June 2014]**

(a) 0.60 (b) 0.90 (c) 0.16 (d) 0.32

14. The ratio of the energy of the first excited state E_1, to that of the ground state E_0 of a particle in a three-dimensional rectangular box of sides L, L and $L/2$ is **[CSIR June 2015]**

(a) 3:2 (b) 2:1 (c) 4:1 (d) 4:3

15. A positron is suddenly absorbed by the nucleus of a tritium $({}_1^3\text{H})$ atom to turn the latter into a He$^+$ ion. If the electron in the tritium atom was initially in the ground state, the probability that the resulting He$^+$ ion will be in its ground state is **[CSIR Dec. 2015]**

(a) 1 (b) $\frac{8}{9}$ (c) $\frac{128}{243}$ (d) $\frac{512}{729}$

16. Suppose that the Coulomb potential of the hydrogen atom is changed by adding an inverse-square term such that the total potential is $V(\vec{r}) = -\frac{ze^2}{r} + \frac{g}{r^2}$, where g is a constant. The energy eigenvalues E_{nlm} in the modified potential **[CSIR June 2016]**

(a) depend on n and l, but not on m (b) depend on n but not on l and m
(c) depend on n and m, but not on l (d) depend explicitly on all three quantum numbers n, l and m

ANSWER KEY

1. (c) 2. (a) 3. (b) 4. (d) 5. (d) 6. (c) 7. (d)
8. (d) 9. (d) 10. (a) 11. (a) 12. (a) 13. (d) 14. (a)
15. (d) 16. ( )

JEST Previous Years' Questions

1. Consider a particle of mass 'm' moving inside a two-dimensional square box whose sides are described by the equations $x = 0,\ x = L,\ y = 0,\ y = L$. What is the lowest eigenvalue of an eigenstate which changes sign under the exchange of x and y? **[JEST 2012]**

   (a) $\frac{\hbar^2}{mL^2}$ (b) $\frac{3\hbar^2}{2mL^2}$ (c) $\frac{5\hbar^2}{2mL^2}$ (d) $\frac{7\hbar^2}{2mL^2}$

2. The binding energy of the hydrogen atom (electron bound to proton) is 13.6 eV. The binding energy of positronium (electron bound to positron) is **[JEST 2012]**

   (a) 13.6/2 eV (b) 13.6/180 eV (c) 13.6×1810 eV (d) 13.6×2 eV

3. A simple model of a helium-like atom with electron-electron interaction is replaced by Hooke's law force is described by hamiltonian $\frac{-\hbar^2}{2m}\left(\nabla_1^2+\nabla_2^2\right)+\frac{1}{2}m\omega^2\left(r_1^2+r_2^2\right)-\frac{\lambda}{4}m\omega^2\left|\vec{r}_1-\vec{r}_2\right|^2$. What is the exact ground state energy? **[JEST 2013]**

   (a) $E=\frac{3}{2}\hbar\omega\left(1+\sqrt{1+\lambda}\right)$ (b) $E=\frac{3}{2}\hbar\omega\left(1+\sqrt{\lambda}\right)$

   (c) $E=\frac{3}{2}\hbar\omega\sqrt{1-\lambda}$ (d) $E=\frac{3}{2}\hbar\omega\left(1+\sqrt{1-\lambda}\right)$

4. If a proton were ten times lighter, then the ground state energy of the electron in a hydrogen atom would have been **[JEST 2013, 2014]**

   (a) Less (b) More (c) The same (d) Depends on the electron mass

5. A particle moving under the influence of a potential $V(r)=kr^2/2$ has a wavefunction $\psi(r,t)$. If the wavefunction changes to $\psi(\alpha r,t)$, the ratio of the average final kinetic energy to the initial kinetic energy will be,

   (a) $\frac{1}{\alpha^2}$ (b) α (c) $\frac{1}{\alpha}$ (d) α^2 **[JEST 2015]**

6. The wavefunction of a hydrogen atom is given by the following superposition of energy eigen-functions $\psi_{nlm}(\vec{r})$ (n, l, m are the usual quantum numbers); **[JEST 2016]**

$$\psi(\vec{r})=\frac{\sqrt{2}}{\sqrt{7}}\psi_{100}(\vec{r})-\frac{3}{\sqrt{14}}\psi_{210}(\vec{r})+\frac{1}{\sqrt{14}}\psi_{322}(\vec{r})$$

   The ratio of expectation value of the energy to the ground state energy and the expectation value of L^2 are, respectively:

   (a) $\frac{229}{504}$ and $\frac{12\hbar^2}{7}$ (b) $\frac{101}{504}$ and $\frac{12\hbar^2}{7}$ (c) $\frac{101}{504}$ and $\hbar^2$ (d) $\frac{229}{504}$ and $\hbar^2$

7. The Hamiltonian of a quantum particle of mass m confined to a ring of unit radius is: **[JEST 2016]**

$$H=\frac{\hbar^2}{2m}\left(-i\frac{\partial}{\partial\theta}-\alpha\right)^2,$$

   where θ is the angular coordinate, α is a constant. The energy eigenvalues and eigenfunctions of the particle are (n is an integer):

   (a) $\psi_n(\theta)=\frac{e^{in\theta}}{\sqrt{2\pi}}$ and $E_n=\frac{\hbar^2}{2m}(n-\alpha)^2$ (b) $\psi_n(\theta)=\frac{\sin(n\theta)}{\sqrt{2\pi}}$ and $E_n=\frac{\hbar^2}{2m}(n-\alpha)^2$

(c) $\psi_n(\theta)=\dfrac{\cos(n\theta)}{\sqrt{2\pi}}$ and $E_n=\dfrac{\hbar^2}{2m}(n-\alpha)^2$ (d) $\psi_n(\theta)=\dfrac{e^{in\theta}}{\sqrt{2\pi}}$ and $E_n=\dfrac{\hbar^2}{2m}(n+\alpha)^2$

8. In the ground state of hydrogen atom, the most probable distance of the electron from the nucleus, in units of Bohr radius a_0 is: **[JEST 2016]**

(a) $\dfrac{1}{2}$ (b) 1 (c) 2 (d) $\dfrac{3}{2}$

ANSWER KEY

1. (c) **2. (a)** **3. ( )** **4. (b)** **5. (b)** **6. (a)** **7. (a)**

8. (b)

TIFR Previous Years' Questions

1. The normalized wavefunctions of a Hydrogen atom are denoted by $\psi_{n,\ell,m}(\vec{x})$, where n, ℓ and m are, respectively, the principal, azimuthal and magnetic quantum numbers respectively. Now consider an electron in the mixed state

$$\Psi(\vec{x}) = \frac{1}{3}\psi_{1,0,0}(\vec{x}) + \frac{2}{3}\psi_{2,1,0}(\vec{x}) + \frac{2}{3}\psi_{3,2,-2}(\vec{x})$$ **[TIFR 2012]**

The expectation value $\langle E\rangle$ of the energy of this electron, in electron-Volts (eV) will be approximately

(a) –1.5 (b) –3.7 (c) –13.6 (d) –80.1 (e) +13.6

2. An energy eigenstate of the Hydrogen atom has the wave function

$$\psi_{n\ell m}(r,\theta,\varphi) = \frac{1}{81\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2}\sin\theta\cos\theta\exp\left[-\left(\frac{r}{3a_0}+i\varphi\right)\right]$$ **[TIFR 2013]**

where a_0 is the Bohr radius. The principal (n), azimuthal (ℓ) and magnetic (m) quantum numbers corresponding to this wave function are

(a) $n=3, \ell=2, m=1$ (b) $n=2, \ell=1, m=1$

(c) $n=3, \ell=2, m=-1$ (d) $n=2, \ell=1, m=\pm1$

3. The velocity of an electron in the ground state of a hydrogen atom is v_H. If v_p be the velocity of an electron in the ground state of positronium, then **[TIFR 2013]**

(a) $v_p = v_H$ (b) $v_p = 2v_H$ (c) $v_p = \frac{v_H}{2}$ (d) $v_p = \sqrt{2}v_H$ **Ans. (a)**

4. A rigid rotator is in a quantum state described by the wavefunction

$$\psi(\theta,\varphi) = \sqrt{\frac{3}{4\pi}}\sin\theta\sin\varphi$$ **[TIFR 2014]**

where θ and φ are the usual polar angles. If two successive measurements of L_z are made on this rotator, the probability that the second measurement will yield the value $+\hbar$ is

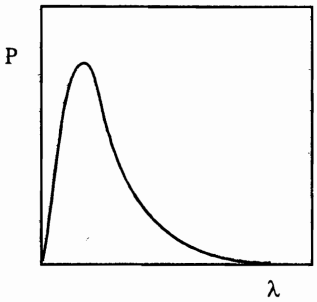
(a) 0.25 (b) 0.33 (c) 0.5 (d) negligible

5. A particle in the 2s state of hydrogen has the wave function

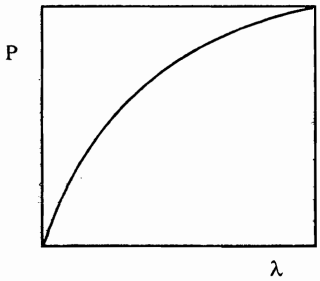
$$\psi_{2s}(r) = \frac{1}{4\sqrt{2}\,\pi}\left(\frac{1}{a_0}\right)^{3/2}\left(2-\frac{r}{a_0}\right)\exp\left(-\frac{r}{2a_0}\right)$$

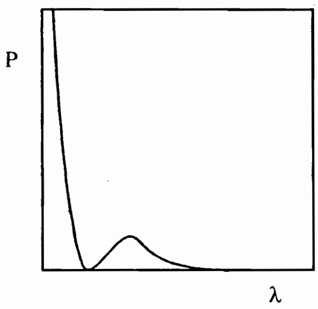
where 'r' is the radial coordinate w.r.t. the nucleus as origin and a_0 is the Bohr radius. The probability P of finding the electron somewhere inside a sphere of radius λa_0 centred at the nucleus, is best described by the graph **[TIFR 2014]**

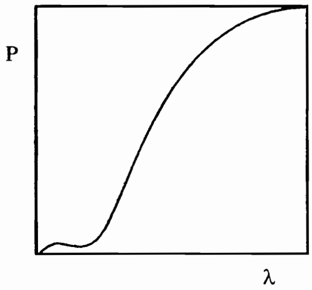
(a)



(b)



(c) 

(d) 

6. The ground state energy of a particle of mass m in a three dimensional cubical box of side ℓ is not zero but $\frac{3h^2}{8m\ell^2}$. This is because **[TIFR 2015]**
   (a) The ground state has no nodes in the interior of the box.
   (b) This is the most convenient choice of the zero level of potential energy.
   (c) Position and momentum cannot be exactly determined simultaneously.
   (d) The potential at the boundaries is not really infinite, but just very large.

7. A quantum mechanical plane rotator consists to two rigidly connected particles of mass m and connected by a massless rod of length d is rotating in the xy-plane about their centre of mass. Suppose that the initial state of the rotor is given by $\psi(\varphi, t=0) = A\cos^2\varphi$, where φ is the angle between one mass and the x-axis, while A is normalization constant. Find the exception value of $3\hat{L}_z^2$ in this state, in units of $\hbar^2$. **[TIFR 2016]**

ANSWER KEY

1. (b)	2. (c)	3. (a)	4. (c)	5. (d)	6. (c)	7. (004)

Other Examinations Previous Years' Questions

1. Consider a quantum particle of mass 'm' in a three-dimensional isotropic simple harmonic potential $V(x,y,z)=\frac{1}{2}m\omega^2\left(x^2+y^2+z^2\right)$. It is known that the particle is in an energy eigenstate with eigenvalue $E=7\hbar\omega/2$. Which of the following cannot be the wave function of the particle? (In the following $\alpha=\sqrt{m\omega/\hbar}$ and $H_n(\xi)$ is the nth Hermite polynomial)

(a) $H_2(\alpha x)\exp\left(-\alpha\left(y^2+z^2\right)\right)$ (b) $H_2(\alpha x)\exp\left(-\alpha\left(x^2+y^2+z^2\right)\right)$

(c) $H_1(\alpha y)H_1(\alpha z)\exp\left(-\alpha\left(x^2+y^2+z^2\right)\right)$ (d) $H_1(\alpha x)H_1(\alpha z)\exp\left(-\alpha\left(x^2+y^2+z^2\right)\right)$

2. A particle of mass m is moving in a three-dimensional potential

$$V(x,y,z)=\frac{1}{2}m\omega^2\left(x^2+2y^2+4z^2\right)$$

The energy of the particle in the ground state (lowest energy quantum state) is

(a) $\frac{\sqrt{7}}{2}\hbar\omega$ (b) $\frac{3}{2}\hbar\omega$ (c) $\frac{7}{2}\hbar\omega$ (d) $\frac{\left(3+\sqrt{2}\right)}{2}\hbar\omega$

3. Given that the ionization potential of hydrogen is 13.6 eV, that of positronium which is a composite state made of one electron and one position is,

(a) 0 eV (b) 6.8 eV (c) 13.6 eV (d) 27.2 eV

4. Which of the following is not an allowed set of quantum numbers for an electron in a Hydrogen atom?

(a) $n=1, l=0, m=0, s=\frac{1}{2}$ (b) $n=2, l=2, m=1, s=\frac{1}{2}$

(c) $n=3, l=1, m=-1, s=-\frac{1}{2}$ (d) $n=4, l=3, m=-2, s=\frac{1}{2}$

5. A hydrogen beam is prepared in the state $\psi(\bar{r},t)=\frac{\sqrt{3}}{2}\exp\left(i\frac{E_1t}{\hbar}\right)\psi_1(\bar{r})+\frac{1}{2}\exp\left(i\frac{E_2t}{\hbar}\right)\psi_2(\bar{r})$

Where E_1 and E_2 are the energies of the ground state and the first excited state of the hydrogen atom and $\psi_1(\bar{r})$ and $\psi_2(\bar{r})$ are their wave functions respectively. The beam is incident on a detector which measures their energy. Let E_0 be the ionization energy of the hydrogen atom. The average energy measured by the detector is given by

(a) $-E_0$ (b) $-\frac{13}{16}E_0$ (c) $-\frac{3}{4}E_0$ (d) $-\frac{15}{16}E_0$

6. A hydrogen beam is prepared in the state

$$\Psi(\bar{r},t)=\frac{1}{\sqrt{14}}\exp\left(i\frac{E_1t}{\hbar}\right)\Psi_1(\bar{r})+\sqrt{\frac{2}{7}}\exp\left(i\frac{E_2t}{\hbar}\right)\Psi_2(\bar{r})+\frac{3}{\sqrt{14}}\exp\left(i\frac{E_3t}{\hbar}\right)\Psi_3(\bar{r})$$

Where E_1, E_2, E_3 are the energies of the ground state and the first excited and second excited states of the hydrogen atom and $\Psi_1(\bar{r}),\Psi_2(\bar{r}),\Psi_3(\bar{r})$ are their normalized wave functions respectively. The beam is incident on a detector which measures their energy. Let E_0 be the ionization energy of the hydrogen atom. The average energy measured by the detector is given by

(a) $-E_0$ (b) $-\frac{3}{14}E_0$ (c) $-\frac{1}{7}E_0$ (d) $-\frac{1}{14}E_0$

7. A quantum mechanical particle of mass m is confined to move in a circle of radius R. The energy levels are

(a) $E=\frac{\hbar^2}{2mR^2}\ell(\ell+1)$. $\ell\in\{0,1,2,3,.....\}$ (b) $E=\frac{\hbar^2}{2mR^2}\ell^2$ $\ell\in\{...,-3,-2,-1,0,1,2,3.....\}$

(c) $E\frac{\hbar^2}{2mR^2}\ell(\ell-1)$, $\ell\in\{0,1,2,3,...\}$. (d) $E=\frac{\hbar^2R^2}{2m}\ell^2$, $\ell\in\{....,-3,-2,-1,0,1,2,3,...\}$.

8. The ground state energy of the Hydrogen atom is –13.6 eV. The energy of the second excited state is:

(a) –27.2 eV (b) –6.8 eV (c) –1.5 eV (d) –4.5 eV

9. The degeneracy of energy for a hydrogen-like atom with principal quantum number n and orbital quantum number *l* is:

(a) n (b) n^2 (c) $2n^2$ (d) $l(l+1)$

10. In a central force field, the angular part of the solution of the Schrodinger equation is in general

(a) Independent of θ and ϕ (b) A constant

(c) A spherical harmonic (d) A Bessel function

11. Consider 1s and 2s states for hydrogen atom, then $\int\psi_{100}(\vec{r})\psi_{200}(\vec{r})\,d^3r$, is

(a) –ve (b) +ve (c) zero (d) 1.0

12. The wave function $\psi(\vec{r})=e^{i\vec{p}.\vec{r}}$ is an eigen function for a Hamiltonian with

(a) A constant potential $V\geq 0$

(b) A zero potential only

(c) A potential V(r) (not constant) which is spherically symmetric.

(d) Any potential which is periodic with periodicity $\vec{R}$.

13. The azimuthal wave function for the hydrogen atom is: $\phi(\varphi)=Ae^{im\phi}$. Where m is the magnetic quantum number. If $\phi(\varphi)$ is normalized, the value of the normalization constant A is:

(a) $\frac{1}{2\sqrt{\pi}}$ (b) $\frac{1}{2\sqrt{m\pi}}$ (c) $\frac{1}{\sqrt{2\pi}}$ (d) $\frac{1}{\sqrt{2m\pi}}$

14. An electron in the hydrogen atom is in a state whose energy is -3.4 eV. The degeneracy of the state is

(a) 4 (b) 8 (c) 2 (d) 1

15. The quantum states of a particle in a Coulomb potential is labelled by the principal, orbital and azimuthal quantum numbers n, *l* and m respectively. From the rotational symmetry of the Hamiltonian alone we may conclude that the energy eigenvalues are

(a) independent of n only (b) independent of *l* only

(c) independent of *l* and m (d) independent of m only

16. The degeneracy of the state of energy $E_n=\hbar\omega(n+1)$, where n is an integer, in a two-dimensional isotopic harmonic oscillator with potential $V(x,y)=\frac{1}{2}m\omega^2(x^2+y^2)$ is

(a) $\frac{n(n-1)}{2}$ (b) $n+1$ (c) $n-1$ (d) n

17. If E_H is the binding energy of the electron in the Bohr model of the Hydrogen atom, and both the charge and mass of the electron are doubled, the binding energy becomes

(a) $32\ E_H$ (b) $2\ E_H$ (c) $4\ E_H$ (d) $16\ E_H$

Statement for Linked Answer type Q. 18 and Q. 19.

Consider a three dimensional harmonic oscillator with Hamiltonian

$$H=\frac{p_x^2}{2m}+\frac{p_y^2}{2m}+\frac{p_z^2}{2m}+\frac{1}{2}m\omega^2\left(x^2+y^2+z^2\right)$$

18. The number of distinct eigenvalues with energy eigenvalue $\frac{5}{2}\hbar\omega$ is:

(a) 5 (b) 3 (c) 2 (d) 1

19. The eigenvalue of L^2 (where $\vec{L}$ is the angular momentum operator) in the ground state is:

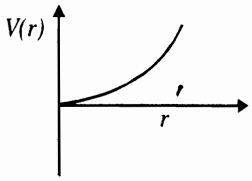
(a) $\hbar^2$ (b) $2\hbar^2$ (c) 0 (d) $6\hbar^2$

20. The number of nodes of the radial wave function for an atomic electron, $R(r)=A\left[27-18r+2r^2\right]e^{-r/3}$ is:

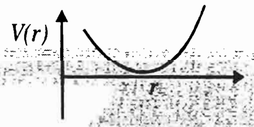
(a) 0 (b) 1 (c) 2 (d) 3

21. Which of the following figure represents the potential, $V(r)=-\frac{A}{r}+Br^2$

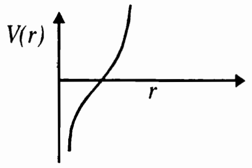
(a)



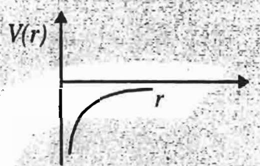
(b)



(c)



(d)

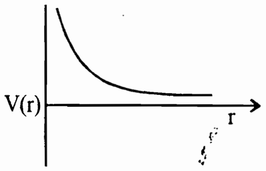


22. The effective radial potential for the motion of the electron in the hydrogen atom is of the form

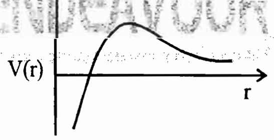
$$V(r)=-\frac{K}{r}+\frac{\ell(\ell+1)\hbar^2}{2mr^2}$$

Wheren K is strength of interaction, ℓ is angular momentum quantum number and m is mass of the particle. The variation of V(r) with r is given by

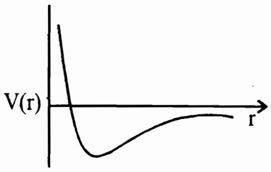
(a)



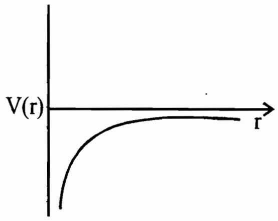
(b)



(c)



(d)



23. A quantum particle of mass m moves in two dimensions in an anisotropic harmonic oscillator potential $V(x,y)=\frac{1}{2}m\omega^2x^2+2m\omega^2y^2$. The energy eigenvalues are (n is a positive integer or zero)

(a) $\hbar\omega(2n+1)$ (b) $\hbar\omega(n+1)$ (c) $2\hbar\omega(n+1)$ (d) $\hbar\omega\left(n+\frac{3}{2}\right)$

ANSWER KEY

1. (a) 2. (d) 3. (b) 4. (b) 5. (b) 6. (b) 7. (b)
8. (c) 9. (c) 10. (c) 11. (c) 12. (a) 13. (c) 14. (b)
15. (c) 16. (b) 17. (a) 18. (b) 19. (c) 20. (c) 21. (c)
22. (c) 23. (d)

GATE Previous Years' Questions

1. Which one of the following is not a Boson? **[GATE 1994]**

(a) ${}_{2}^{4}\text{He}$ (b) ${}_{1}^{1}\text{H}$ (c) ${}_{2}^{3}\text{He}$ (d) photon

2. The energy of a single particle in an infinite potential box is E_0n^2 where $E_0 = \hbar^2\pi^2/2ma^2, n = 1, 2, 3, ...$ If three non-interacting bosons are in such a potential box, the ground state energy of the system will be

(a) $3E_0$ (b) $4E_0$ (c) $6E_0$ (d) $9E_0$ **[GATE 1998]**

3. Two spin -1/2 fermions having spins $\vec{S}_1$ and $\vec{S}_2$ interact via a potential $V(r) = \vec{S}_1 \cdot \vec{S}_2 V_0(r)$. The contributions of this potential in the singlet and triplet states, respectively, are **[GATE 2004]**

(a) $-\frac{3}{2}V_0(r)$ and $\frac{1}{2}V_0(r)$ (b) $\frac{1}{2}V_0(r)$ and $-\frac{3}{2}V_0(r)$

(c) $\frac{1}{4}V_0(r)$ and $-\frac{3}{4}V_0(r)$ (d) $-\frac{3}{4}V_0(r)$ and $\frac{1}{4}V_0(r)$

4. The wavefunctions of two identical particles in state n and s are given by $\phi_n(r_1)$ and $\phi_s(r_2)$, respectively. The particles obey Maxwell-Boltzmann statistics. The state of the combined two-particle system is expressed as **[GATE 2006]**

(a) $\phi_n(r_1)+\phi_s(r_2)$ (b) $\frac{1}{\sqrt{2}}\left[\phi_n(r_1)+\phi_s(r_2)+\phi_n\phi_s(r_1)\right]$

(c) $\frac{1}{\sqrt{2}}\left[\phi_n(r_1)+\phi_s(r_2)-\phi_n\phi_s(r_1)\right]$ (d) $\phi_n(r_1)\phi_s(r_2)$

5. Consider the wavefunction $\psi = \psi(\vec{r}_1, \vec{r}_2)\chi_s$ for a fermionic system consisting of two spin-half particles. The spatial part of the wavefunction is given by **[GATE 2012]**

$$\psi(\vec{r}_1,\vec{r}_2)=\frac{1}{\sqrt{2}}\left[\phi_1(\vec{r}_2)\phi_2(\vec{r}_2)+\phi_2(\vec{r}_1)\phi_1(\vec{r}_2)\right]$$

Where ϕ_1 and ϕ_2 are single particle states. The spin part χ_s of the wavefunction with spin states $\alpha\left(+\frac{1}{2}\right)$ and $\beta\left(-\frac{1}{2}\right)$ should be

(a) $\frac{1}{\sqrt{2}}(\alpha\beta+\beta\alpha)$ (b) $\frac{1}{\sqrt{2}}(\alpha\beta-\beta\alpha)$ (c) $\alpha\alpha$ (d) $\beta\beta$

6. The ground state and the first excited state wave functions of a one dimensional infinite potential well are ψ_1 and ψ_2, respectively. When two spin-up electrons are placed in this potential, which one of the following, with x_1 and x_2 denoting the position of the two electrons, correctly represents the space part of the ground state wave function of the system? **[GATE 2014]**

(a) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_1)-\psi_1(x_2)\psi_2(x_2)\right]$ (b) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2)+\psi_1(x_2)\psi_2(x_1)\right]$

(c) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_1)+\psi_1(x_2)\psi_2(x_2)\right]$ (d) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2)-\psi_1(x_2)\psi_2(x_1)\right]$

7. Which one of the following is a fermion? **[GATE 2014]**

(a) $\alpha -$particle (b) ${}_4Be^7$ nucleus (c) Hydrogen atom (d) Deuteron

8. The Pauli matrices for three spin-$\frac{1}{2}$ particles are $\vec{\sigma}_1, \vec{\sigma}_2$ and $\vec{\sigma}_3$, respectively. The dimension of the Hilbert space required to define an operator $\hat{O} = \vec{\sigma}_1 \cdot \vec{\sigma}_2 \times \vec{\sigma}_3$ is______ **[GATE 2015]**

9. Let the Hamiltonian for two spin-$\frac{1}{2}$ particles of equal masses m, momenta $\vec{p}_1$ and $\vec{p}_2$ and positions $\vec{r}_1$ and $\vec{r}_2$ be $H = \frac{1}{2m}p_1^2 + \frac{1}{2m}p_2^2 + \frac{1}{2}m\omega^2\left(r_1^2 + r_2^2\right) + k\vec{\sigma}_1 \cdot \vec{\sigma}_2$, where $\vec{\sigma}_1$ and $\vec{\sigma}_2$ denote the corresponding Pauli matrices, $\hbar\omega = 0.1eV$ and $k = 0.2eV$. If the ground state has net spin zero, then the energy (in eV) is______ **[GATE 2015]**

10. Consider a system of eight non-interacting, identical quantum particles of spin-$\frac{3}{2}$ in a one dimensional box of length L. The minimum excitation energy of the system, in units of $\frac{\pi^2\hbar^2}{2mL^2}$ is______ **[GATE 2015]**

11. If $\vec{s}_1$ and $\vec{s}_2$ are the spin operators of the two electrons of a He atom, the value of $\langle \vec{s}_1 \cdot \vec{s}_2 \rangle$ for the ground state is **[GATE 2016]**

(a) $-\frac{3}{2}\hbar^2$ (b) $-\frac{3}{4}\hbar^2$ (c) 0 (d) $\frac{1}{4}\hbar^2$

12. A two-dimensional square rigid box of side L contains six non-interacting electrons at $T = 0$ K. The mass of the electron is m. The ground state energy of the system of electrons, in units of $\frac{\pi^2\hbar^2}{2mL^2}$ is ______. **[GATE 2016]**

ANSWER KEY

1. (c) **2. (a)** **3. (d)** **4. (d)** **5. (b)** **6. (d)** **7. (b)**

8. (8) **9. (-0.3)** **10. (5)** **11. (b)** **12. (24)**

CSIR-UGC-NET Previous Years' Questions

1. In a system consisting of two spin $\frac{1}{2}$ particles labeled 1 and 2, let $\vec{S}^{(1)} = \frac{\hbar}{2}\vec{\sigma}^{(1)}$ and $\vec{S}^{(2)} = \frac{\hbar}{2}\vec{\sigma}^{(2)}$ denote the corresponding spin operators. Here $\vec{\sigma} \equiv (\sigma_x, \sigma_y, \sigma_z)$ and $\sigma_x, \sigma_y, \sigma_z$ are the three Pauli matrices.

[CSIR June 2011]

A. In the standard basis the matrices for the operators $S_x^{(1)} S_y^{(2)}$ and $S_y^{(1)} S_x^{(2)}$ respectively,

(a) $\frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ (b) $\frac{\hbar^2}{4}\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$

(c) $\frac{\hbar^2}{4}\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$ (d) $\frac{\hbar^2}{4}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

B. These two operators satisfy the relation

(a) $\left\{S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\right\} = S_z^{(1)}S_z^{(2)}$ (b) $\left\{S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\right\} = 0$

(c) $\left[S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\right] = i\,S_z^{(1)}S_z^{(2)}$ (d) $\left[S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\right] = 0$

2. The minimum energy of a collection of 6 non-interacting electrons of spin $-\frac{1}{2}$ placed in a one dimensional infinite square well potential of width L is

[CSIR Dec 2012]

(a) $14\pi^2\hbar^2 / mL^2$ (b) $91\pi^2\hbar^2 / mL^2$ (c) $7\pi^2\hbar^2 / mL^2$ (d) $3\pi^2\hbar^2 / mL^2$

3. Consider a system of two non-interacting identical fermions, each of mass *m* in an infinite square well potential of width *a*. (Take the potential inside the well to be zero and ignore spin). The composite wavefunction for the system with total energy

$$E = \frac{5\pi^2\hbar^2}{2ma^2}$$

is

[CSIR June 2014]

(a) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) - \sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(b) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) + \sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(c) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{3\pi x_2}{2a}\right) - \sin\left(\frac{3\pi x_1}{2a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(d) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\cos\left(\frac{\pi x_2}{a}\right) - \sin\left(\frac{\pi x_2}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

ANSWER KEY

1_A(c) **1_B(b)** **2. (a)** **3. (a)**

JEST Previous Years' Questions

1. The spatial part of a two-electron state is symmetric under exchange. If $|\uparrow\rangle$ and $|\downarrow\rangle$ represent the spin-up and spin-down states respectively of each particle, the spin-part of the two-particle state is **[JEST 2012]**

(a) $|\uparrow\rangle|\uparrow\rangle$ (b) $|\uparrow\rangle|\downarrow\rangle$

(c) $\left(|\downarrow\rangle|\uparrow\rangle-|\uparrow\rangle|\downarrow\rangle\right)/\sqrt{2}$ (d) $\left(|\downarrow\rangle|\uparrow\rangle+|\uparrow\rangle|\downarrow\rangle\right)/\sqrt{2}$

2. Consider a system of two spin-1/2 particles with total spin $\vec{S}=\vec{S}_1+\vec{S}_2$, where S_1 and S_2 are in terms of Pauli matrices σ_i. The spin triplet projection operator is **[JEST 2012]**

(a) $\frac{1}{4}+s_1.s_2$ (b) $\frac{3}{4}-s_1.s_2$ (c) $\frac{3}{4}+s_1.s_2$ (d) $\frac{1}{4}-s_1.s_2$

3. The grond state energy of 5 identical spin-1/2 particles which are subject to a one-dimensional simple harmonic oscillator potential of frequency ω is **[JEST 2012]**

(a) $\frac{15}{2}\hbar\omega$ (b) $\frac{13}{2}\hbar\omega$ (c) $\frac{1}{2}\hbar\omega$ (d) $5\hbar\omega$

4. Two electrons are confined in a one dimensional box of length L. The one-electron states are given by $\psi_n(x)=\sqrt{2/L}\sin(n\pi x/L)$. What would be the ground state wave function $\psi(x_1,x_2)$ if both electrons are arranged to have the same spin state? **[JEST 2013]**

(a) $\psi(x_1,x_2)=\frac{1}{\sqrt{2}}\left[\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)+\frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)\right]$

(b) $\psi(x_1,x_2)=\frac{1}{\sqrt{2}}\left[\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)-\frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)\right]$

(c) $\psi(x_1,x_2)=\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

(d) $\psi(x_1,x_2)=\frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)$

ANSWER KEY

1. (c) **2. (c)** **3. (b)** **4. (b)**

TIFR Previous Years' Questions

1. Two identical non-interacting particles, each of mass m and spin 1/2, are placed in a one-dimensional box of length L. In quantum mechanics, the lowest possible value of the total energy of these two particles is ε_0. If, instead, four such particles are introduced into a similar one-dimensional box of length 2L, then the lowest possible value of their total energy will be **[TIFR 2011]**

(a) $2\varepsilon_0$ (b) $5\varepsilon_0/4$ (c) $3\varepsilon_0/2$ (d) ε_0

2. Consider two spin-1/2 identical particles A and B, separated by a distance r, interacting through a potential $V(r)=\frac{V_0}{r}\vec{S}_A.\vec{S}_B$ where V_0 is a positive constant and the spins are $\vec{S}_{A,B}=\vec{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ in terms of the Pauli spin matrices. The exception values of this potential in the spin-singlet and triplet states are

(a) $\text{Singlet}:-\frac{V_0}{3r}, \text{Triplet}:\frac{V_0}{r}$ (b) $\text{Singlet}:-\frac{3V_0}{r}, \text{Triplet}:\frac{V_0}{r}$ **[TIFR 2016]**

(c) $\text{Singlet}:\frac{3V_0}{r}, \text{Triplet}:-\frac{V_0}{r}$ (d) $\text{Singlet}:-\frac{V_0}{r}, \text{Triplet}:\frac{3V_0}{r}$

ANSWER KEY

1. (b) **2. (b)**

Other Examinations Previous Years' Questions

1. A system consists of two identical particles, each of which can be in any one of the two quantum states. The allowed states are

(a) 3 symmetric and antisymmetric (b) 1 symmetric and 3 antisymmetric

(c) 1 symmetric and 1 antisymmetric (d) 3 symmetric and 3 antisymmetric.

2. Consider a one dimensional infinite square well potential $V(x)=\begin{cases}0, \text{ for } 0\le x\le L\\ \infty, \text{ otherwise}\end{cases}$.

If two identical non- interacting bosons occupy the lowest two energy levels, the unnormalized wavefunction of the combined system is given by

(a) $\psi(x_1,x_2)=\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

(b) $\psi(x_1,x_2)=\frac{1}{2}\left[\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)-\sin\left(\frac{\pi x_2}{L}\right)\sin\left(\frac{2\pi x_1}{L}\right)\right]$

(c) $\psi(x_1,x_2)=\frac{1}{2}\left[\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)+\sin\left(\frac{\pi x_2}{L}\right)\sin\left(\frac{2\pi x_1}{L}\right)\right]$

(d) $\psi(x_1,x_2)=\frac{1}{2}\left[\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)+\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)\right]$

3. Consider a one-dimensional infinite square well potential $V(x)=\begin{cases}0 & \le x\le L\\ \infty & \text{otherwise}\end{cases}$

If each of the lowest two energy levels are occupied by identical non-interacting bosonic particles (one in each level), then the unnormalized wave function of the combined system is:

(a) $\psi(x_1,x_2)=\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

(b) $\psi(x_1,x_2)=\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)+\sin\left(\frac{\pi x_2}{L}\right)\sin\left(\frac{2\pi x_1}{L}\right)$

(c) $\psi(x_1,x_2)=\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)+\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

(d) $\psi(x_1,x_2)=\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)+\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

4. Lithium has three electrons in the state $\psi_1(\vec{r}_1)$; $\psi_2(\vec{r}_2)$ and $\psi_3(\vec{r}_3)$.

The wavefunction of the three electrons (together) can be written as

(a) $\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\psi_3(\vec{r}_3)$ (b) $(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)-\psi_1(\vec{r}_1)\psi_2(\vec{r}_1)\psi_3(\vec{r}_3))$

(c) $(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)+\psi_1(\vec{r}_1)\psi_2(\vec{r}_1)\psi_3(\vec{r}_3))$ (d) $\frac{1}{\sqrt{6}}\begin{vmatrix}\psi_1(\vec{r}_1) & \psi_1(\vec{r}_2) & \psi_1(\vec{r}_3)\\ \psi_2(\vec{r}_1) & \psi_2(\vec{r}_2) & \psi_3(\vec{r}_3)\\ \psi_3(\vec{r}_1) & \psi_3(\vec{r}_2) & \psi_3(\vec{r}_3)\end{vmatrix}$

5. Consider a system of two spin-half particles, in a state with total spin quantum number $S=0$. The eigenvalue of the spin Hamiltonian $H=A\,\vec{S}_1\cdot\vec{S}_2$ (A is a positive constant) in this state is:

(a) $A\hbar^2/4$ (b) $-A\hbar^2/4$ (c) $3A\hbar^2/4$ (d) $-3A\hbar^2/4$

6. What is the ground state energy of six non-interacting electrons in a 2-dimensional, isotropic, simple harmonic oscillator characterized by ω?

(a) $3\hbar\omega$ (b) $5\hbar\omega$ (c) $10\,\hbar\omega$ (d) $12\,\hbar\omega$

ANSWER KEY

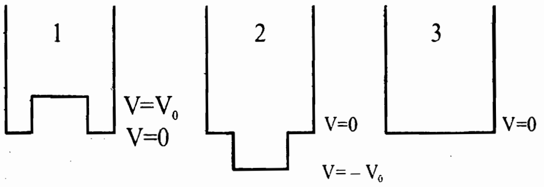
1. (a) 2. (c) 3. (b) 4. (d) 5. (d) 6. (c)

GATE Previous Years' Questions

1. Let $E'_n\,(n = 0,1,2,....)$ be the energy eigenvalues for a particle of mass m placed in an anharmonic potential $V(x)=\frac{1}{2}m\omega^2x^2+ax^4,(a>0)$. Let $E_n \equiv \left(n+\frac{1}{2}\right)\hbar\omega$. Then according to the first order perturbation theory: **[GATE 1996]**

(a) $E'_0 = E_0$ (b) $E'_0 > E_0$ (c) $E'_0 < E_0$ (d) $E'_0 < E_0$ for all n

2. Let E_1, E_2, E_3 be the respective ground state energies of the following potentials: Which one of the following is correct? **[GATE 2000]**



(a) $E_1 < E_2 < E_3$ (b) $E_3 < E_1 < E_2$ (c) $E_2 < E_3 < E_1$ (d) $E_2 < E_1 < E_3$.

3. A quantum harmonic oscillator is in the energy eigenstate $|n\rangle$. A time independent perturbation $\lambda\left(a^\dagger a\right)^2$ acts on the particle, where λ is a constant of suitable dimensions and a and $a^\dagger$ are lowering and raising operators respectively. Then the first order energy shift is gien by **[GATE 2001]**

(a) λn (b) $\lambda^2 n$ (c) λn^2 (d) $(\lambda n)^2$

4. The wave function of a one-dimensional harmonic oscillator is $\Psi_0 = A\exp\left(\frac{-\alpha^2x^2}{2}\right)$ for the ground state $E_0\,(\alpha x/10)^4$, the first order change in the ground state energy is: [Given: $\left[\Gamma(x+1)=\int_0^\infty t^x\exp(-t)dt\right]$ **[GATE 2004]**

(a) $\left(\frac{1}{2}E_0\right)10^{-4}$ (b) $(3E_0)10^{-4}$ (c) $\left(\frac{3}{4}E_0\right)10^{-4}$ (d) $(E_0)10^{-4}$

Common data for Q. 5, Q. 6 and Q.7

An unperturbed two-level system has energy eigenvalues E_1 and E_2, and eigen functions $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$.

When perturbed, its Hamiltonian is represented by $\begin{pmatrix}E_1 & A\\ A^* & E_2\end{pmatrix}$ **[GATE 2006]**

5. The first-order correction to E_1 is:

(a) 4A (b) 2A (c) A (d) 0

6. The second-order correction to E_1 is

(a) 0 (b) A (c) $\frac{A^2}{E_2-E_1}$ (d) $\frac{|A|^2}{E_1-E_2}$

7. The first-order correction to the eigenfunction $\begin{pmatrix}1\\0\end{pmatrix}$ is:

(a) $\begin{pmatrix}0\\A^*/(E_1-E_2)\end{pmatrix}$ (b) $\begin{pmatrix}0\\1\end{pmatrix}$ (c) $\begin{pmatrix}A^*/(E_1-E_2)\\0\end{pmatrix}$ (d) $\begin{pmatrix}1\\1\end{pmatrix}$

8. A particle of mass m is confined in an infinite potential well $V(x)=\begin{cases}0 & \text{if } 0<x<L,\\ \infty & \text{otherwise}\end{cases}$

It is subjected to a perturbing potential $V_p(x)=V_0\sin\left(\frac{2\pi x}{L}\right)$ within the well. Let $E^{(1)}$ and $E^{(2)}$ be the corrections to the ground state energy in the first and second order in V_0, respectively. Which of the following are true? **[GATE 2010]**

(a) $E^{(1)}=0;\ E^{(2)}<0$ (b) $E^{(1)}>0;\ E^{(2)}=0$

(c) $E^{(1)}=0;\ E^{(2)}$ depends on the sign of V_0 (d) $E^{(1)}<0;\ E^{(2)}<0$

9. The normalized eigenstates of a particle in a one-dimensional potential well

$$V(x)=\begin{cases}0 & \text{if } 0<x<a\\ \infty & \text{otherwise}\end{cases}$$

are given by

$$\psi_n(x)=\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$$

The particle is subjected to a perturbation

$$\begin{aligned}V'(x)&=V_0\cos\left(\frac{\pi x}{a}\right) && \text{for } 0\le x\le\frac{a}{2}\\ &=0 && \text{otherwise}\end{aligned}$$

[GATE 2011]

The shift in the ground state energy due tothe perturbation, in the first order perturbation theory,

(a) $\frac{2V_0}{3\pi}$ (b) $\frac{V_0}{3\pi}$ (c) $-\frac{V_0}{3\pi}$ (d) $-\frac{2V_0}{3\pi}$

10. Consider a system in the unperturbed state described by the Hamiltonian, $H_0=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$. The system is subjected to a perturbation of the form $H'=\begin{pmatrix}\delta & \delta\\ \delta & \delta\end{pmatrix}$, where $\delta<<1$. The energy eigenvalues of the perturbed system using the first order perturbation approximation are **[GATE 2012]**

(a) 1 and $(1+2\delta)$ (b) $(1+\delta)$ and $(1-\delta)$ (c) $(1+2\delta)$ and $(1-2\delta)$ (d) $(1+\delta)$ and $(1-2\delta)$

Common Data for Q. 11 and Q. 12:

To the given unperturbed Hamiltonian $\begin{bmatrix}5 & 2 & 0\\ 2 & 5 & 0\\ 0 & 0 & 2\end{bmatrix}$

we add a small perturbation given by $\varepsilon\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & -1\\ 1 & -1 & 1\end{bmatrix}$ **[GATE 2013]**

where ε is a small quantity.

11. The ground state eigen vector of the unperturbed Hamiltonian is

(a) $(1/\sqrt{2},1/\sqrt{2},0)$ (b) $(1/\sqrt{2},-1/\sqrt{2},0)$ (c) (0, 0, 1) (d) (1, 0, 0)

12. A pair of eigen values of the perturbed Hamiltonian, using first order perturbation theory, is

(a) $3+2\varepsilon, 7+2\varepsilon$ (b) $3+2\varepsilon, 2+\varepsilon$ (c) $3, 7+2\varepsilon$ (d) $3, 2+2\varepsilon$

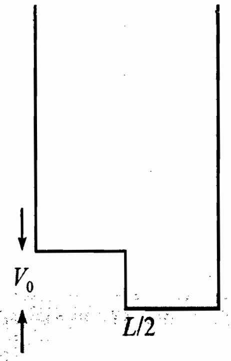
13. A particle is confined in a one-dimensional potential box with the potential

$$V(x)=\begin{cases}0 & \text{if } 0<x<a\\ \infty & \text{otherwise}\end{cases}$$

If the particle is subjected to a perturbation, within the box, $W=\beta x$, where β is a small constant, the first order correction to the ground state energy is [GATE 2014]

(a) 0 (b) $\frac{a\beta}{4}$ (c) $\frac{a\beta}{2}$ (d) $a\beta$

14. A particle is confined in a box of length L as shown below. **[GATE 2015]**



If the potential V_0 is treated as a perturbation, including the first order correction, the ground state energy is

(a) $E=\frac{\hbar^2\pi^2}{2mL^2}+V_0$ (b) $E=\frac{\hbar^2\pi^2}{2mL^2}-\frac{V_0}{2}$ (c) $E=\frac{\hbar^2\pi^2}{2mL^2}+\frac{V_0}{4}$ (d) $E=\frac{\hbar^2\pi^2}{2mL^2}+\frac{V_0}{2}$

15. A hydrogen atom is in its ground state. In the presence of a uniform electric field $\vec{E}=E_0\hat{z}$, the leading order change in its energy is proportional to $(E_0)^n$. The value of the exponent n is_______. **[GATE 2016]**

ANSWER KEY

1. (b)	**2.(c)**	**3. (c)**	**4. (c)**	**5. (d)**	**6. (d)**	**7. (a)**
8. (a)	**9. (a)**	**10. (a)**	**11. (c)**	**12. (c)**	**13. (c)**	**14. (d)**
15. (2)						

CSIR-UGC-NET Previous Years' Questions

1. If the perturbation $H' = ax$, where a is a constant; is added to the infinite square well potential

$$V(x) = \begin{cases} 0 \text{ for } & 0 \le x \le \pi \\ \infty & \text{otherwise} \end{cases}$$

The first order correction to ground state energy is: **[CSIR June 2011]**

(a) $\frac{a\pi}{2}$ (b) $a\pi$ (c) $\frac{a\pi}{4}$ (d) $\frac{a\pi}{\sqrt{2}}$

2. The perturbation $H' = bx^4$, where 'b' is a constant, is added to the one dimensional harmonic oscillator potential $V(x) = \frac{1}{2}m\omega^2x^2$. Which of the following denotes the correction to the ground state energy to first order in b ? **[CSIR Dec 2011]**

[Hint : The normalized ground state wave function of the one dimensional harmonic oscillator potential is

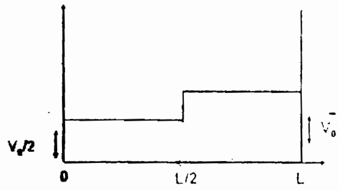
$\psi_0 = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} e^{-m\omega x^2/2\hbar}$.,You may use the following integral $\int_{-\infty}^{\infty} x^{2n}e^{-ax^2}dx = a^{-n-\frac{1}{2}}\Gamma\left(n+\frac{1}{2}\right)$

(a) $\frac{3b\hbar^2}{4m^2\omega^2}$ (b) $\frac{3b\hbar^2}{2m^2\omega^2}$ (c) $\frac{3b\hbar^2}{2\pi m^2\omega^2}$ (d) $\frac{b\hbar}{4m\omega}$

3. A constant perturbation as shown in the figure below acts on a particle of mass *m* confined in a infinite potential well between 0 and *L*. The first-order correction to the ground state energy of the particle is **[CSIR Dec 2011]**

(a) $\frac{V_0}{2}$ (b) $\frac{3V_0}{4}$

(c) $\frac{V_0}{4}$ (d) $\frac{3V_0}{2}$



4. Consider an electron in a box of lenght L with periodic boundary condition $\psi(x) = \psi(x+L)$. If the electron is in the $\psi_k(x) = \frac{1}{\sqrt{L}}e^{jkx}$ with energy $\varepsilon_k = \frac{\hbar^2k^2}{2m}$, what is the correction to its energy, to second order of perturbation theory, when it is subjected to weak periodic potential $V(x) = V_0\cos gx$, where g is an integral multiple of the $2\pi/L$? **[CSIR June 2012]**

(a) $V_0^2\,\varepsilon_g/\varepsilon_k^2$ (b) $-\frac{mV_0^2}{2\hbar^2}\left(\frac{1}{g^2+2kg}+\frac{1}{g^2-2kg}\right)$

(c) $V_0^2\left(\varepsilon_k-\varepsilon_g\right)/\varepsilon_g^2$ (d) $V_0^2/\left(\varepsilon_k+\varepsilon_g\right)$

9. The motion of a particle of mass m in one dimension is described by the Hamiltonian $H=\frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2+\lambda x$. What is the difference between the (quantized) energies of the first two levels? (In the following, $\langle x\rangle$ is the expectation value of x in the ground state)

[CSIR Dec 2013]

(a) $\hbar\omega-\lambda\langle x\rangle$ (b) $\hbar\omega+\lambda\langle x\rangle$ (c) $\hbar\omega+\frac{\lambda^2}{2m\omega^2}$ (d) $\hbar\omega$

10. The ground state eigenfunction for the potential $V(x)=-\delta(x)$, where $\delta(x)$ is the delta function, is given by $\psi(x)=Ae^{-\alpha|x|}$, where A and $\alpha>0$ are constants. If a perturbation $H'=bx^2$ is applied, the first order correction to the energy of the ground state will be **[CSIR June 2014]**

(a) $\frac{b}{\sqrt{2}\alpha^2}$ (b) $\frac{b}{\alpha^2}$ (c) $\frac{2b}{\alpha^2}$ (d) $\frac{b}{2\alpha^2}$

11. The Hamiltonian H_0 for a three-state quantum system is given by the matrix $H_0=\begin{pmatrix}1&0&0\\0&2&0\\0&0&2\end{pmatrix}$. When perturbed by $H'=\in\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix}$ where $\in\ll 1$, the resulting shift in the energy eigenvalue $E_0=2$ is

[CSIR Dec 2014]

(a) $\in,-2\in$ (b) $-\in,2\in$ (c) $\pm\in$ (d) $\pm 2\in$

12. A hydrogen atom is subjected to the perturbation $V_{pert}(r)=\in\cos 2r/a_0$ where a_0 is the Bohr radius. The change in the ground state energy to first order in $\in$ is **[CSIR Dec. 2015]**

(a) $\in/4$ (b) $\in/2$ (c) $-\in/2$ (d) $-\in/4$

13. Consider a particle of mass m in a potential $V(x)=\frac{1}{2}m\omega^2x^2+g\cos kx$. The change in the ground state energy, compared to the simple harmonic potential $\frac{1}{2}m\omega^2x^2$, to first order in g is **[CSIR June 2016]**

(a) $g\exp\left(-\frac{k^2\hbar}{2m\omega}\right)$ (b) $g\exp\left(\frac{k^2\hbar}{2m\omega}\right)$ (c) $g\exp\left(-\frac{2k^2\hbar}{m\omega}\right)$ (d) $g\exp\left(-\frac{k^2\hbar}{4m\omega}\right)$

ANSWER KEY

1. (a) **2. (a)** **3. (b)** **4. (b)** **5. (a)** **6. (d)** **7. (b)**

8. (c) **9. (d)** **10. (d)** **11. (c)** **12. (d)** **13. ( )**

JEST Previous Years' Questions

1. Consider a spin-$\frac{1}{2}$ particle characterized by the Hamiltonian $H = \omega S_z$. Under a perturbation $H' = gS_x$, the second order correction to the ground state energy is given by, **[JEST 2015]**

(a) $-\frac{g^2}{4\omega}$ (b) $\frac{g^2}{4\omega}$ (c) $-\frac{g^2}{2\omega}$ (d) $\frac{g^2}{2\omega}$

2. A particle of mass m is confined in a potential well given by $V(x) = 0$ for $-L/2 < x < L/2$ and $V(x) = \infty$ elsewhere. A perturbing potential $H'(x) = \alpha x$ has been applied to the system. Let the first and second order corrections to the ground state be $E_0^{(1)}$ and $E_0^{(2)}$, respectively. Which one of the following statements is correct? **[JEST 2015]**

(a) $E_0^{(1)} < 0$ and $E_0^{(2)} > 0$ (b) $E_0^{(1)} = 0$ and $E_0^{(2)} > 0$

(c) $E_0^{(1)} > 0$ and $E_0^{(2)} < 0$ (d) $E_0^{(1)} = 0$ and $E_0^{(2)} < 0$

3. Consider a quantum particle of mass m in one dimension in an infinite potential well, i.e., $V(x) = 0$ for $-a/2 < x < a/2$, and, $V(x) = \infty$ for $|x| \geq a/2$. A small perturbation, $V'(x) = 2 \in |x|/a$, is added. The change in the ground state energy to $O(\varepsilon)$ is: **[JEST 2016]**

(a) $\frac{\varepsilon}{2\pi^2}(\pi^2 - 4)$ (b) $\frac{\varepsilon}{2\pi^2}(\pi^2 + 4)$ (c) $\frac{\varepsilon\pi^2}{2}(\pi^2 + 4)$ (d) $\frac{\varepsilon\pi^2}{2}(\pi^2 - 4)$

ANSWER KEY

1. (a) 2. (d) 3. (a)

TIFR Previous Years' Questions

1. A charged particle is in the ground state of a one-dimensional harmonic oscillator potential, generated by electrical means. If the power is suddenly switched off, so that the potential disappears, then, according to quantum mechanics, **[TIFR 2010]**

(a) the particle will shoot out of the well and move out towards infinity in one of the two possible direction as

(b) the particle will stop oscillating and as time increases it may be found farther and farther way from the centre of the well

(c) the particle will keep oscillating about the same mean position but with increasing amplitude as time increases

(d) the particle will undergo a transition to one of the higher excited states of the harmonic oscillator

2. A particle of mass 'm' and charge 'e' is in the ground state of a one-dimensional harmonic oscillator potential in the presence of a uniform external electric field E. The total potential felt by the particle is

$$V(x)=\frac{1}{2}kx^2-eEx$$ **[TIFR 2014]**

If the electric field is suddenly switched off, then the particle will

(a) Make a transition to any harmonic oscillator state with $x=-eE/k$ as origin without emitting any photon.

(b) Make a transition to any harmonic oscillator state with $x=0$ as origin and absorb a photon.

(c) Settle into the harmonic oscillator ground state with $x=0$ as origin after absorbing a photon.

(d) Oscillate back and forth with initial amplitude eE/k, emitting multiple photons as it does so.

ANSWER KEY

1. (a) **2. (b)**

Other Examinations Previous Years' Questions

1. The validity of Fermi's golden rule which connects two states with energy difference ΔE, implies that it is true for

(a) All times t (b) $t\geq\frac{\hbar}{\Delta E}$ (c) $t\leq\frac{\hbar}{\Delta E}$ (d) $\Delta E=0$

2. A one-dimensional harmonic oscillator whose Hamiltonian is $H_0=\frac{1}{2m}\hat{p}^2+\frac{1}{2m}\omega^2\hat{x}^2$ is perturbed such that the total Hamiltonian is $H=H_0+\varepsilon\hat{x}$. The first order correction to the ground state is

(a) $2\varepsilon\hbar\omega$ (b) $\varepsilon\hbar\omega/2$ (c) $\varepsilon\hbar\omega$ (d) 0

3. The first-order correction to the eigenvalues of the Hamiltonian $\hat{H}_0=\frac{\hat{p}^2}{2m}+\frac{1}{2}k\hat{x}^2$ due to a perturabation $\hat{H}_1=\alpha\hat{x}$ are

(a) 0 (b) α/k (c) $2\alpha/k$ (d) $-2\alpha/k$

ANSWER KEY

1. (d) **2. (d)** **3. (a)**

GATE Previous Years' Questions

1. A particle is scatterd by a central potential. If the dominant contribution to the scattering is from the p-wave, the differential cross-section is **[GATE 2000]**

(a) isotropic
(b) proportional to $\cos^2\theta$
(c) stabilize the magnetization
(d) cause critical magnetic fluctuations

2. If σ is the total cross-section and $f(\theta)$ is the scattering amplitude (θ being the angle of scattering) for a quantum mechanical elastic scattering by a spherically symmetric potential, then which of the following is true? Note that k is the magnitude of the wave vector along the $\hat{z}$ direction. **[GATE 2002]**

(a) $\sigma = |f(\theta)|^2$
(b) $\sigma = \frac{4\pi}{k}|f(\theta=0)|^2$
(c) $\sigma = \frac{4\pi}{k} \times$ imaginary part of $[f(\theta=0)]$
(d) $\sigma = \frac{4\pi}{k}|f(\theta)|^2$

Common data for Q. 3 and Q. 4.

A particle is scattered by a spherically symmetric potential. In the centre of mass (CM) frame the wavefunction of the incoming particle is $\psi = Ae^{ikz}$ where k is the wavevector and A is a constant. **[GATE 2005]**

3. If $f(\theta)$ is an angular function then in the asymptotic region the scattered wavefunction has the form

(a) $\frac{Af(\theta)e^{ikr}}{r}$
(b) $\frac{Af(\theta)e^{-ikr}}{r}$
(c) $\frac{Af(\theta)e^{ikr}}{r^2}$
(d) $\frac{Af(\theta)e^{-ikr}}{r^2}$

4. The differential scattering cross section $\sigma(\theta)$ in CM frame is:

(a) $\sigma(\theta) = |A|^2\frac{|f(\theta)|^2}{r^2}$
(b) $\sigma(\theta) = |A|^2|f(\theta)|^2$
(c) $\sigma(\theta) = |f(\theta)|^2$
(d) $\sigma(\theta) = |A||f(\theta)|$

5. The scattering of particles by a potential can be analyzed by Born approximation. In particular, if the scattered wave is replaced by an appropriate plane wave, the corresopnding Born approximation is known as the first Born approximation. Such as approximation is valid for **[GATE 2016]**

(a) large incident energies and weak scattering potentials
(b) large incident energies and strong scattering potentials.
(c) small incident energies and weak scattering potentials
(d) small incident energies and strong scattering potentials.

6. Consider an elastic scattering of particles in $l = 0$ states. If the corresponding phase shift δ_0 is 90° and the magnitude of the incident wave vector is equal to $\sqrt{2\pi}$ fm^{-1} then the total scattering cross section in units of fm^2 is ____________ **[GATE 2016]**

ANSWER KEY

1. (b) **2. (c)** **3. (a)** **4. (c)** **5. (a)** **6. (2)**

CSIR-UGC-NET Previous Years' Questions

1. A free particle described by a plane wave and moving in the positive z-direction undergoes scattering by a potential $V(r)=\begin{cases} V_0 & \text{if } r \le R \\ 0 & \text{if } r > R \end{cases}$ if V_0 is changes to $2V_0$, keeping R fixed, then the differential scattering cross-section, in the born approximation, **[CSIR June 2012]**

(a) increases to four times the original value (b) increases to twice the original value
(c) decreases to half the original value (d) decreases to one fourth the original value

2. In the Born approximation, the scattering amplitude **f**(θ) for the Yukawa potential $V(r)=\frac{\beta e^{-\mu r}}{r}$ is given by: (in the following $b = 2k\sin\frac{\theta}{2}$, $E=\hbar^2k^2/2m$) **[CSIR June 2013]**

(a) $-\frac{2m\beta}{\hbar^2(\mu^2+b^2)^2}$ (b) $-\frac{2m\beta}{\hbar^2(\mu^2+b^2)}$ (c) $-\frac{2m\beta}{\hbar^2\sqrt{\mu^2+b^2}}$ (d) $-\frac{2m\beta}{\hbar^2(\mu^2+b^2)^3}$

3. The scattering amplitude $f(\theta)$ for the Yukawa potential $V(r)=\beta e^{-\mu r}$, where β and μ are positive constants, is given, in the Born approximation by (in the following $b=2k\sin\frac{\theta}{2}$, $E=\hbar^2k^2/2m$) **[CSIR June 2014]**

(a) $-\frac{4m\beta\mu}{\hbar^2\left(b^2+\mu^2\right)^2}$ (b) $-\frac{4m\beta\mu}{\hbar^2b^2\left(b^2+\mu^2\right)}$ (c) $-\frac{4m\beta\mu}{\hbar^2\sqrt{b^2+\mu^2}}$ (d) $-\frac{4m\beta\mu}{\hbar^2\left(b^2+\mu^2\right)^3}$

4. The differential cross-section for scattering by a target is given by $\frac{d\sigma(\theta,\varphi)}{d\Omega}=a^2+b^2\cos^2\theta$. If N is the flux of the incoming particles, the number of particles scattered per unit time is **[CSIR June 2015]**

(a) $\frac{4\pi}{3}N(a^2+b^2)$ (b) $4\pi N\left(a^2+\frac{1}{6}b^2\right)$

(c) $4\pi N\left(\frac{1}{2}a^2+\frac{1}{3}b^2\right)$ (d) $4\pi N\left(a^2+\frac{1}{3}b^2\right)$

5. A particle of energy E scatters off a repulsive spherical potential **[CSIR June 2015]**

$$V(r)=\begin{cases} V_0 & \text{for} \quad r<a \\ 0 & \text{for} \quad r \ge a \end{cases}$$

where V_0 and a are positive constants. In the low energy limit, the total scattering cross-section is $\sigma = 4\pi a^2\left(\frac{1}{ka}\tanh ka - 1\right)^2$, where $k^2=\frac{2m}{\hbar^2}(V_0-E)>0$. In the limit $V_0 \to \infty$ the ratio of σ to the classical scattering cross-section off a sphere of radius a is

(a) 4 (b) 3 (c) 1 (d) 1/2

6. In the scattering of some elementary particles, the scattering cross-section σ is found to depend on the total energy E and the fundamental constants h (Planck's constant) and c (the speed of light in vacuum). Using dimensional analysis, the dependence of σ on these quantities is given by **[CSIR Dec. 2015]**

(a) $\sqrt{\frac{hc}{E}}$ (b) $\frac{hc}{E^{3/2}}$ (c) $\left(\frac{hc}{E}\right)^2$ (d) $\frac{hc}{E}$

ANSWER KEY

1. (a) **2. (b)** **3. (a)** **4. (d)** **5. (a)** **6. (c)**

Other Examinations Previous Years' Questions

1. For the scattering under a central force field, the asymptotic form of the total wave function after scattering is:

(a) $\psi(r) = e^{ikr} + e^{-ikr}$ (b) $\psi(r) = f(\theta)\dfrac{e^{-ik.r}}{r}$

(c) $\psi(r) = e^{ik.r} + f(\theta)e^{ik.r.}$ (d) $\psi(r) = e^{ik.r} + f(\theta)\dfrac{e^{ik.r}}{r}$

2. Born approximation is justified for
(a) Particle moving very slowly. (b) Very strong potential fields.
(c) Fast particles in a weak potential. (d) Hydrogen atom.

3. A beam of quantum mechanical particles of momentum k moving along the z-direction scatters off a spherically symmetric potential

$$V(r) = \begin{cases} V_0, & \text{for } 0 \le r \le R \\ 0, & \text{for } r > R \end{cases}$$

The leading term of the wave function of the scattered particles is of the form

(a) e^{ikr} (b) e^{ikz} (c) $\dfrac{e^{ikr}}{r}$ (d) $\dfrac{e^{ikr}}{r^2}$

4. Let k be the wave number of the incident plane wave in a scattering experiment. If the scattering is purely a p-wave with the phase shift $\delta_1 = \pi/4$, then the total scattering cross-section is:

(a) $2\pi/k^2$ (b) $6\pi/k^2$ (c) 0 (d) k^2

5. For a spherically symmetric potential, $V(r) = g\delta(\vec{r})$ where $\delta(\vec{r})$ is a three dimensional δ – function and g is a constant. The scattering amplitude $f(\theta)$ in the Born approximation then varies as

(a) g (b) g^2 (c) 1/g (d) $1/g^2$.

6. A beam of particles of linear momentum $\hbar k$ is scattered by a short-range potential of range a_1 where $ak << 1$. Which of the following statements is correct?
(a) All phase shifts are zero (b) There is no scattering
(c) The scattering is spherically symmetric (d) All phase shifts are equal.

7. Consider the scattering of two identical fermions. If the two particles are in a spin singlet state, the differential cross section relates to the scattering amplitude as

(a) $\dfrac{d\sigma}{d\Omega} = |f(\theta)|^2 + |f(\pi-\theta)|^2$ (b) $\dfrac{d\sigma}{d\Omega} = |f(\theta)|^2 - |f(\pi-\theta)|^2$

(c) $\dfrac{d\sigma}{d\Omega} = |f(\theta) - f(\pi-\theta)|^2$ (d) $\dfrac{d\sigma}{d\Omega} = |f(\theta) + f(\pi-\theta)|^2$

ANSWER KEY

1. (d) **2. (c)** **3. (c)** **4. (b)** **5. (a)** **6. (c)** **7. (d)**

GATE Previous Years' Questions

1. Let the hermitian operator H be the Hamiltonian of a quantum mechanical particle, E_0 its ground state energy, and $\psi(\vec{r})$ an arbitrary normalized trial wavefunction. The quantity **[GATE 1991]**

$$\int \psi^*(\vec{r}) H \psi(\vec{r}) dV$$

(a) must necessarily be less than or equal to E_0.
(b) must necessarily be greater than or equal to E_0.
(c) may be any real number – greater than, equal to, or less tahn E_0
(d) is purely imaginary.

ANSWER KEY

1. (b)

CSIR-UGC-NET Previous Years' Questions

1. A particle in one dimension moves under the influence of a potential $V(x) = ax^n$, where a is a real constant. For large n the quantized energy level E_n depends on n as: **[CSIR June 2011]**

(a) $E_n \sim n^3$ (b) $E_n \sim n^{4/3}$ (c) $E_n \sim n^{6/5}$ (d) $E_n \sim n^{3/2}$

2. A variational calculation is done with the normalized trial wavefunction $\psi(x) = \frac{\sqrt{15}}{4a^{5/2}}\left(a^2 - x^2\right)$ for the one dimensional potential well $V(x) = \begin{cases} 0 \text{ if } |x| \le a \\ \infty \text{ if } |x| > a \end{cases}$

The ground state energy is estimated to be **[CSIR June 2012]**

(a) $\frac{5\hbar^3}{3ma^2}$ (b) $\frac{3\hbar^2}{2ma^2}$ (c) $\frac{3\hbar^2}{5ma^2}$ (d) $\frac{5\hbar^2}{4ma^2}$

3. What would be the ground state energy of the Hamiltonian

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x)$$

if variational principle is used to estimate it with the trial wavefunction $\psi(x) = A\, e^{-bx^2}$ with b as the variational parameter?

[Hint: $\int\limits_{-\infty}^{\infty} x^{2n}e^{-2bx^2}dx = (2b)^{-n-\frac{1}{2}}\Gamma\left(n+\frac{1}{2}\right)$] **[CSIR Dec 2012]**

(a) $-m\alpha^2/2\hbar^2$ (b) $-2m\alpha^2/\pi\hbar^2$ (c) $-m\alpha^2/\pi\hbar^2$ (d) $m\alpha^2/\pi\hbar^2$

4. The bound on the ground state energy of the Hamiltonian with an attractive delta-function potential, namely

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x)$$

using the variational principle with the trial wavefunction $\psi(x) = A\exp(-bx^2)$ is

[CSIR June 2013]

(a) $-m\alpha^2/4\pi\hbar^2$ (b) $-m\alpha^2/2\pi\hbar^2$ (c) $-m\alpha^2/\pi\hbar^2$ (d) $-m\alpha^2/\sqrt{5}\pi\hbar^2$

5. Consider a particle of mass m in the potential $V(x) = a|x|, a > 0$. The energy eigen-values $E_n (n = 0, 1, 2, ...)$, in the WKB approximation, are **[CSIR Dec 2014]**

(a) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{1/3}$ (b) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{2/3}$ (c) $\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)$ (d) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{4/3}$

6. The ground state energy of the attractive delta function potential $V(x) = -b\delta(x)$, where $b > 0$, calculated with the variational trial function $\psi(x) = \begin{cases} A\cos\frac{\pi x}{2a}, & \text{for } -a < x < a, \\ 0, & \text{otherwise,} \end{cases}$ is **[CSIR Dec 2014]**

(a) $-\frac{mb^2}{\pi^2\hbar^2}$ (b) $-\frac{2mb^2}{\pi^2\hbar^2}$ (c) $-\frac{mb^2}{2\pi^2\hbar^2}$ (d) $-\frac{mb^2}{4\pi^2\hbar^2}$

7. The ground state energy of a particle in the potential $V(x) = g\,|\,x\,|$, estimated using the trial wavefunction

$$\psi(x) = \begin{cases} \sqrt{\dfrac{c}{a^5}}(a^2 - x^2), & x < |\,a\,| \\ 0, & x \geq |\,a\,| \end{cases}$$ (where g and c are constants) is **[CSIR Dec. 2015]**

(a) $\dfrac{15}{16}\left(\dfrac{\hbar^2 g^2}{m}\right)^{1/3}$ (b) $\dfrac{5}{6}\left(\dfrac{\hbar^2 g^2}{m}\right)^{1/3}$ (c) $\dfrac{3}{4}\left(\dfrac{\hbar^2 g^2}{m}\right)^{1/3}$ (d) $\dfrac{7}{8}\left(\dfrac{\hbar^2 g^2}{m}\right)^{1/3}$

8. The energy levels for a particle of mass m in the potential $V(x) = \alpha\,|x|$, determined in the WKB approximation

$\sqrt{2m}\int_a^b \sqrt{E - V(x)}\,dx = \left(n + \dfrac{1}{2}\right)\hbar\pi$, where a, b are the turning points and $n = 0, 1, 2, \ldots$), are

(a) $E_n = \left[\dfrac{\hbar\pi\alpha}{4\sqrt{m}}\left(n + \dfrac{1}{2}\right)\right]^{2/3}$ (b) $E_n = \left[\dfrac{3\hbar\pi\alpha}{4\sqrt{2m}}\left(n + \dfrac{1}{2}\right)\right]^{2/3}$ **[CSIR June 2016]**

(c) $E_n = \left[\dfrac{3\hbar\pi\alpha}{4\sqrt{m}}\left(n + \dfrac{1}{2}\right)\right]^{2/3}$ (d) $E_n = \left[\dfrac{\hbar\pi\alpha}{4\sqrt{2m}}\left(n + \dfrac{1}{2}\right)\right]^{2/3}$

9. The ground state energy of a particle of mass m in the potential $V(x) = \dfrac{\hbar^2\beta}{6m}x^4$, estimated using the normalized

trial wavefunction $\psi(x) = \left(\dfrac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$ is **[CSIR June 2016]**

[Use $\sqrt{\dfrac{\alpha}{\pi}}\int_{-\infty}^{\infty} dx\,x^2\,e^{-\alpha x^2} = \dfrac{1}{2\alpha}$ and $\sqrt{\dfrac{\alpha}{\pi}}\int_{-\infty}^{\infty} dx\,x^4\,e^{-\alpha x^2} = \dfrac{3}{4\alpha^2}$].

(a) $\dfrac{3}{2m}\hbar^2\beta^{1/3}$ (b) $\dfrac{8}{3m}\hbar^2\beta^{1/3}$ (c) $\dfrac{2}{3m}\hbar^2\beta^{1/3}$ (d) $\dfrac{3}{8m}\hbar^2\beta^{1/3}$

ANSWER KEY

1. (d) 2. (d) 3. (c) 4. (c) 5. (b) 6. (b) 7. (a)

8. ( ) 9. ( )

7. The ground state energy of a particle in the potential $V(x) = g\,|\,x\,|$, estimated using the trial wavefunction

$$\psi(x) = \begin{cases} \sqrt{\dfrac{c}{a^5}}(a^2 - x^2), & x < |\,a\,| \\ 0, & x \geq |\,a\,| \end{cases}$$ (where g and c are constants) is **[CSIR Dec. 2015]**

(a) $\dfrac{15}{16}\left(\dfrac{\hbar^2 g^2}{m}\right)^{1/3}$ (b) $\dfrac{5}{6}\left(\dfrac{\hbar^2 g^2}{m}\right)^{1/3}$ (c) $\dfrac{3}{4}\left(\dfrac{\hbar^2 g^2}{m}\right)^{1/3}$ (d) $\dfrac{7}{8}\left(\dfrac{\hbar^2 g^2}{m}\right)^{1/3}$

8. The energy levels for a particle of mass m in the potential $V(x) = \alpha\,|x|$, determined in the WKB approximation

$\sqrt{2m}\int_a^b \sqrt{E - V(x)}\,dx = \left(n + \dfrac{1}{2}\right)\hbar\pi$, where a, b are the turning points and $n = 0, 1, 2, \ldots$), are

(a) $E_n = \left[\dfrac{\hbar\pi\alpha}{4\sqrt{m}}\left(n + \dfrac{1}{2}\right)\right]^{2/3}$ (b) $E_n = \left[\dfrac{3\hbar\pi\alpha}{4\sqrt{2m}}\left(n + \dfrac{1}{2}\right)\right]^{2/3}$ **[CSIR June 2016]**

(c) $E_n = \left[\dfrac{3\hbar\pi\alpha}{4\sqrt{m}}\left(n + \dfrac{1}{2}\right)\right]^{2/3}$ (d) $E_n = \left[\dfrac{\hbar\pi\alpha}{4\sqrt{2m}}\left(n + \dfrac{1}{2}\right)\right]^{2/3}$

9. The ground state energy of a particle of mass m in the potential $V(x) = \dfrac{\hbar^2\beta}{6m}x^4$, estimated using the normalized trial wavefunction $\psi(x) = \left(\dfrac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$ is **[CSIR June 2016]**

[Use $\sqrt{\dfrac{\alpha}{\pi}}\int_{-\infty}^{\infty} dx\,x^2\,e^{-\alpha x^2} = \dfrac{1}{2\alpha}$ and $\sqrt{\dfrac{\alpha}{\pi}}\int_{-\infty}^{\infty} dx\,x^4\,e^{-\alpha x^2} = \dfrac{3}{4\alpha^2}$].

(a) $\dfrac{3}{2m}\hbar^2\beta^{1/3}$ (b) $\dfrac{8}{3m}\hbar^2\beta^{1/3}$ (c) $\dfrac{2}{3m}\hbar^2\beta^{1/3}$ (d) $\dfrac{3}{8m}\hbar^2\beta^{1/3}$

ANSWER KEY

1. (d) **2. (d)** **3. (c)** **4. (c)** **5. (b)** **6. (b)** **7. (a)**

8. ( ) **9. ( )**

Other Examinations Previous Years' Questions

1. Let E_T be the energy obtained variationally by using trial function $Ae^{-\alpha r^2}$ (with α as a variational parameter) for Hydrogen atom. If the exact ground state is E_H then,

   (a) $E_T = E_H$ (b) $E_T > E_H$ (c) $E_T \geq E_H$ (d) $E_T < E_H$

2. For a harmonic oscillator of mass $m = \frac{1}{2}$ and energy E, moving in a potential $V(x) = \frac{1}{2}kx^2$, the WKB quantization condition reads

   (a) $\int_{-\sqrt{2E/k}}^{\infty} dx\sqrt{E - \frac{1}{2}kx^2} = \left(n + \frac{1}{2}\right)\pi\hbar$ (b) $\int_{\infty}^{\sqrt{2E/k}} dx\sqrt{E - \frac{1}{2}kx^2} = \left(n + \frac{1}{2}\right)\pi\hbar$

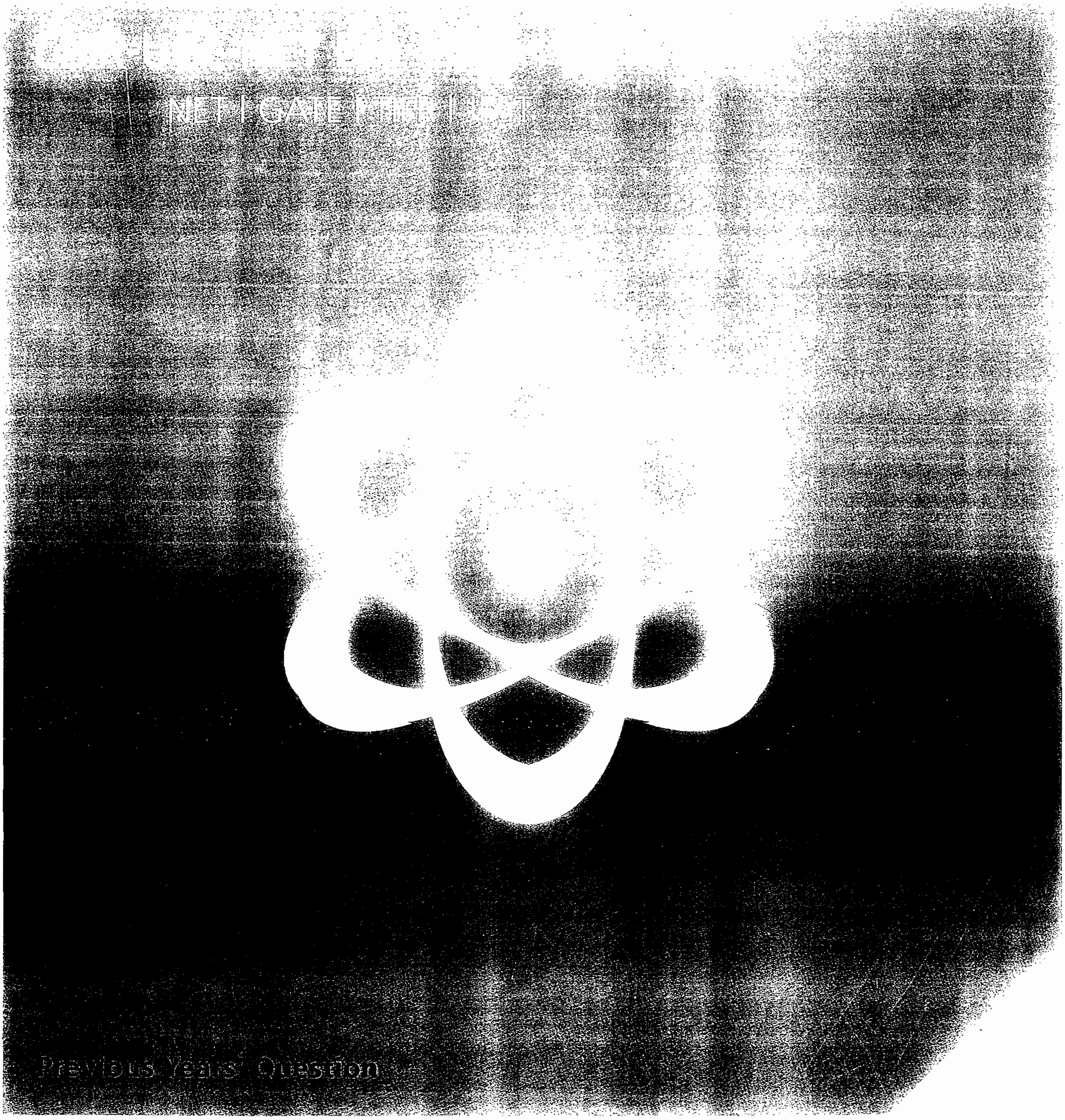
   (c) $\int_{-\sqrt{2E/k}}^{\sqrt{2E/k}} dx\sqrt{E - \frac{1}{2}kx^2} = \left(n + \frac{1}{2}\right)\pi\hbar$ (d) $\int_{-\infty}^{\infty} dx\sqrt{E - \frac{1}{2}kx^2} = \left(n + \frac{1}{2}\right)\pi\hbar$

ANSWER KEY

1. (b) **2. (c)**

[illegible]

NET | GATE | [illegible]



Previous Years' Question

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Inu Kumar

SOLID STATE PHYSICS

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CSIR-UGC-NET/JRF|GATE-PHYSICS

SOLID STATE PHYSICS

Assignment-1

1. The packing fraction of a simple cubic lattice is approximately:
(a) 0.74 (b) 0.68 (c) 0.52 (d) 0.34

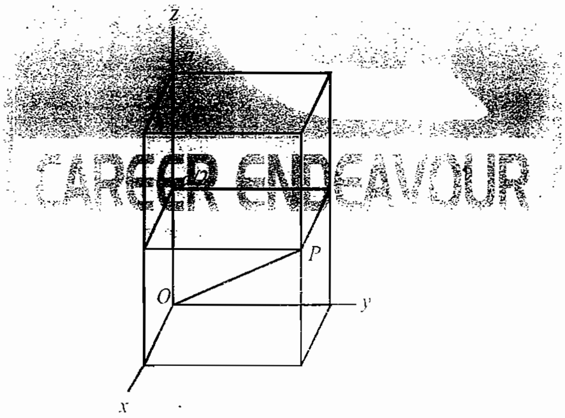
2. Given that r is the radius of the atoms and a is the lattice constant of a solid having a cubic structure, which one of the following relations is true for a body centred cubic structure?
(a) a = 2r (b) $a = 2r\sqrt{2}$ (c) $a = 2r\sqrt{3}$ (d) $a = 4r/\sqrt{3}$

3. What is the second nearest-neighbour distance in a face centred cubic lattice whose conventional unit cell parameter is a?
(a) $a/\sqrt{2}$ (b) a/2 (c) a (d) $\sqrt{2}/a$

4. The C/a ratio for an ideal hexagonal closed packed structure is:
(a) $\frac{2}{\sqrt{3}}$ (b) $\sqrt{8}$ (c) $\sqrt{5}$ (d) $\sqrt{\frac{8}{3}}$

5. For a closed packed BCC structure of hard spheres, the lattice constant a is related to the sphere radius R as:
(a) $a = 4R/\sqrt{3}$ (b) $a = 4R\sqrt{3}$ (c) $a = 4R\sqrt{2}$ (d) $a = 2R\sqrt{2}$

6. In crystallographic notation the vector $\overline{OP}$ in the cubic cell shown in the figure is:



(a) [2 2 1] (b) [1 2 2] (c) [1 2 1] (d) [1 1 2]

7. The de-Broglie wavelength λ for an electron of energy 150 eV is:
(a) 10^{-8} m (b) 10^{-10}m (c) 10^{-12}m (d) 10^{-14}m

8. Which is loose packed structure?
(a) Body centred cubic structure (b) Hexagonal close-packed structure
(c) Face-centred cubic structure (d) None of these

9. The value of a primitive unit cell defined by axes $(\hat{k}+\hat{i})$Å, $(\hat{i}+\hat{j})$Å, $(\hat{j}+\hat{k})$Å, is

(a) 1Å^3 (b) 3Å^3 (c) 2Å^3 (d) 4Å^3

10. The Miller indices of a plane passing through the points $3\hat{a}$, $4\hat{b}$ and $5\hat{c}$, where $\hat{a}$, $\hat{b}$ and $\hat{c}$, are lattice vectors are

(a) (12, 15, 20) (b) (20, 15, 12) (c) (5, 4, 3) (d) (3, 4, 5)

11. A simple cubic crystal with lattice parameter a_c undergoes transition into a tetragonal structure with lattice parameter $a_t = b_t = \sqrt{2}\,a_c$ and $c_t = 2a_c$, below a certain temperature. The ratio of the interplanar spacings of (101) planes for the cubic and the tetragonal structure is

(a) $\sqrt{\frac{1}{6}}$ (b) $\frac{1}{6}$ (c) $\sqrt{\frac{3}{8}}$ (d) $\frac{3}{8}$

12. The inter-planar spacing for (110) plane of a monoatomic FCC crystal of lattice constant 2Å is Corre Ans is $\frac{1}{\sqrt{2}}$ Å.

(a) 1/2Å (b) 2Å (c) $\sqrt{2}$ Å (d) $\sqrt{3}$ Å

13. Which one of the following diffraction techniques has most surface sensitivity?

(a) X-ray (b) Electron (b) Neutron (d) Optical

14. By using X-ray diffraction one cannot obtain

(a) crystal structure (b) type of bonding (c) residual stress (d) grain size

15. Band gap of a semiconductor nanocrystal (keeping the same lattice structure)

(a) increases with decreasing size (b) decreases with decreasing size
(c) is same as that of bulk (d) is always materials dependent

16. The position of an X-ray diffraction peak may be shifted with respect to the peak of a perfect crystal due to

(a) homogenous strain (b) [illegible]neous strain
(c) small unstrained crystal[illegible] (d) [illegible]usly strained nano-[illegible]rystallites

7. A flat surface is covered with non-overlapping disks of same size. What is the largest fraction of the area that can be covered?

(a) $\frac{3}{\pi}$ (b) $\frac{5\pi}{6}$ (c) $\frac{6}{7}$ (d) $\frac{\pi}{2\sqrt{3}}$

8. A metal suffers a structural phase transition from face-centred cubic (FCC) to the simple cubic (SC) structure. It is observed that this phase transition does not involve any change of volume. The nearest neighbour distances d_{fc} and d_{sc} for the FCC and the SC structures respectively are in the ratio (d_{fc}/d_{sc}) [Given $2^{1/3} = 1.26$]

(a) 1.029 (b) 1.122 (c) 1.374 (d) 1.130

9. Consider the crystal structure of sodium chloride which is modeled as a set of touching spheres. Each sodium atom has a radius r_1 and each chlorine atom has a radius r_2. The centres of the spheres form a simple cubic lattice. The packing fraction of this system is

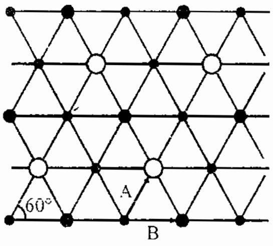
(a) $\pi\left[\left(\frac{r_1}{r_1+r_2}\right)^3+\left(\frac{r_2}{r_1+r_2}\right)^3\right]$ (b) $\frac{2\pi}{3}\frac{r_1^3+r_2^3}{(r_1+r_2)^3}$ (c) $\frac{r_1^3+r_2^3}{(r_1+r_2)^3}$ (d) $\pi\frac{r_1^3+r_2^3}{2(r_1+r_2)^3}$

9→c 11-c 12→c 13→b 14→b 15-a 16-a 17-d
18-b

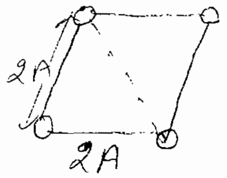
0 · 4472a

20. The lattice parameters a, b, c of an orthorhombic crystal are related by $a = 2b = 3c$. In units of a, the interplanar separation between the (110) planes is ____________ (upto three decimal places) **[GATE 2015]**

21. What is the area of the irreducible Brillouin zone of the crystal structure as given in the figure ?

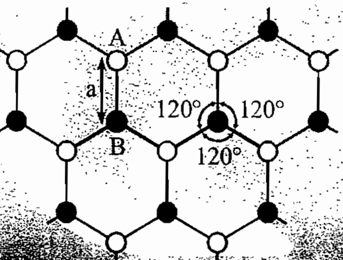


$|A| = |B| = A$



(a) $\frac{2\pi^2}{\sqrt{3}A^2}$ (b) $\frac{\sqrt{3}\pi^2}{2A^2}$ (c) $\frac{2\pi^2}{A^2}$ (d) $\frac{\pi^2}{\sqrt{3}A^2}$ **[JEST 2015]**

22. For a 2-dimensional honeycomb lattice as shown in the figure, the first Bragg spot occurrs for the grazing angle θ_1, while sweeping the angle from 0°. The next Bragg spot is obtained at θ_2 given by,



(a) $\sin^{-1}(3\sin\theta_1)$ (b) [illegible] $\sin^{-1}\left(\frac{\sqrt{3}}{2}\sin\theta_1\right)$ [illegible] $^{-1}(\sqrt{3}\sin\theta_1)$

23. In a simple cubic lattice of lattice constant 0.287 nm, the number of atoms per mm² along the 111 plane is **[TIFR 2016]**

(a) 2.11×10^{13} (b) 1.73×10^{13} (c) 1.29×10^{13} (d) 1.21×10^{13}

24. The crystal structure of diamond is:

(a) fcc with two atom basis of (000) and $\frac{a}{4}(\hat{i}+\hat{j}+\hat{k})$

(b) simple cubic with two atom basis of (000) and $\frac{a}{2}(\hat{i}+\hat{j}+\hat{k})$

(c) fcc with two atoms basis of (000) and $\frac{a}{2}(\hat{i}+\hat{j}+\hat{k})$

(d) bcc with one atom basis

25. Which of the following crystals be represented by interpenetrating lattices:

(a) Sodium [✓ (b) Sodium chloride]

[✓ (c) Cesium chloride] [✓ (d) Diamond]

21-a 22-d 24→a

For an NaCl crystal, the cell-edge a = 0.542 nm. The smallest angle at which Bragg reflection can occur corresponds to a set of planes whose indices are

(a) 100 (b) 110 (c) 111 (d) 200

The target of an X-ray tube is subjected to an excitation voltage V. The wavelength of the emitted X-rays is proportional to:

(a) $\frac{1}{\sqrt{V}}$ (b) $\sqrt{V}$ (c) $\frac{1}{V}$ (d) V

Consider the atomic packing factor (APF) of the following crystal structures:

P. Simple Cubic **Q.** Body Centred Cubic
R. Face Centred Cubic **S.** Diamond
T. Hexagonal Close Packed

Which two of the above structures have equal APF?

(a) P and Q (b) S and T (c) R and S (d) R and T

A metal with body centered cubic (bcc) structure shows the first (i.e. smallest angle) diffraction peak at Bragg angle of $\theta = 30°$. The wavelength of X-ray used is 2.1 Å. The volume of the PRIMITIVE unit cell of the metal is:

(a) 26.2 $(Å)^3$ (b) 13.1 $(Å)^3$ (c) 9.3 $(Å)^3$ (d) 4.6 $(Å)^3$.

Silicon has diamond structure with unit-cell-edge a = 0.542 nm. The interatomic separation is:

(a) 0.122 nm (b) 0.234 nm (c) 0.383 nm (d) 0.542 nm.

Consider X-ray diffraction from a crystal with a face-centered-cubic (fcc) lattice. The lattice. The lattice plane for which there is NO diffraction peak is:

(a) (2, 1, 2) (b) (1, 1, 1) (c) (2, 0, 0) (d) (3, 1, 1)

The $\theta - 2\theta$ X-ray powder diffraction pattern of ionic crystals KCl and KBr both of which form fcc structure will have:

(a) same number of diffraction lines with equal intensities
(b) pattern of KBr will have more lines
(c) Pattern of KCl will have more lines
(d) number of lines will be same but their intensities will be different

Statement for Linked Answer Q.33 and Q.34.

The powder diffraction pattern of a body centred cubic crystal is recorded by using Cuk$_\alpha$ X-rays of wavelength 1.54Å.

If the (002) planes diffract at 60°, then lattice parameter is:

1·77

(a) 2.67 Å (b) 3.08 Å (c) 3.56 Å (d) 5.34 Å

Assuming the atomic mass of the constituent atoms to be 50.94 amu, then density of the crystal in units of kg m^{-3} is:

(a) 3.75×10^3 (b) 4.45×10^3 (c) 5.79×10^3 (d) 8.89×10^3

In a powder differaction pattern recorded from a face-centred cubic sample using x-rays, the first peak appears at 30°. The second peak will appear at:

(a) 32.8° (b) 33.7° (c) 34.8° (d) 35.3°

Statement for Linked Answer Q. 36 and Q. 37.

Lead has atomic weight of 207.2 amu and density of 11.35 gm cm^{-3}.

Number of atoms per cm^3 for lead is:

(a) 1.1×10^{25} (b) 3.3×10^{22} (c) 1.1×10^{22} (d) 3.3×10^{25}

26→c 27-c 28-d 30-b 31-a 32-b 36-b

33-b 34-c. 35-c

37. If the energy of vacancy formation in lead is 0.55 eV/atom, the number of vacancies cm^3 at 500K is:
(a) 3.2×10^{16} (b) 3.2×10^{19} (c) 9.5×10^{19} (d) 9.5×10^{16}

Statement for Linked Answer Q. 38 and Q. 39:
The primitive translatin vectors of the face centered cubic (fcc) lattice are

$$\hat{a}_1 = \frac{a}{2}\left(\hat{j}+\hat{k}\right);\ \hat{a}_2 = \frac{a}{2}\left(\hat{i}+\hat{k}\right);\ a_3 = \frac{a}{2}\left(\hat{i}+\hat{j}\right)$$

38. The primitive translation vectos of the fcc reciprocal lattice are

(a) $\hat{b}_1 = \left(\frac{2\pi}{a}\right)\left(-\hat{i}+\hat{j}+\hat{k}\right); \hat{b}_2 = \left(\frac{2\pi}{a}\right)\left(\hat{i}-\hat{j}+\hat{k}\right); \hat{b}_3 = \left(\frac{2\pi}{a}\right)\left(\hat{i}+\hat{j}-\hat{k}\right)$

(b) $\hat{b}_1 = \left(\frac{\pi}{a}\right)\left(-\hat{i}+\hat{j}+\hat{k}\right); \hat{b}_2 = \left(\frac{\pi}{a}\right)\left(\hat{i}-\hat{j}+\hat{k}\right); \hat{b}_3 = \left(\frac{\pi}{a}\right)\left(\hat{i}+\hat{j}-\hat{k}\right)$

(c) $\hat{b}_1 = \left(\frac{\pi}{2a}\right)\left(-\hat{i}+\hat{j}+\hat{k}\right); \hat{b}_2 = \left(\frac{\pi}{2a}\right)\left(\hat{i}-\hat{j}+\hat{k}\right); \hat{b}_3 = \left(\frac{\pi}{2a}\right)\left(\hat{i}+\hat{j}-\hat{k}\right)$

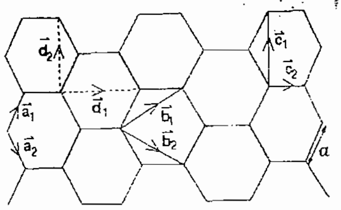
(d) $\hat{b}_1 = \left(\frac{3\pi}{a}\right)\left(-\hat{i}+\hat{j}+\hat{k}\right); \hat{b}_2 = \left(\frac{3\pi}{a}\right)\left(\hat{i}-\hat{j}+\hat{k}\right); \hat{b}_3 = \left(\frac{3\pi}{a}\right)\left(\hat{i}+\hat{j}-\hat{k}\right)$

39. The volume of the primitive cell of the fcc reciprocal lattice is:

(a) $4\left(\frac{2\pi}{a}\right)^3$ (b) $4\left(\frac{\pi}{a}\right)^3$ (c) $4\left(\frac{\pi}{2a}\right)^3$ (d) $4\left(\frac{3\pi}{a}\right)^3$

40. A narrow beam of X-rays with wavelength 1.5Å is reflected from an ionic crystal with an fcc lattice structure with a density of 3.32 g cm^{-3}. The molecular weight is 108 AMU (1 AMU= 1.66×10^{-24} g).
A. The lattice constant is:
(a) 6.00Å (b) 4.56Å (c) 4.00Å (d) 2.56Å
B. The sine of the angle corresponding to (111) reflection is:
(a) $\sqrt{3}/4$ (b) [illegible] (c) $1/4$ (d) [illegible]8

41. The two dimensional lattice of graphene is an arrangement of Carbon atoms forming a honeycomb lattice of lattice spacing a, as shown below. The carbon atoms occupy the vertices.



A. The Wigner-Seitz cell has an area of

(a) $2a^2$ (b) $\frac{\sqrt{3}}{2}a^2$ (c) $6\sqrt{3}\,a^2$ (d) $\frac{3\sqrt{3}}{2}a^2$

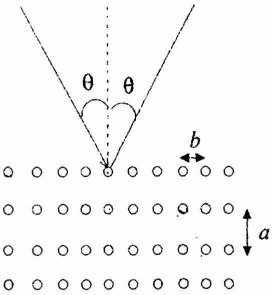
B. The Bravais lattice for this array is a
(a) Rectangular lattice with basis vectors $\overline{d}_1$ and $\overline{d}_2$
(b) Rectangular lattice with basis vectors $\overline{c}_1$ and $\overline{c}_2$
(c) Hexagonal lattice with basis vectors $\overline{a}_1$ and $\overline{a}_2$
(d) Hexagonal lattice with basis vectors $\overline{b}_1$ and $\overline{b}_2$.

$41-B \to d$ $40_a \to a$ $40_b \to b$

42. The total number of Cl^- ions in a unit cell of NaCl crystal is
(a) 4 (b) 2 (c) 8 (d) 10

43. The co-ordination number of Na^+ and Cl^- in the rock salt structure is respectively
(a) 8 and 6 (b) 6 and 8 (c) 6 and 6 (d) 4 and 4

44. A monochromatic beam of X-ray with wavelength λ is incident at an angle θ on a crystal with lattice spacing a and b as sketched in the figure below.



A condition for there to a maximum in the diffracted X-ray intensity is
(a) $2\sqrt{a^2+b^2}\sin\theta = \lambda$ (b) $2b\cos\theta = \lambda$
(c) $2a\cos\theta = \lambda$ (d) $(a+b)\sin\theta = \lambda$

45. Metallic Copper is known to from cubic crystals and the lattice constant is measured from X-ray diffraction studies to be about 0.36nm. If the specific gravity of Copper is 8.96 and its atomic weight is 63.5, one can conclude that
(a) the crystals are of simple cubic type (b) the crystals are of b.c.c. type
(c) the crystals are of f.c.c. type (d) the crystals are a mixture of f.c.c. and b.c.c. types.

46. The basis vectors of a lattice [illegible] $2\hat{j}$ and $\hat{k}$. The basis [illegible] procal lattice are
(a) $\frac{2}{\pi}(\hat{i}+\hat{j}), \frac{2}{\pi}\hat{j}$ and $\frac{2}{\pi}\hat{k}$ [illegible] $\pi(2\hat{i}-\hat{j}), \pi\hat{j}$ an[illegible]
(c) $\frac{\pi}{2}(2\hat{i}-\hat{j}), \pi\hat{j}$ and $2\pi\hat{k}$ (d) [illegible] $\hat{k}$

4

47. The experimental method used most frequently for detecting magnetic structure of solids is
(a) electron diffraction (b) neutron diffraction
(c) x-ray diffraction (d) photo-emission spectroscopy

4

48. When the sample is heated the X-ray diffraction pattern of a powder changes as follows:
(a) The line spacing decreases and line intensity increases
(b) The line spacing and line intensity both increases
(c) The line spacing increases and line intensity decreases
(d) The line spacing and line intensity both decrease

49. The most suitable experimental method for studying the dispersion relation of phonons is
(a) X-ray diffraction (b) elastic scattering of neutorns
(c) inelastic scattering of thermal neutrons (d) photo-electron emission.

50. A beam of x-rays is incident on a BCC crystal. If the difference between the incident and the scattered wave vectors is $\vec{K} = h\hat{x} + k\hat{y} + \ell\hat{z}$ where $\hat{x}, \hat{y}, \hat{z}$ are the unit vectors of the associate cubic lattice, the necessary condition for the scattered beam to given a Laue maximum is
(a) h + k + l = even (b) h = k = 1 (c) h, k, l are all distinct (d) h + k + l = odd

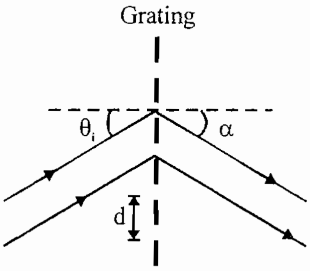
Laue = Bragg

46 – c 48 – d 50 – a

51. the second order maximum in the diffraction of X-rays of 0.20 nanometer wavelength from a simple cubic crystal is found to occur at an angle of thirty degrees with respect to the crystal plane. The distance between the lattice planes is

(a) 1 Angstrom (b) 2 Angstroms (c) 4 Angstroms (d) 8 Angstroms

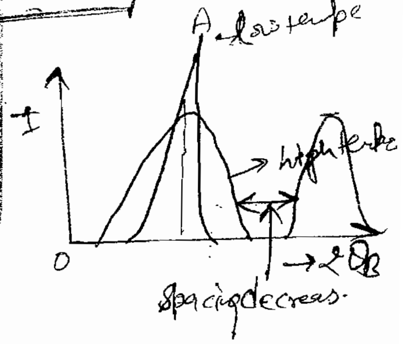
52. A parallel beam of light of wavelength λ is incident on a transmission grating with groove spacing d, at an angle θ_i, as shown in the figure on the left. The plane of incidence is normal to the grooves. After diffraction, the transmitted beam is seen to be at an angle α relative to the normal. Which of the following conditions must be satisfied for this to happen?



(a) $d(\sin\theta_i - \sin\alpha) = n\lambda$ (b) $2d\sin(\theta_i - \alpha) = n\lambda$

(c) $d(\sin\theta_i + \sin\alpha) = n\lambda$ (d) $2d(\alpha + \theta_i) = n\lambda$

48 → By increasing the temperature XRD peak intensity decrease and Broadness of the peak increases.



49 → inelastic scattering of thermal neutron.

except - 21, 22, 29

51 → C 52 - c

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SOLID STATE PHYSICS

Assignment-2

1. Atoms, which can be assumed to be hard spheres of radius R, are arranged in an *fcc* lattice with lattice constant a, such that each atom touches its nearest neighbours. Take the center of one of the atoms as the origin. Another atom of radius r (assumed to be hard sphere) is to be accommodated at a position $(0, a/2, 0)$ without distorting the lattice. The maximum value of r/R is ___________. (Give your answer upto two decimal places). **[GATE 2016]**

2. Consider a regular arrangement of identical spheres in a face-centred cubic (fcc) structure in which the centres of the respective spheres are located at each of the eight corners and the centres of the six surfaces of a unit cube. The fraction of each cubic unit cell occupied by the spheres in the close-pack configuration is
(a) 0.50 (b) 0.62 (c) 0.74 (d) 0.88

3. The number of different Bravais lattices possible in two dimension is:
(a) 2 (B) 3 (c) 5 (d) 6 **[JEST 2016]**

4. Which of the following statements is correct for NaCl crystal structure?
(a) It is a simple cubic lattice with one atom basis.
(b) It is a face-centered cubic lattice with one atom basis
(c) It is a simple cubic lattice with two atom basis
(d) It is a face-centered cubic lattice with two atom basis.

5. The value of θ at which the first-order peak in X-ray $(\lambda = 1.53\,\text{Å})$ diffraction corresponding to (111) plane of a simple cubic structure with the lattice constant, a = 2.65 Å, is approximately
(a) 15° (b) 30° (c) 45° (d) 60°

6. The number of second-nearest neighbour ions to a Na^+ ion in NaCl crystal is ____12____

7. The ratio of the second-neighbour distance to the nearest-neighbour distance ~~to the nearest-neighbour distance~~ in an fcc lattice is
(a) $2\sqrt{2}$ (b) 2 (c) $\sqrt{3}$ (d) $\sqrt{2}$

8. An X-ray beam of wavelength 1.54Å is diffracted from the (110) planes of a solid with a cubic lattice of lattice constant 3.08Å. The first-order Bragg diffraction occurs at
(a) $\sin^{-1}\left(\frac{1}{4}\right)$ (b) $\sin^{-1}\left(\frac{1}{2\sqrt{2}}\right)$ (c) $\sin^{-1}\left(\frac{1}{2}\right)$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

9. The fraction of volume unoccupied in the unit cell of the body centered cubic lattice is
(a) $\frac{8-\sqrt{3}\pi}{8}$ (b) $\frac{\sqrt{3}\pi}{8}$ (c) $\frac{6-\sqrt{2}\pi}{6}$ (d) $\frac{\pi}{3\sqrt{2}}$

0. Diamond lattice can be considered as a combination of two fcc lattices displaced along the body diagonal by one quarter of its length. There are eight atoms per unit cell. The packing fraction of the diamond structure is
(a) 0.48 (b) 0.74 (c) 0.34 (d) 0.68

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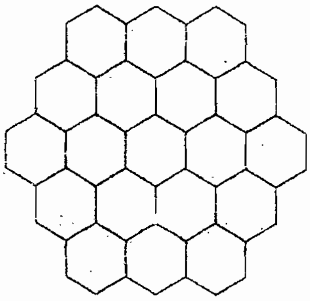
11. Thermal neutrons (energy = 300 k_B = 0.025eV) are sometimes used for structural determination of materials. The typical lattice spacing of a material for which these can be used is

(a) 0.01 nm (b) 0.05 nm (c) 0.1 nm (d) 0.15 nm

12. In a single layer of graphite, called graphene, the carbon atoms from a hexagonal lattice (see figure). How many carbon atoms are there in a unit cell of the lattice?



(a) one (b) two (c) three (d) six

13. An X-ray machine operates at a potential of 5000 V. What is the minimum possible value of wavelength present in the radiations?

(a) 0.0124 nm (b) 0.0245 nm (c) 0.124 nm (d) 0.248 nm

14. Helium atoms at low temperatures make a perfect closed pack structure of hexagonal lattice with parameters a = 0.36 nm and c = 0.59 nm. The density of the crystal is approximately

(a) 2000 kg/m^3 (b) 100 kg/m^3 (c) 123 kg/m^3 (d) 200 kg/m^3

15. Monochromatic X rays of wavelength 1Å are incident on a simple cubic crystal. The first order Brag reflection from (311) plane occurs at an angle of 30° from the plane. The lattice parameter of the crystal in Å is:

(a) 1 (b) $\sqrt{3}$ (c) $\sqrt{\frac{11}{2}}$ (d) $\sqrt{11}$

16. X-ray diffraction of cubic crystal gives an intensity maximum for Bragg angle of 20° corresponding to the (110) plane. The lattice p[illegible]ystal is ______ nm.
.116
(Consider wavelength o[illegible]

17. The second order maximum in the diffraction of X-rays of 0.20 nanometer wavelength from a simple cubic crystal is found to occur at an angle [illegible] degrees with respect to the [illegible] plane. The distance between the lattice planes is

(a) 1 Angstrom (b) 2 Angstroms (c) 4 Angstroms (d) 8 Angstroms

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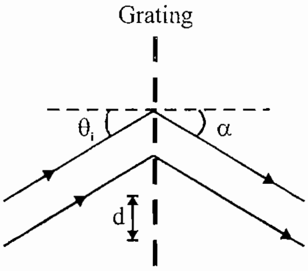
18. A parallel beam of light of wavelength λ is incident on a transmission grating with groove spacing d, at an angle θ_i, as shown in the figure on the left. The plane of incidence is normal to the grooves. After diffraction, the transmitted beam is seen to be at an angle α relative to the normal. Which of the following conditions must be satisfied for this to happen?



(a) $d(\sin\theta_i - \sin\alpha) = n\lambda$ (b) $2d\sin(\theta_i - \alpha) = n\lambda$

(c) $d(\sin\theta_i + \sin\alpha) = n\lambda$ (d) $2d(\alpha + \theta_i) = n\lambda$

upto 20 -

9. A lattice is characterized by the following primitive vectors (in angstroms) :
$\vec{a} = 2(\hat{i}+\hat{j}), \vec{b} = 2(\hat{j}+\hat{k}), \vec{c} = 2(\hat{k}+\hat{i})$. The reciprocal lattice corresponding to the above is:
(a) body centered cubic lattice with cube edge p Å^{-1}
(b) body centered cubic lattice with cube edge 2 p Å^{-1}
(c) face centered cubic lattice with cube edge p Å^{-1}
(d) face centered cubic lattice with cube edge 2 p Å^{-1}

0. A lattice has the following primitive vectors (in A): = $\vec{a} = 2(\hat{j}+\hat{k}), \vec{b} = 2(\hat{k}+\hat{i}), \vec{c} = 2(\hat{i}+\hat{j})$. The reciprocal lattice corresponding to the above lattice is

(a) BCC lattice with cube edge of $\left(\frac{\pi}{2}\right)A^{-1}$ (b) BCC lattice with cube edge of $(2\pi)A^{-1}$

(c) FCC lattice with cube edge of $\left(\frac{\pi}{2}\right)A^{-1}$ (d) FCC lattice with cube edge of $(2\pi)A^{-1}$

1. Neutrons moving with speed 10^3 m/s are used for the determination of crystal structure. If the Bragg angle for the first order diffraction is 30°, the interplanarspacing of the crystal is ________Å.
(Given: $m_n = 1.675\times10^{-27}\,kg, h = 6.626\times10^{-34}\,J.s$)

2. If $\vec{k}$ is the wavefactor of incident light ($|\vec{k}| = 2\pi/\lambda$, λ is the wavelength of light) and $\vec{G}$ is a reciprocal lattice vector, then the Bragg's law can be written as: **[JEST 2016]**
(a) $\vec{k}+\vec{G}=0$ (b) $2\vec{k}\cdot\vec{G}+G^2=0$ (c) $2\vec{k}\cdot\vec{G}+k^2=0$ (d) $\vec{k}\cdot\vec{G}=0$

3. The first order diffraction peak of a crystalline solid occurs at a scattering angle of 30° when the diffraction pattern is recorded using an x-ray beam of wavelength 0.15nm. If the error in measurements of the wavelength and the angle are 0.01nm and 1° respectively, then the error in calculating the inter-planar spacing will approximately be **[NET Dec. 2015]**
(a) 1.1×10^{-2} nm (b) 1.3×10^{-4} nm (c) 2.5×10^{-3} nm (d) 2.0×10^{-3} nm

4. The primitive translation vectors of the body centered cubic lattice are $\vec{a} = \frac{a}{2}(\hat{x}+\hat{y}-\hat{z})$, $\vec{b} = \frac{a}{2}(\hat{x}+\hat{y}+\hat{z})$, and $\vec{c} = \frac{a}{2}(\hat{x}-\hat{y}+\hat{z})$. The primitive translation vectors $A, B\,and\,C$ of the reciprocal lattice are:

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(a) $\vec{A} = \frac{2\pi}{a}(\hat{x}-\hat{y}); \vec{B} = \frac{2\pi}{a}(\hat{y}+\hat{z}); \vec{C} = \frac{2\pi}{a}(\hat{x}+\hat{z})$

(b) $\vec{A} = \frac{2\pi}{a}(\hat{x}+\hat{y}); \vec{B} = \frac{2\pi}{a}(\hat{y}-\hat{z}); \vec{C} = \frac{2\pi}{a}(\hat{x}+\hat{z})$

(c) $\vec{A} = \frac{2\pi}{a}(\hat{x}+\hat{y}); \vec{B} = \frac{2\pi}{a}(\hat{y}+\hat{z}); \vec{C} = \frac{2\pi}{a}(\hat{x}-\hat{z})$

(d) $\vec{A} = \frac{2\pi}{a}(\hat{x}+\hat{y}); \vec{B} = \frac{2\pi}{a}(\hat{y}+\hat{z}); \vec{C} = \frac{2\pi}{a}(\hat{x}+\hat{z})$

25. The structure factor of a single cell of identical atoms of form factor f is given by $Shkl = f\sum_{!}\exp\left(-i2\pi\left(x_j h + y_j k + z_j l\right)\right)$ where (x_j, y_j, $z_j l$) is the coordinate of an atom, and hkl are the Miller indices. Which one of the following statement is correct for the diffraction peaks of body centered cubic (BCC) and face centered cubic (FCC) lattices?

(a) BCC : (200; (110); (222)
FCC : (111); (311); (400)

(b) BCC : (210); (110); (222)
FCC : (111); (311); (400)

(c) BCC : (200); (110); (222)
FCC : (111); (211); (400)

(d) BCC : (200); (210); (222)
FCC : (111); (211); (400)

26. Which one of the following axes of rotational symmetry is NOT permissible in single crystals?
(a) two fold axis (b) three-fold axis (d) four-fold axis (d) five-fold axis

27. X-rays with a wave vector $\vec{K}$ are scattered from a simple cubic lattice with lattice spacing a = 2p. The scattered X-rays have wave vector $\vec{K}$. The possible values of $DK_1 = K_x - K'_x$ for which there are peaks in the scattered intensity are:
(a) $0 \leq \Delta K_1 \leq p/4$
(b) ΔK_x = integer
(c) ΔK_x = integer multiple of 2p
(d) ΔK_x = integer/2p

Common Data for Q. 28 and Q. 29.

A crystal belongs to a face centered cubic lattice with four atoms in the unit cell. The size of the crystal is 1 cm and its unit cell dimension is 1 nm. f is the scattering factor of the atom.

28. The number of atoms in the crystal is:
(a) 2×10^{21} (b) 4×10^{21} (c) 2×10^{23} (d) 4×10^{24}

29. The structure factors for (0 1 0) and (2 0 0) reflections respectively are:
(a) 2 f and zero (b) zero and 4 f (c) 2 f and 2 f (d) zero and zero

30. The total number of Na^+ and Cl^- ions per unit cell of NaCl is, **[JEST 2015]**
(a) 2 (b) 4 (c) 6 (d) 8

31. Which of the following bo[illegible]ional:
(a) Ionic bonds (b) [illegible] (c) Metalic (d) Vander Waals

32. If there are n-atoms in the primitive cell, the phonon dispersion relation has optical branches:-
(a) 3n – 3 (b) 3n – 2 (c) [illegible] (d) none

33. In a cubic crystal, atoms of mass M_1 lie on one set of planes and atoms of mass M_2 lie on planes interleaved between those of the first set. If C is the force constant between nearest neighbour planes, the frequency of lattice vibrations for the optical phonon branch with wavevector k = 0 is:

(a) $\sqrt{2C\left(\frac{1}{M_1}+\frac{1}{M_2}\right)}$ (b) $\sqrt{C\left(\frac{1}{M_1}+\frac{1}{M_2}\right)}$ (c) $\sqrt{C\left(\frac{1}{M_1}+\frac{1}{2M_2}\right)}$ (d) 0

34. At low temperature the specific heat of insulating crystals varies as:
(a) AT^3
(b) $BT + CT^3$
(c) D exp (E/T)
(d) will not change with temperature

35. A cubic cell consists of two atoms of masses m_1 and m_2 ($m_1 > m_2$) with m_1 and m_2 atoms situated on alternate planes. Assuming only nearest neighbour interactions, the center of mass of the two atoms:
(a) moves with the atoms in the optical mode and remains fixed in the acoustic mode
(b) remains fixed in the optical mode and moves with the atoms in the acoustic mode
(c) remains fixed in both optical and acoustic modes
(d) moves with the atoms in both optical and acoustic modes

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5. A linear diatomic lattice of lattice constant a with masses M and m (M > m) are coupled by a force constant C. The dispersion relation is given by:

$$\omega_{\pm}^{2} = C\left(\frac{M+m}{Mm}\right) \pm \left[C^{2}\left(\frac{M+m}{M}\right)^{2} - \frac{4C^{2}}{Mm}\sin^{2}\frac{ka}{2}\right]^{1/2}$$

Which one of the following statements is incorrect?

(a) The atoms vibrating in transverse mode correspond to the optical branch

(b) The maximum frequency of the acoustic branch depends on the mass of the lighter atom m.

(c) The dispersion of frequency in the optical branch is smaller than that in the acoustic branch.

(d) No normal modes exist in the acoustic branch for any frequency greater than the maximum frequency at $k = \frac{\pi}{a}$.

7. The solid phase of an element follows van der Waals bonding with iner-atomic potential $V(r) = -\frac{P}{r^6} + \frac{Q}{r^{12}}$, where P and Q are constants. The bond length can be expressed as

(a) $\left(\frac{2Q}{P}\right)^{-6}$ (b) $\left(\frac{Q}{P}\right)^{-6}$ (c) $\left(\frac{P}{2Q}\right)^{-6}$ (d) $\left(\frac{P}{Q}\right)^{-6}$

The total number of Cl^- ions in a unit cell of NaCl crystal is

(a) 4 (b) 2 (c) 8 (d) 10

At the equilibrium spacing of a diatomic molecule, the resultant force is

(a) zero (b) minimum (c) maximum (d) unity

The potential energy in the above said spacing is

(a) zero (b) minimum (c) maximum (d) unity

Which of the interatomic bonds are directional?

(a) covalent (b) metallic (c) ionic (d) van der Waals

The potencial of a diatomic molecule as a function of the distance between the atoms is given by $V(r) = -\frac{a}{r^6} + \frac{b}{r^{12}}$. The value of the potential at equilibrium separation between the atoms is :

(a) $-4a^2/b$ (b) $-2a^2/b$ (c) $-a^2/2b$ (d) $-a^2/4b$

The number of phonon branch in graphite, with four atoms in the primitive cell, is

(a) 8, out of which two are acoustic, the rest optical

(b) 9, out of which three are acoustic, the rest optical

(c) 12, out of which three are acoustic, the rest optical

(d) 12, out of which two are acoustic, the rest optical

The excitation of a three-dimensional solid are bosonic in nature with their frequency ω and wave-number k are related by $\omega \propto k^2$ in the large wavelength limit. If the chemical potential is zero, the behaviour of the specific heat of the system at low temperature is proportional to

(a) $T^{1/2}$ (b) T (c) $T^{3/2}$ (d) T^3

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45. The nature of the interaction that gives rise to the van der Waals force in a molecular crystal is
(a) dipolar (b) nuclear (c) magnetic (d) quadrupolar

46. The specific heat of a normal metal at low temperatures is dominated by the contribution of
(a) conduction electrons (b) acoustic phonons
(c) core electrons (d) optical phonons

47. According to the Debye model, the internal energy of a solid a at temperature T close to absolute zero varies as
(a) T^2 (b) T^4 (c) T^3 (d) T

48. Let $\Phi(\vec{q}) = \int d\vec{r}\, e^{i\vec{q}.\vec{r}} V(\vec{r})\ q \neq 0$, where $V(\vec{r})$ is the potential seen by an electron in a crystal lattice. If $\vec{G}$ is a reciprocal lattice vector, then $\Phi(\vec{q})$ is nonzero,
(a) only if $\vec{q} = \vec{G}$ (b) for $\vec{q} = \frac{1}{2}\vec{G}$ (c) for all $\vec{q}$ (d) for all $\vec{q}$ within the Brillouin zone.

49. In the continuum model of an elastic solid, the frequency of sound waves
(a) is constant in some directions and proportional to the wave-vector in other directions
(b) is proportional to the square of the wave-vector
(c) is a constant independent of the wave-vector
(d) is proportional to the wave-vector in all directions.

50. The number of acoustic phonon branches in a three-dimensional crystal with 3 atoms per primitive cell is
(a) 6 (b) 9 (c) 3 (d) 12

51. Suppose the frequency of phonons in a one-dimensional chain of atoms is proportional to the wavevector. If *n* is the number density of atoms and *c* is the speed of the phonons, then the Debye frequency is
(a) $2\pi cn$ (b) $\sqrt{2}\pi cn$ (c) $\sqrt{3}\pi cn$ (d) $\frac{\pi cn}{2}$ **[NET June 2016]**

52. 72. The free energy difference between the superconducting and the normal states of a material is given by $\Delta F = F_S - F_N = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4$, where ψ is an order parameter and α and β are constants such that $\alpha > 0$ in the normal and $\alpha < 0$ in the superconducting state, while $\beta > 0$ always. The minimum value of ΔF in the superconducting state is
(a) $-\alpha^2/\beta$ (b) $-\alpha^2/2\beta$ (c) $-3\alpha^2/2\beta$ (d) $-5\alpha^2/2\beta$

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SOLID STATE PHYSICS

Assignment-3

1. Which one of the following CANNOT be explained by considering a harmonic approximation for the lattice vibrations in solids ?
(a) Debye's T^3 law (b) Dulong petit's law
(c) Optical baranches in lattices (d) Thermal expansion

Common data for Q. 2 and Q. 3

The dispersion relation for a one dimensional monatomic crystal with lattice spacing a, which interacts via nearest neighbour harmonic potential is given by $\omega = A\left|\sin\frac{Ka}{2}\right|$, where A is a constant of a appropriate unit.

2. The group velocity at the boundary of the first Brillouin zone is
(a) 0 (b) 1 (c) $\sqrt{\frac{Aa^2}{2}}$ (d) $\frac{1}{2}\sqrt{\frac{Aa^2}{2}}$

3. The force constant between the nearest neighbour of the lattice is (M is the mass of the atom)
(a) $\frac{MA^2}{4}$ (b) $\frac{MA^2}{2}$ (c) MA^2 (d) $2MA^2$

4. The Dulong Petit law fails near room temperature (300 K) for many light elements (such as boron and beryllium) because their Debye [illegible]
(a) $\gg 300$ K (b) [illegible] (c) $\ll 300$ K (d) [illegible] K

5. The dispersion relation of phonons in solids is given by $\omega^2(k) = \omega_0^2\left(3 - \cos k_x a - \cos k_y a - \cos k_z a\right)$
The velocity of the phonons at large wavelenght is
(a) $\omega_0 a/\sqrt{3}$ (b) $\omega_0 a$ (c) $\sqrt{3}\omega_0 a$ (d) $\omega_0 a/\sqrt{2}$

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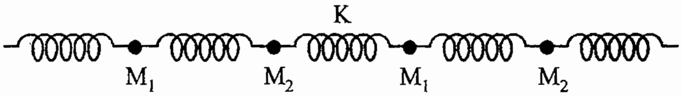
6. The total energy of an ionic solid is given by an expression $E = -\frac{\alpha e^2}{4\pi\varepsilon_0 r} + \frac{B}{r^9}$ where α is Madelung constant, r is the distance between the nearest neighbours in the crystal and B is a constant. If r_0 is the equilibrium separation between the nearest neighbours then the value of B is
(a) $\frac{\alpha e^2 r_0^8}{36\pi\epsilon_0}$ (b) $\frac{\alpha e^2 r_0^8}{4\pi\epsilon_0}$ (c) $\frac{2\alpha e^2 r_0^{10}}{9\pi\epsilon_0}$ (d) $\frac{\alpha e^2 r_0^{10}}{36\pi\epsilon_0}$

7. The phonon dispersion for the following one-dimensional diatomic lattice with masses M_1 and M_2 (as shown in the figure)



is given by

$$\omega^2(q)=K\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\left[1\pm\sqrt{1-\frac{4M_1M_2}{(M_1+M_2)^2}\sin^2\left(\frac{qa}{2}\right)}\right]$$

where a is the lattice parameter and K is spring constant. The velocity of sound is

(a) $\sqrt{\frac{K(M_1+M_2)}{2M_1M_2}}a$ (b) $\sqrt{\frac{K}{2(M_1+M_2)}}a$

(c) $\sqrt{\frac{K(M_1+M_2)}{M_1M_2}}a$ (d) $\sqrt{\frac{KM_1M_1}{2(M_1+M_2)^3}}a$

8. The position of an X-ray diffraction peak may be shifted with respect to the peak of a perfect crystal due to
(a) homogenous strain (b) inhomogeneous strain
(c) small unstrained crystallites (d) inhomogeneously strained nano-sized crystallites

9. Using the frequency-dependent Drude formula, what is the effective kinetic inductance of a metallic wire that is to be used as a transmission line? [In the following, the electron mass is m, density of electrons is n, and the length and cross-sectional area of the wire are ℓ and A respectively]

(a) $mA/(ne^2\ell)$ (b) zero

(c) $m\ell/(ne^2A)$ (d) $m\sqrt{A}/(ne^2\ell^2)$

10. A typical semiconductor such as Ge is formed by what kind of bonds?
(a) Covalent (b) Ionic (c) van der Waals (d) Metallic

11. The lattice specific heat C of a crystalline solid can be obtained using the Dulong Petit model, Einstein model and Debye model. At low temperature $\hbar\omega >> K_BT$, which one of the following statements is true (a and A are constants)

(a) Dulong Petit: $C \propto$ ex[illegible]tant, Debye: $C \propto$ [illegible]

(b) Dulong Petit: C = .co[illegible]; Einstein: $C \propto$ [illegible]; Debye: $C \propto \exp$ [illegible]

(c) Dulong Petit: C = .constant; Einstein: $C \propto \frac{e^{\,\cdot}}{T^2}$; Debye: $C \propto \left(\frac{T}{A}\right)$

(d) Dulong Petit: $C \propto \left(\frac{T}{A}\right)^3$; Einstein: $C \propto \frac{e^{-a/T}}{T^2}$; Debye: C=.constant

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12. A uniform linear monoatomic chain is modeled by a spring-mass system of masses m separated by nearest neighbor distance a and spring constant $m\omega_0^2$. The dispersion relation for this system is

(a) $\omega(k)=2\omega_0\left(1-\cos\left(\frac{ka}{2}\right)\right)$ (b) $\omega(k)=2\omega_0\sin^2\left(\frac{ka}{2}\right)$

(c) $\omega(k)=2\omega_0\sin\left(\frac{ka}{2}\right)$ (d) $\omega(k)=2\omega_0\tan\left(\frac{ka}{2}\right)$

3. At low temperature the specific heat of insulating crystals varies as
(a) AT^3 (b) $BT + CT^3$
(c) D exp (E/T) (d) will not change with temperature.

4. Consider two crystalline solids, one of which has a simple cubic structure, and the other has a tetragonal structure. The effective spring constant between atoms in the *c*-direction is half the effective spring constant between atoms in the *a* and *b* directions. At low temperatures, the behaviour of the lattice contribution to the specific heat will depend as a function of temperature *T* as

(a) T^2 for the tetragonal solid, but as T^3 for the simple cubic solid
(b) T for the tetragonal solid and as T^3 for the simple cubic solid
(c) T for both solids
(d) T^3 for both solids

5. A one-dimensional linear chain of atoms contains two types of atoms of masses m_1 and m_2 (where $m_2 > m_1$), arranged alternately. The distance between successive atoms is the same. Assume that the harmonic approximation is valid. At the first Brillouin zone boundary, which of the following statements is correct?
(a) The atoms of mass m_2 are at rest in the optical mode, while they vibrate in the acoustical mode.
(b) The atoms of mass m_1 are at rest in the optical mode, while they vibrate in the acoustical mode.
(c) Both types of atoms vibrate with equal amplitudes in the optical as well as in the acoustical modes.
(d) Both types of atoms vibrate, but with unequal, non-zero amplitudes in the optical as well as in the acoustical modes. **[GATE 2016]**

6. The dispersion relation for phonons in a one dimensional monatomic Bravais lattice with lattice spacing a and consisting of ions of masses M is given by, $\omega(k) = \sqrt{\frac{2C}{M}\left[1 - \cos(ka)\right]}$, where ω is the frequency of oscillation, k is the wavevector and C is the spring constant. For the long wavelength modes $(\lambda >> a)$, the ratio of the phase velocity to the group velocity is ________ **[GATE 2015]**

7. The ratio of the specific heat capacity and temperature, C_υ / T, of Cu is plotted as a function of T^2, the square of the absolute temperature in the graph below:

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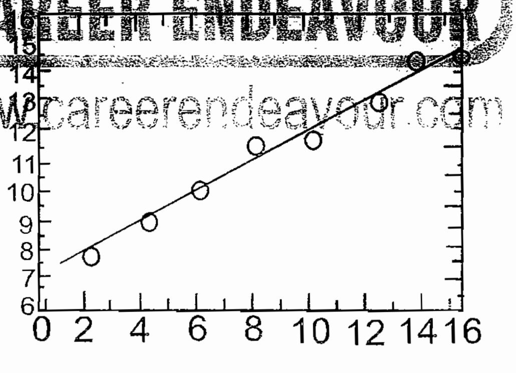

The values of γ and β (the coefficients corresponding to the electronic and the vibrational components of the specific heat) are, approximately

(a) $\gamma = 7.0 \times 10^{-4}\,\text{Jmol}^{-1}\text{K}^{-2}$ and $\beta = 5.0 \times 10^{-5}\,\text{Jmol}^{-1}\text{k}^{-4}$
(b) $\gamma = 5.0 \times 10^{-5}\,\text{J mol}^{-1}\,\text{k}^{-2}$ and $\beta = 7.0 \times 10^{-4}\,\text{J mol}^{-1}\text{k}^{-4}$
(c) $\gamma = 1.4 \times 10^{-3}\,\text{J mol}^{-1}\,\text{k}^{-2}$ and $\beta = 7.0 \times 10^{-4}\,\text{J mol}^{-1}\text{k}^{-4}$
(d) $\gamma = 5.0 \times 10^{-4}\,\text{J mol}^{-1}\,\text{k}^{-2}$ and $\beta = 7.0 \times 10^{-5}\,\text{J mol}^{-1}\text{k}^{-4}$

18. The Fermi energies of two metals X and Y are 5eV and 7eV and their Debye temperatures are 170 K and 340 K, respectively. The molar specific heats of these metals at constant volume at low temperatures can be written as $(C_V)_X = \gamma_X T + A_X T^3$ and $(C_V)_Y = \gamma_Y T + A_Y T^3$, where γ and A are constants. Assuming that the thermal effective mass of the electrons in the two metals are same, which of the following is correct?

(a) $\frac{\gamma_X}{\gamma_Y} = \frac{7}{5}, \frac{A_X}{A_Y} = 8$ (b) $\frac{\gamma_X}{\gamma_Y} = \frac{7}{5}, \frac{A_X}{A_Y} = \frac{1}{8}$ **[GATE 2016]**

(c) $\frac{\gamma_X}{\gamma_Y} = \frac{5}{7}, \frac{A_X}{A_Y} = \frac{1}{8}$ (d) $\frac{\gamma_X}{\gamma_Y} = \frac{5}{7}, \frac{A_X}{A_Y} = 8$

19. The binding energy per molecule of NaCl (lattice parameter is 0.563 nm) is 7.95 eV. The repulsive term of the potential is of the form $\frac{K}{r^9}$, where K is *a* constant. The value of Madelung constant is ____________ (upto three decimal places). **[GATE 2015]**

(Electron charge $e = -1.6\times10^{-19}C; \varepsilon_0 = 8.854\times10^{-12}\ C^2N^{-1}m^{-2}$).

20. The lattice constant of a material is of the order of a μm, and its bond energies are of the order of an eV. The bulk modulus of such a material, in Pascals, is of the order of **[TIFR 2016]**

(a) 10^{1} (b) 10^{-3} (c) 10^{-6} (d) 10^{9}

21. The origin of van der Waals interaction of molecular crystals is:
(a) nuclear (b) magnetic (c) ionic (d) fluctuating dipolar

22. In a cube system with cell edge a, two phonos with wave vectors $\vec{q}_1$ and $\vec{q}_2$ collide and produce a third phonon with a wave vector $\vec{q}_3$ such that $\vec{q}_1 + \vec{q}_2 = \vec{q}_3 + \vec{R}$,
Where $\vec{R}$ is a lattice vector. Such a collision process will lead to:
(a) finite thermal resistance
(b) zero thermal resistance
(c) an infinite thermal resistance
(d) a finite thermal resistance for certain $\vec{R}$ only.

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23. Consider the Fermi-Dirac distribution function f(E) at room temprature (300 K) where E refers to energy. If E_F is the Fermi energy, which of the following is true?
(a) f(E) is a step function
(b) f(EF) has a value of 1/2
(c) States with $E<E_F$ are filled completely
(d) f(E) is large and tends to infinity as E decreases much below E_F

24. Consider the energy E in the first Brillouin zone as a function of the magnitude of the wave vector k for a crystal of lattice constant a. Then
(a) the slope of E versus k is proportional to the group velocity
(b) the slope of E versus k has its maximum value at $|k| = \pi/a$
(c) the plot of E versus k will be parabolic in the interval $(-\pi/a) < k < (\pi/a)$
(d) the slope of E versus k is non-zero for all k the interval $(-\pi/a) < k\ (\pi/a)$

Statement for Linked Answer Q. 25 and Q. 26.

The atomic density of a solid is $5.85\times10^{28}\ \text{m}^{-3}$. Its electrical resistivity is $1.6\times10^{-8}\ \Omega$ m. Assume that electrical conduction is described by the Drude model (classical theory), and that each atom contributes one conduction electron.

The drift mobility (in $\text{m}^2\ \text{V}^{-1}\text{s}^{-1}$) of the conduction electrons is:

(a) 6.67×10^{-3} (b) 6.67×10^{-6} (c) 7.63×10^{-3} (d) 7.63×10^{-6}

The relaxation time (mean free time), in seconds, of the conduction electrons is:

(a) 3.98×10^{-15} (b) 3.79×10^{-14} (c) 2.84×10^{-12} (d) 2.64×10^{-11}

Metallic monovalent sodium crystallizes in body centered cubic structure. If the length of the unit cell is 4×10^{-8} cm, the concentration of conduction electrons in metallic sodium is:

(a) $6.022\times10^{23}\ \text{cm}^{-3}$ (b) $3.125\times10^{22}\ \text{cm}^{-3}$

(c) $2.562\times10^{21}\ \text{cm}^{-3}$ (d) $1.250\times10^{20}\ \text{cm}^{-3}$

The kinetic energy of a free electron at a corner of the first Brillouin zone of a two dimensional square lattice is larger than that of an electron at the mid-point of a side of the zone by a factor b. The value of b is:

(a) $b=\sqrt{2}$ (b) b = 2 (c) b = 4 (d) b = 8

The valence electrons do not directly determine the following property of a metal.

(a) Electrical conductivity (b) Thermal conductivity

(c) Shear modulus (d) metallic lustre.

For a two-dimensional free electron gas, the electronic density n, and the Fermi energy E_F, are related by

(a) $n=\dfrac{(2mE_F)^{3/2}}{3\pi^2\hbar^3}$ (b) $n=\dfrac{mE_F}{\pi\hbar^2}$ (c) $n=\dfrac{mE_F}{2\pi\hbar^2}$ (d) $n=\dfrac{2^{3/2}(mE_F)^{1/2}}{\pi\hbar}$

The resistivity of metals:

(a) Always varies as T^5

(b) is independent of temperature

(c) increases liniearly with [illegible] room temperature and above

(d) attains a constant value [illegible] to absolute zero

For a Fermi gas of N particles in three dimensions at T = 0 K the Fermi energy, E_F

(a) $N^{2/3}$ (b) $N^{3/2}$ (c) N^3 (d) N^2

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For an energy state E of a photon gas, the density of states is proportional to:

(a) $\sqrt{E}$ (b) E (c) $E^{3/2}$ (d) E^2

For a perfect free-electron gas in a metal, the magnitudes of phase velocity (v_p) and group velocity (v_g) are such that:

(a) $v_p=v_g$ (b) $v_p=\frac{1}{2}v_g$ (c) $v_p=\sqrt{2}v_g$ (d) $v_p=2\ v_g$

In simple metals the phonon contribution to the electrical resistivity at temperature T is:

(a) directly proportional to T above Debye temperature and to T^3 below it.

(b) inversely proportional to T for all temperatures

(c) independent of T for all temperatures

(d) directly proportional to T above Debye temperature and to T^5 below it

36. Density of states of free electrons in a solid moving with an energy 0.1 eV is given by 2.15×10^{21} eV^{-1} cm^{-3}. The density of states in eV^{-1} cm^{-3}) for electrons moving with an energy of 0.4 eV will be:

(a) 1.07×10^{21} (b) 1.52×10^{21} (c) 3.04×10^{21} (d) 4.30×10^{21}

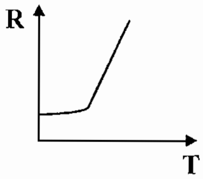
37. Which one of the following statements is NOT correct about the Brillouin zones (BZ) of a square lattices with constant a?

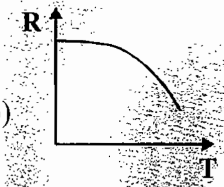
(a) The first BZ is a square of size 2p/a in k_x–k_y plane
(b) The areas of the first BZ and third BZ are the same
(c) The k-points are equidistant in k_x as well as in k_y directions
(d) The area of the second BZ is twice that of the first BZ

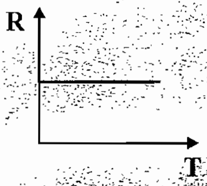
38. An electron confined within a thin layer of semiconductor may be treated as a free particle inside an infinitely deep one-dimensional potential well. If the difference in energies between the first and the second energy levels is δE, then the thickness of the layer is: **[JEST 2016]**

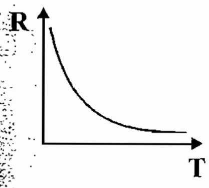
(a) $\sqrt{\dfrac{3\hbar^2\pi^2}{2m\delta E}}$ (b) $\sqrt{\dfrac{2\hbar^2\pi^2}{3m\delta E}}$ (c) $\sqrt{\dfrac{\hbar^2\pi^2}{2m\delta E}}$ (d) $\sqrt{\dfrac{\hbar^2\pi^2}{m\delta E}}$

39. Temperature dependence of resistivity of a metal can be described by

(a)


(b)


(c)


(d)


40. A copper wire is 1 metre long and has a uniform cross-section of 0.1 mm^2. The resistance of the wire at room temperature is 0.171 ohm. What is the resistivity of the material?

(a) 1.71×10^{-6} Ω.m (b) 1.71×10^{-7} Ω.m (c) 1.71×10^{-8} Ω.m (d) 1.71×10^{-9} Ω.m

41. As the Fermi energy of sil[illegible]ule, the velocity of the fastest electron in the silver at 0 K. (Given: Rest mass of electron = 9.[illegible]

(a) 3.33×10^{5} m/s (b) [illegible] (c) 1.39×10^{6} m/s (d) [illegible]$\times10^{8}$ m/s

42. The temperature dependence [illegible] is shown in the plot below. This material is a

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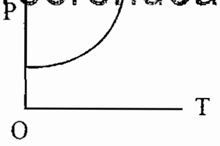
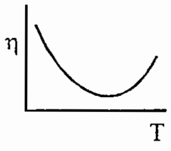
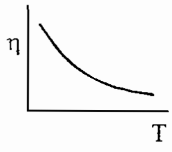
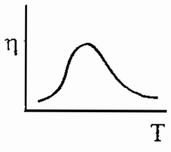


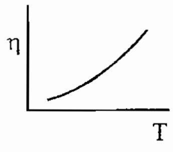
(a) insulator (b) semiconductor (c) normal metal (d) superconductor

43. Which of the following figures represents the temperature dependence of the thermal conductivity η of a solid according to the Drude model.

(a)


(b)


(c)


(d)


For Li metal, the resistivity at 77K is $1.01\ \mu\Omega-\text{cm}$ and that at 273K is $8.57\ \mu\Omega-\text{cm}$. The ratio of the mean collision times, $\frac{\tau_{77}}{\tau_{273}}$ is approximately,

(a) 3 (b) 1/9 (c) 9 (d) 1/3

The real part of the ac conductivity of a metal at a large frequency ω ($\omega\tau >> 1, \tau =$ mean collision time).

(a) increases as ω (b) decreases $\frac{1}{\omega^2}$

(c) becomes independent of ω (d) varies as $\sqrt{\omega}$

For an ideal Fermi gas in three dimensions, the electron velocity V_F at he Fermi surface is related to electron concentration n as,

(a) $V_F \propto n^{2/3}$ (b) $V_F \propto n$ (c) $V_F \propto n^{1/2}$ (d) $V_F \propto n^{1/3}$

The radius of the Fermi sphere of free electrons in a monovalent metal with an fcc structure, in which the volume of the unit cell is a^3, is

(a) $\left(\frac{12\pi^2}{a^3}\right)^{1/3}$ (b) $\left(\frac{3\pi^2}{a^3}\right)^{1/3}$ (c) $\left(\frac{\pi^2}{a^3}\right)^{1/3}$ (d) $\frac{1}{a}$

Which one of the statements about density of states is not true?
(a) Variation in the energy dependence of density of states is dimension independent.
(b) Variation in the energy dependence of density of states in 3D follows $E^{1/2}$
(c) Variation in the energy dependence of density of states in 2D follows E^0
(d) Variation in the energy dependence of density of states in 1D follows $E^{-1/2}$

The fraction of electrons excited across the energy gap of 0.7 eV in Ge, at room temperature is

(a) 0 (b) 1.4×10^{-6} (c) 1.6×10^{-8} (d) 1.03×10^{-12}

What is the value of potential energy V in a free electron gas theory of metals?

(a) A constant (b) Infinite
(c) It varies with the distance (d) Zero

Which of the following statement best explains why the specific heat of electrons in metals is much smaller than that expected in a non-interacting (free) electron gas model?

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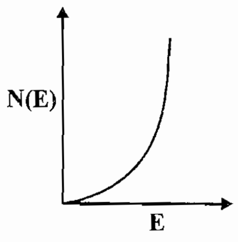
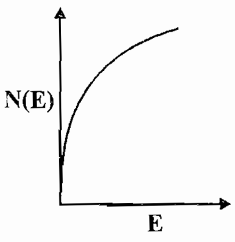
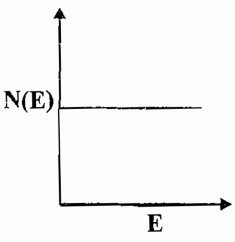
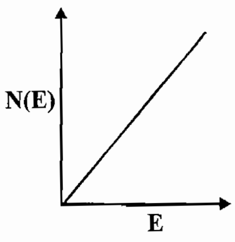
(a) The mass of electron is much smaller than that of the ions in the crystal.
(b) The Pauli exclusion principle restricts the number of electrons which can absorb thermal energy
(c) Electron spin can take only two different values.
(d) Electrons in a metal cannot be modelled as non-interacting.

For a free electron gas in two dimensions, the variation of the density of states, N(E) as a function of energy E, is the best represented by

(a)


(b)


(c)


(d)


53. The energy dependence of the density of states for a two dimensional non-relativistic electron gas is given by, $g(E) = CE^n$, where C is constant. The value of n is ________ **[GATE 2015]**

54. Given that the Fermi energy of gold is 5.54 eV, the number density of electrons is ________ $\times 10^{28}$ m^{-3} (upto one decimal places) **[GATE 2015]**
(Mass of electron = 9.11×10^{-31} kg; h = 6.626×10^{-34} J.s; 1 eV = 1.6×10^{-19} J)

55. A thin metal film of dimension 2 mm × 2 mm contains 4×10^{12} electrons. The magnitude of the Fermi wavevector of the system, in the free electron approximation, is **[NET Dec. 2015]**
(a) $2\sqrt{\pi} \times 10^7 \text{cm}^{-1}$ (b) $\sqrt{2\pi} \times 10^7 \text{cm}^{-1}$ (c) $\sqrt{\pi} \times 10^7 \text{cm}^{-1}$ (d) $2\pi \times 10^7 \text{cm}^{-1}$

56. The potential in a divalent solid at a particular temperature is represented by a one-dimensional periodic model. The solid should behave electrically as:
(a) semiconductor (b) a conductor
(c) an insulator (d) a superconductor

57. A metal has free-electron density n = 10^{29} m^{-3}. Which of the following wavelengths will excite plasma oscillations?
(a) 0.033 μm (b) 0.330 μm (c) 3.300 μm (d) 33.000 μm

58. In a one-dimensional Kroning Penny model, the total number of possible wave functions is equal to:
(a) twice the number of unit cells
(b) number of unit cells
(c) half the number of unit cells
(d) independent of the number of unit cells

59. In a crystal of N primitive cells, each cell contains two monovalent atoms. The highest occupied energy band of the crystal is:
(a) one-fourth filled (b) one-third filled (c) half filled (d) completely filled

60. The Bloch theore states that within a crystal, the wavefunction, $\psi(\vec{r})$, of an electron has the form

(a) $\psi(\vec{r}) = u(\vec{r})e^{i\vec{k}.\vec{r}}$ where $u(\vec{r})$ is an arbitrary function and $\vec{k}$ is an arbitrary vector.

(b) $\psi(\vec{r}) = u(\vec{r})e^{i\vec{G}.\vec{r}}$ where $u(\vec{r})$ is an arbitrary function and $\vec{G}$ is a reciprocal lattice vector.

(c) $\psi(\vec{r}) = u(\vec{r})e^{i\vec{G}.\vec{r}}$ where $u(\vec{r}) = u(\vec{r}+\vec{A})$, $\vec{A}$ is a lattice vector and $\vec{G}$ is a reciprocal lattice vector.

(d) $\psi(\vec{r}) = u(\vec{r})e^{i\vec{k}.\vec{r}}$ where $u(\vec{r}) = u(\vec{r}+\vec{A})$, $\vec{A}$ is a lattice vector and $\vec{k}$ is an arbitrary vector.

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61. The lattice specific heat C of a crystalline solid can be obtained using the Dulong Petit model. Einstein modle and Debye model. At low tesmperature hw >> k_BT, which one of the following statements is true (a and A are constants)

(a) Dulong Petit : $C \propto \exp\left(\frac{-a}{T}\right)$; Einstein: C = constant; Debye : $C \propto \left(\frac{T}{A}\right)^3$

(b) Dulong Petit : C = constant; Einstein: $C \propto \left(\frac{T}{A}\right)^3$ Debye : $C \propto \left(\frac{-a}{T}\right)$

(c) Dulong Petit : C = constant; Einstein: $C \propto \frac{e^{-a/T}}{T^2}$ Debye : $C \propto \left(\frac{T}{A}\right)^3$

(d) Dulong Petit : $C \propto \left(\frac{T}{A}\right)^3$; Einstein: $C \propto \frac{e^{-a/T}}{T^2}$ Debye : C = constant

Common data Q. 62 and Q.63:

The tight binding energy dispersion (E–k) relation for electrons in a one-dimensional array of atoms having lattice constant a and total length L is:

$$E = E_0 - \beta - 2\gamma\cos(ka)$$

Where E_0, β and γ are constants and k is the wave-vector.

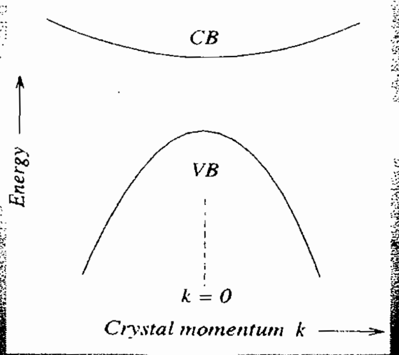
2. The density of states of electrons (including spin degeneracy) in the band is given by

(a) $\dfrac{L}{\pi\gamma a\sin(ka)}$ (b) $\dfrac{L}{2\pi\gamma a\sin(ka)}$ (c) $\dfrac{L}{2\pi\gamma a\cos(ka)}$ (d) $\dfrac{L}{\pi\gamma a\cos(ka)}$

3. The effective mass of electrons in the band is given by

(a) $\dfrac{\hbar^2}{\gamma a^2\cos(ka)}$ (b) $\dfrac{\hbar^2}{2\gamma a^2\cos(ka)}$ (c) $\dfrac{\hbar^2}{\gamma a^2\sin(ka)}$ (d) $\dfrac{\hbar^2}{2\gamma a^2\cos(ka)}$

4. Suppose the energy band of a certain pure crystalline solid is as shown in the figure below, where the energy (E) varies with crystal momentum (k) as $E \propto k^2$



At finte temperatures the b[illegible]ion band (CB) is partia[illegible]d with electrons (e) and the top of the valence band (VB) is [illegible] f an electric field is [illegible]d to this solid, both e and h will start moving. If the time between collisions is the same for both e and h then

(a) e and h will move with the same speed in opposite directions
(b) h will on an average achieve higher speed than e
(c) e will on an average achieve higher speed than h
(d) e and h will recombine and after a while there will be no flow of charges.

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. In a period potential given by $V(\vec{r}) = V(\vec{r}+\vec{R})$, where $\vec{R}$ is the lattice vector, the eigenstates of the Hamiltonian $H = \dfrac{-\hbar^2}{2m}\nabla^2 + V(\vec{r})$ are $\psi_k(\vec{r}) = e^{i\vec{k}.\vec{r}}u_{\vec{k}}(\vec{r})$, where

(a) $u_{\vec{k}}(\vec{r}) = e^{i\vec{k}.\vec{R}}(\vec{k} \neq \vec{k}')$ (b) $u_{\vec{k}}(\vec{r}) = e^{i\vec{k}.\vec{R}}\, u_{\vec{k}}\left(\vec{r}+\dfrac{\vec{R}}{3}\right)$

(c) $u_{\vec{k}}(\vec{r}) = u_{\vec{k}}\left(\vec{r}+\dfrac{\vec{R}}{2}\right)$ (d) $u_{\vec{k}}(\vec{r}) = u_{\vec{k}}(\vec{r}+\vec{R})$

66. The Fermi level in an intrinsic semiconductor at zero temperature lies
(a) in the conduction band
(b) at the top of the valence band
(c) at the bottom of the conduction band
(d) in the energy gap between the valence and the conduction bands

67. In a band structure calculation, the dispersion relation for electrons is found to be

$$\varepsilon_k = \beta\left(\cos k_x a + \cos k_y a + \cos k_z a\right)$$

where β is a constant and a is the lattice constant. The effective mass at the boundary of the first Brillouin zene is

(a) $\frac{2\hbar^2}{5\beta a^2}$ (b) $\frac{4\hbar^2}{5\beta a^2}$ (c) $\frac{\hbar^2}{2\beta a^2}$ (d) $\frac{\hbar^2}{3\beta a^2}$

68. The energy of an electron in a band as a function of its wave vector **k** is given by
$E(k) = E_0 - B(\cos k_x a + \cos k_y a + \cos k_z a)$, where E_0, B and a are constants. The effective mass of the electron near the bottom of the band is

(a) $\frac{2\hbar^2}{3Ba^2}$ (b) $\frac{\hbar^2}{3Ba^2}$ (c) $\frac{\hbar^2}{2Ba^2}$ (d) $\frac{\hbar^2}{Ba^2}$

69. Melting temperature of strained nanoparticles having the same structure as that of bulk is
(a) higher than that of bulk (b) lower than that of bulk
(c) does not change (d) always 2/3 times that of bulk

70. Which one of the following diffraction peaks will be absent in a powder diffraction pattern for BCC-crystalline powder such as Fe
(a) (110) (b) (111) (c) (200) (d) (211)

71. The energy, ε_k for band el[illegible] of the wave vector, k in [illegible] Brillouin zone $\left(-\frac{\pi}{a} \le k \le \frac{\pi}{a}\right)$ of a one dimensional monatomic lattice is shown as (a is lattice constant)

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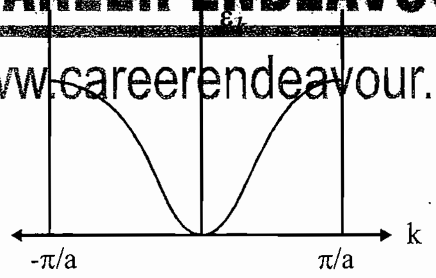
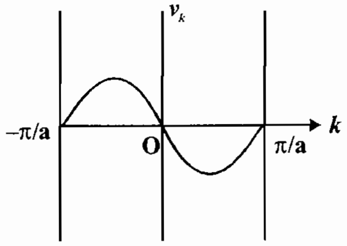
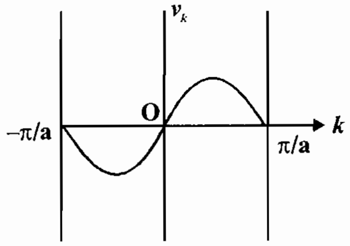
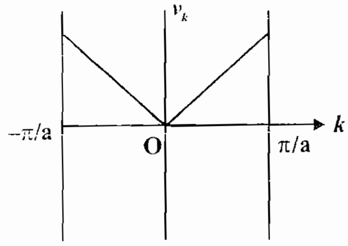


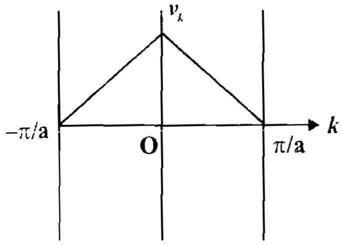
(a)


(b)


(c)



(d)



2. Consider an electron in a b.c.c. lattice with lattice constant a. A single particle wavefunction that satisfies the Bloch theorem will have the form $f(\bar{r})\exp(i\bar{k}.\bar{r})$, with $f(\bar{r})$ being

(a) $1+\cos\left[\frac{2\pi}{a}(x+y-z)\right]+\cos\left[\frac{2\pi}{a}(-x+y+z)\right]+\cos\left[\frac{2\pi}{a}(x-y+z)\right]$

(b) $1+\cos\left[\frac{2\pi}{a}(x+y)\right]+\cos\left[\frac{2\pi}{a}(y+z)\right]+\cos\left[\frac{2\pi}{a}(z+x)\right]$

(c) $1+\cos\left[\frac{\pi}{a}(x+y)\right]+\cos\left[\frac{\pi}{a}(y+z)\right]+\cos\left[\frac{\pi}{a}(z+x)\right]$

(d) $1+\cos\left[\frac{\pi}{a}(x+y-z)\right]+\cos\left[\frac{\pi}{a}(-x+y+z)\right]+\cos\left[\frac{\pi}{a}(x-y+z)\right]$

3. The dispersion relation for electrons in an f.c.c. crystal is given, in the tight binding approximation by

$$\varepsilon(k)=-4\varepsilon_0\left[\cos\frac{k_x a}{2}\cos\frac{k_y a}{2}+\cos\frac{k_y a}{2}\cos\frac{k_z a}{2}+\cos\frac{k_z a}{2}\cos\frac{k_x a}{2}\right]$$

where 'a' is the lattice constant and ε_0 is a constant with the dimension of energy. The x-component of the velocity of the electrons at $\left(\frac{\pi}{a},0,0\right)$ is

(a) $-\frac{2\varepsilon_0 a}{\hbar}$ (b) $\frac{2\varepsilon_0 a}{\hbar}$ (c) $-\frac{4\varepsilon_0 a}{\hbar}$ (d) $\frac{4\varepsilon_0 a}{\hbar}$

4. Which of the following is the best technique for measuring the effective mass of an electron in a semiconductor?
(a) Resistivity measurements (b) X-rays diffraction experiment
(b) Cyclotron resonance (d) Millikan's oil drop experiment

5. In the basic band structure theory of crystalline solids which of the following leads to energy gaps in the allowed electronic energy values ?
(a) Electron spin (b) Bragg reflection
(c) Electron-electron interacion (d) Electron-phonon interaction

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6. The material inside a box is either a metal or a semiconductor. If R (= 1 Ω) is the resistance of the material, which of the following experiments CANNOT distinguish whether it is a metal or a semiconductor ?
(a) Measurement of R using power supplies of different frequencies.
(b) Measurement of absorption spectrum in the energy range 0.1 – 2 eV.
(c) Measurement of R at different temperatures.
(d) Measurement of R in the presence of different magnetic fields.

7. Given that tight binding dispersion relation $E(k)=E_0+A\sin^2\left(\frac{ka}{2}\right)$, where E_0 and A are constant and a is the lattice parameter. What is the group velocity of an electron at the second Brillouin zone boundary?

(a) 0 (b) $\frac{a}{h}$ (c) $\frac{2a}{h}$ (d) $\frac{a}{2h}$ **[JEST 2015]**

78. The energy vs. wave vector $(E-k)$ relationship near the bottom of a band for a solid can be approximated as $E = A(ka)^2 + B(ka)^4$, where the lattice constant $a = 2.1$ Å. The values of A and B are 6.3×10^{-19} J and 3.2×10^{-20} J, respectively. At the bottom of the conduction band, the ratio of the effective mass of the electron to the mass of free electron is ________. (Give your answer upto two decimal places) **[GATE 2016]**
(Take $\hbar = 1.05 \times 10^{-34}$ J-s, mass of free electron $= 9.1 \times 10^{-31}$ kg).

79. The dispersion relation for electrons in the conduction band of a n-type semiconductor has the form $E(k) = ak^2 + b$ where a and b are constant. It was observed that the cyclotron resonance frequency of such electrons is $\omega_0 = 1.8 \times 10^{11}$ rad s^{-1}, when placed in a magnetic field $B = 0.1$ W m^{-2}. It follows that the constant a must be about **[TIFR 2016]**
(a) 10^{-36} (b) 10^{-28} (c) 10^{-32} (d) 10^{-38}

80. The dispersion relation of electrons in a 3-dimensional lattice in the tight binding approximation is given by,
$\varepsilon_k = \alpha \cos k_x a + \beta \cos k_y a + \gamma \cos k_z a$ where a is the lattice constant and α, β, γ are constants with dimension of energy. The effective mass tensor at the corner of the first Brillouin zone $\left(\frac{\pi}{a}, \frac{\pi}{a}, \frac{\pi}{a}\right)$ is

(a) $\frac{\hbar^2}{a^2}\begin{pmatrix} -\frac{1}{\alpha} & 0 & 0 \\ 0 & -\frac{1}{\beta} & 0 \\ 0 & 0 & \frac{1}{\gamma} \end{pmatrix}$ (b) $\frac{\hbar^2}{a^2}\begin{pmatrix} -\frac{1}{\alpha} & 0 & 0 \\ 0 & -\frac{1}{\beta} & 0 \\ 0 & 0 & -\frac{1}{\gamma} \end{pmatrix}$ **[NET Dec. 2015]**

(c) $\frac{\hbar^2}{a^2}\begin{pmatrix} \frac{1}{\alpha} & 0 & 0 \\ 0 & \frac{1}{\beta} & 0 \\ 0 & 0 & \frac{1}{\gamma} \end{pmatrix}$ (d) $\frac{\hbar^2}{a^2}\begin{pmatrix} \frac{1}{\alpha} & 0 & 0 \\ 0 & \frac{1}{\beta} & 0 \\ 0 & 0 & -\frac{1}{\gamma} \end{pmatrix}$

81. For an electron moving through a one-dimensional periodic lattice of periodicity a, which of the following corresponds to an energy eigenfunction consistent with Bloch's theorem? **[NET Dec. 2015]**

(a) $\psi(x) = A\exp\left(i\left[\frac{\pi x}{a} + \cos\left(\frac{\pi x}{2a}\right)\right]\right)$ (b) $\psi(x) = A\exp\left(i\left[\frac{\pi x}{a} + \cos\left(\frac{2\pi x}{a}\right)\right]\right)$

(c) $\psi(x) = A\exp\left(i\left[\frac{2\pi x}{a} + i\cosh\left(\frac{2\pi x}{a}\right)\right]\right)$ (d) $\psi(x) = A\exp\left(i\left[\frac{\pi x}{2a} + i\left|\frac{\pi x}{2a}\right|\right]\right)$

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82. Consider electrons in graphene, which is a planar monatomic layer of carbon atoms. If the dispersion relation of the electrons is taken to be $\varepsilon(k) = ck$ (where c is constant) over the entire k-space, then the Fermi energy ε_F depends on the number density of electrons ρ as **[NET June 2016]**

(a) $\varepsilon_F \propto \rho^{1/2}$ (b) $\varepsilon_F \propto \rho$ (c) $\varepsilon_F \propto \rho^{2/3}$ (d) $\varepsilon_F \propto \rho^{1/3}$

83. The band energy of an electron in a crystal for a particular k-direction has the form $\varepsilon(k) = A - B\cos 2ka$, where A and B are positive constants and $0 < ka < \pi$. The electron has a hole-like behaviour over the following range of k: **[NET June 2016]**

(a) $\frac{\pi}{4} < ka < \frac{3\pi}{4}$ (b) $\frac{\pi}{2} < ka < \pi$ (c) $0 < ka < \frac{\pi}{4}$ (d) $\frac{\pi}{2} < ka < \frac{3\pi}{4}$

84. The low-energy electronic excitations in a two-dimensional sheet of graphene is given by $E(\vec{k}) = \hbar v k$, where v is the velocity of the excitations. The density of states is proportional to **[NET June 2015]**

(a) E (b) $E^{3/2}$ (c) $E^{1/2}$ (d) E^2

85. Consider a metal which obeys the Sommerfeld model exactly. If E_F is the Fermi energy of the metal at $T = 0\text{K}$ and R_H is its Hall coefficient, which of the following statements is correct? **[GATE 2016]**

(a) $R_H \propto E_F^{3/2}$ (b) $R_H \propto E_F^{2/3}$

(c) $R_H \propto E_F^{-3/2}$ (d) R_H is independent of E_F

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SOLID STATE PHYSICS

Assignment-4

1. The effective mass of an electron in a semiconductor can be:
(a) negative near the bottom of the band (b) a scalar quantity with a small magnitude
(c) zero at the center of the band (d) negative near the top of the band

2. The effective mass of an electron in a semiconductor
(a) can never be positive (b) can never be negative
(c) can be positive or negative (d) depends on its spin

3. An n-type semiconductor has an electron concentration of 3×10^{20} m^{-3}. If the electron drift velocity is 100 ms^{-1} in an electric field of 200 Vm^{-1}, the conductivity (in W^{-1}m^{-1}) of this material is:
(a) 24 (b) 36 (c) 48 (d) 96

4. Which one of the following is NOT a correct statement about semiconductors?
(a) The electrons and holes have different mobilities in a semiconductor
(b) In an n-type semiconductor, the Fermi level lies closer to the conduction band edge
(c) Silicon is a direct band gap semiconductor
(d) Silicon has diamond structure

5. Compared to a p-n junction with $N_A = N_D = 10^{14}/cm^3$, which one of the following statement is TRUE for a p-n junction with $N_A = N_D = 10^{20}/cm^3$?
(a) Reverse breakdown voltage is lower and depletion capacitance is lower.
(b) Reverse breakdown voltage is higher and depletion capacitance is lower.
(c) Reverse breakdown volt[illegible]pletion capacitance is higher.
(d) Reverse breakdown vol[illegible]tion capacitance is hi[illegible]

6. The ratio of the mobility to the diffusion coefficient in a semiconductor has the units:
(a) V^{-1} (b) cmV[illegible] (c) [illegible] (d) $V s$

7. Silicon is doped with boron to a concentration of 4×10^{17} atoms/cm^3. Assume the intrinsic carrier concentration of silicon to be 1.5×10^{10} /cm^3 and the value of kT/q to be 25 mV at 300 K. Compared to un-doped silicon, the Fermi level of doped silicon:
(a) Goes down by 0.13 eVs (b) Goes up by 0.13 eV
(c) Goes down by 0.427 eV (d) Goes up by 0.427 eV

8. The electron and hole concentrations in an intrinsic semiconductor are n_i per cm^3 at 300K. Now, if acceptor impurities are introduced with a concentration of N_A per cm^3 (where $N_A >> n_i$), the electron concentration per cm^3 at 300 K will be:
(a) n_i (b) $n_i + N_A$ (c) $N_A - n_i$ (d) $\frac{n_i^2}{N_A}$

9. In a p$^+$ n junction diode under reverse bias, the magnitude of electric field is maximum at:
(a) The edge of the depletion region on the p-side (b) The edge of the depletion region on the n-side
(c) The p$^+$ n junction (d) The centre of the depletion region on the n-side.

10. A P^+–N junction has a built-in potential of 0.8 V. The depletion layer width at a reverse bias of 1.2 V is 2 μm. For a reverse bias of 7.2 V, the depletion layer width will be:
(a) 4 μm (b) 4.9 μm (c) 8 μm (d) 12 μm

11. The bandgap of silicon at room temperature is:
(a) 1.3 eV (b) 0.7 eV (c) 1.1 eV (d) 1.4 eV

12. A silicon PN junction diode under reverse bias has depletion region of width 10 μm. The relative permitivity of Silicon, $\varepsilon_r = 11.7$ and the permitivity of free space $\varepsilon_a = 8.85\times10^{-12}\ F/m$. The depletion capacitance of the diode per square metter is:
(a) 100 μF (b) 10 μF (c) 1 μF (d) 20 μF

13. In an abrupt p-n junction, the doping concentrations on the p-side and n-side are $N_A = 9\times10^{16}/cm^3$ and $N_D = 1\times10^{16}/cm^3$ respectively. The p-n junction is reverse biased and the total depletion width is 3 μm. The depletion width on the p-side is:
(a) 2.7 μm (b) 0.3 μm (c) 2.25 μm (d) 0.75 μm

14. N-type silicon bar 0.1 cm long and 100 μm^2 in cross-sectional area has a majority carrier concentration of $5\times10^{20}/cm^3$ and the carrier mobility is 0.13 m^2/V-s at 300 K. If the charge of an electron is 1.6×10^{-19} coulomb, then the resistance of the bar is:
(a) 10^6 ohm (b) 10^4 ohm (c) 10^{-1} ohm (d) 10^{-4} ohm

15. A long specimen of p-type semiconductor material:
(a) Is positively charged (b) Is electrically neutral
(c) Has an electric field directed along its length (d) Acts as a dipole

16. The probability that an electron in a metal occupies the Fermi-level at any temperature (> 0 K):
(a) 0 (b) 1 (c) 0.5 (d) 1.0

17. In a P-type silicon sample, the [illegible] concentration is 2.25×10[illegible] intrinsic carrier concentration is $1.5\times10^{10}/cm^3$, the electron [illegible]
(a) Zero (b) [illegible] [illegible] $10^5/cm^3$ [illegible] $\times10^{25}/cm^3$

18. An infra-red LED is usually [illegible]
(a) Ge (b) Si (c) Ga[illegible] [illegible] GaAs P

19. Which of the following statements is/are true for a good conductor of electricity?
1. Its conductivity decreases with increasing temperature.
2. Number of free electrons is around $10^{28}\,m^{-3}$.
3. Its conductivity decreases with addition of impurities.
4. It is a good conductor of heat also.
(a) 1, 2, 3 and 4 (b) 1 only (c) 2 and 3 only (d) 3 and 4 only

20. Consider the following statements:
The conductivity of a metal has negative temperature coefficient since:
1. The electron concentration increases with temperature.
2. The electron mobility decreases with temperature.
3. The electron-lattice scattering rate increases with temperature.
Which of the above statements is/are correct?
(a) 1 only (b) 1 and 2 (c) 2 and 3 (d) 3 only

21. Which one of the following is a trivalent material?
(a) Antimony (b) Phosphorus (c) Arsenic (d) Boron

D. Electron diffusion current 4. dn/dx

(a) A-2, B-1, C-3, D-4 (b) A-3, B-4, C-2, D-1

(c) A-2, B-4, C-3, D-1 (d) A-3, B-1, C-2, D-4

31. A thin resistive film having length (l), width (w) and thickness (t) is used as a resistor (R_1) for conducting current in the direction of l. If the film is connected to serve as a resistor (R_2) for conducting current in the direction of w, then what is the value of R_1/R_2.

(a) l/w (b) w/l (c) l^2/w^2 (d) w^2/l^2.

32. **Match List-I (Type of Material)** with **List-II (Type of Bonding)** and the select the correct answer using the codes given below the lists:

List-I (Typed of Material)	**List-II (Type of Bonding)**
A. Elemental semiconductor	1. Ionic
B. Hydrogen molecule	2. Covalent
C. Copper	3. Van der Waals'
D. AgI	4. Metallic.

Codes:

(a) A-3 B-2 C-1 D-4 (b) A-2 B-3 C-1 D-4
(c) A-2 B-3 C-4 D-1 (d) A-3 B-4 C-2 D-1

33. Depletion capacitance in a diode depends on:
1. Applied junction voltage. 2. Junction built-in potential
3. Current through junction 4. Doping profile across the junction
Select the correct answer using the codes given below:
(a) 1 and 2 (b) 1 and 3 (c) 1, 2 and 4 (d) 2, 3 and 4

34. The depletion layer across a p^+-n junction lies

(a) mostly in the p region (b) mostly in the n region
(c) equally in both p^+ and n regions (d) Entirely in the p^+ region.

35. The Ohm's law for conduction in metals is:
(a) $J = \sigma E$ (b) $J \propto \sigma E$ (c) $J = E/\sigma$ (d) $J \propto E/\sigma$

36. **Match List-I (Crystal type)** with **List-II (Name of the solid)** and select the correct answer using the codes given below the Lists:

List-I	**List-II**
A. Ionic	1. Solid argon
B. Covalent	2. Copper
C. Metallic	3. Silicon
D. Van der Waal's	4. Sodium chloride

Codes:

(a) A-3 B-4 C-2 D-1 (b) A-4 B-3 C-1 D-2
(c) A-3 B-4 C-1 D-2 (d) A-4 B-3 C-2 D-1

37. The carrier mobility in a semiconductor is 0.4 m^2/Vs. Its diffusion constant at 300 K will be (in m^2/s)
(a) 0.43 (b) 0.16 (c) 0.04 (d) 0.01

38. At very high temperatures, the extrinsic semiconductors become intrinsic because
(a) Of drive-in diffusion of dopants and carries
(b) Band to band transition dominates over impurity ionization.
(c) Impurity ionization dominates over band to band transition.
(d) Band to band transition is balanced by impurity ionization

39. The increasing temperature, the electrical conductivity would:
(a) Increase in metals as well as in intrinsic semiconductor
(b) Increase in metals but decrease in intrinsic semiconductor
(c) Decrease in metals but increase in intrinsic semiconductor
(d) Decrease in metals as well as in intrinsic semiconductors.

40. Which one of the following statements with reference to effective mass is incorrect?
(a) It is a function of wave vector K
(b) It can be positive or negative
(c) It is different from free mass because of lattice interaction
(d) Its concept is applicable only to electrons and not to holes.

41. The minority carrier life-time and diffusion constant in a semiconducting material are respectively 100 microsecond and 100 cm^2/s. The diffusion length of the carriers is:
(a) 0.1 cm (b) 0.01 cm (c) 0.0141 cm (d) 1 cm

42. The band gap at 300°K of diamond cubic crystals of C, Si, Ge and Sn is 5.5, 1.2, 0.67 and 0.1 eV respectively. Identify the crystal which will have the highest electrical condictivity at 300 K
(a) Ge (b) Sn (c) C (d) Si

43. The effective density of states at the conduction band edge of Ge is 1.04×10^{19} cm^{-3} at room temeprature 300 K). Ge has an optical bandgap of 0.66 eV. The intrinsic carrier concentration (in cm^{-3}) in Ge at room temperature (300 K) is approximately:

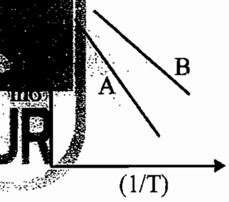
(a) 3×10^{10} (b) 3×10^{13} (c) 3×10^{16} (d) 6×10^{16}

44. An extrinsic semiconductor sample of cross-section A and length L is doped in such a way that the doping concentration varies as $N_D(x) = N_0 \exp\left(-\frac{x}{L}\right)$, where N_0 is a constant. Assume that the mobility m of the majority carriers remains constant. The resistance R of the sample is given by:

(a) $R = \frac{L}{A\mu eN_0}\left[\exp(1.0)-1\right]$ (b) $R = \frac{L}{\mu eN_0}\left[\exp(1.0)-1\right]$

(c) $R = \frac{L}{A\mu eN_0}\left[\exp(-1.0)-1\right]$ (d) $R = \frac{L}{A\mu eN_0}$

45. The temperature dependence of the electrical conductivity σ of two intrinsic semi-conductors A and B is shown in the figure. If E_A and E_B are the band gaps of A and B respectively, which one of the following is TRUE?



(a) $E_A > E_B$
(b) $E_A < E_B$
(c) $E_A = E_B$
(d) E_A and E_B both depend on temperature.

46. An intrinsic semiconductor with mass of a hole m_h and mass of an electron m_C is at a finite temperature T. If the top of the valence band enegy is E_V and the bottom of the conduction band energy is E_C, the Fermi energy of the semiconductor is:

(a) $E_F = \left(\frac{E_V + E_C}{2}\right) - \frac{3}{4}k_B T \ln\left(\frac{m_h}{m_e}\right)$ (b) $E_F = \left(\frac{k_B T}{2}\right) + \frac{3}{4}(E_V + E_C)\ln\left(\frac{m_h}{m_e}\right)$

(c) $E_F = \left(\frac{E_V + E_C}{2}\right) + \frac{3}{4}k_B T \ln\left(\frac{m_h}{m_e}\right)$ (d) $E_F = \left(\frac{k_B T}{2}\right) - \frac{3}{4}(E_V + E_C)\ln\left(\frac{m_h}{m_e}\right)$

47. A silicon sample A is doped with 10^{18} atoms/cm^3 of Boron. Another sample B of identical dimensions is doped with 10^{18} atoms/cm^3 of Phosphorus. The ratio of electron to hole mobility is 3. The ratio of conductivity of the sample A to B is:
(a) 3 (b) 1/3 (c) 2/3 (d) 3/2

48. N-type silicon is obtained by doping silicon with:
(a) Germanium (b) Aluminium (c) Boron (d) Phosphorus

49. The band-gap of silicon at 300 K is:
(a) 1.36 eV (b) 1.10 eV (c) 0.80 eV (d) 6.67 eV

50. The intrinsic carrier density at 300 K is $1.5\times10^{10}/cm^3$, in silicon. For n-type silicon doped to 2.25×10^{15} atoms / cm^3, the equilibrium electron and hole densities are:
(a) $n=1.5\times10^{15}/cm^3,\ p=1.5\times10^{10}/cm^3$ (b) $n=1.5\times10^{10}/cm^3,\ p=2.25\times10^{15}/cm^3$
(c) $n=2.25\times10^{15}/cm^3,\ p=1.0\times10^{5}/cm^3$ (d) $n=1.5\times10^{10}/cm^3,\ p=1.5\times10^{10}/cm^3$

51. The drift velocity of electrons in silicon:
(a) Is proportional to the electric field for all values of electric field.
(b) Is independent of the electric field.
(c) Increases at low values of electric field and decreases at high values of electric field exhibiting negative differential resistance.
(d) Increases linearly with electric field at low values of electric field and gradually saturates at higher values of electric field.

52. A silicon sample is uniformly doped with 10^{16} phosphorous atoms/cm^3 and 2×10^{16} boron atoms/cm^3. If all the dopants are fully ionized, the material is:
(a) n-type with carrier concentration of $10^{16}/cm^3$
(b) p-type with carrier concentration of $10^{16}/cm^3$
(c) p-type with carrier concentration of $2\times10^{16}/cm^3$
(d) T_2-will get damaged and T_1 will be safe.

53. In a uniformly doped abrupt p-n junction, the doping level of n-side is four (4) times the doping level of the p-side. The ratio of the depletion layer widths is:
(a) 0.25 (b) 0.5 (c) 1.0 (d) 2.0

54. The doping concentration on the n-side of a p-n junction diode is enhanced. Which one of the following will get affected?
(a) Width of the depletion re [illegible]
(b) Width of the depletion re [illegible]
(c) Width of the depletion re [illegible]
(d) No change in width of de [illegible]

55. Which one of the following expressions may be used to correctly describe the temperature(T) variation of the intrinsic carrier density (n_i) of a semiconductor?
(a) $n_i(T)=(A/T)\exp(-E_g/kT^2)$ (b) $n_i(T)=A(E_g/kT)^{10}$
(c) $n_i(T)=A\exp(-E_g/2kT)$ (d) $n_i(T)=AT^{3/2}\exp(-E_g/2kT)$

56. An intrinsic semiconductor (intrinsic electron density = 10^{16} m^{-3}) is doped with donors to a level of 10^{22} m^{-3}. What is the hole density assuming all donors to the ionized?
(a) 10^7 m^{-3} (b) 10^8 m^{-3} (c) 10^{10} m^{-3} (d) 10^6 m^{-3}.

57. Consider the following statements in respect of graphite:
1. It has the three coplanar covalent bonds.
2. It has good electrical and thermal conductivity.
3. Sheets of graphite are held together by van der waals interaction.
Which of the statements given above are correct?
(a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

58. The free electron density in a conductor is $(1/1.6)\times10^{22}$ cm^{-3}. The electron mobility is 10 cm^2/V s. What is the value of its resistivity?
(a) 10^{-4} Wm (b) 1.6×10^{-2} Wm (c) 10^{-4} Ωcm (d) 10^{-4} Ω cm^{-1}

59. A conductor carries a current of 4A and if magnitude of charge of an electron $e = 1.6 \times 10^{-19}$ coulomb, then th number of electrons which flow past the cross section per second is:

(a) 2.5×10^{19} (b) 1.6×10^{19} (c) 6.04×10^{19} (d) 0.4×10^{19}

60. Which of the following elements act as donor impurities?

1. Gold 2. Phosphorus 3. Boron 4. Antiomony
5. Arsenic 6. Indium.
Select the corrent answer using the codes given below:
(a) 1, 2 and 3 (b) 1, , 4 and 5 (c) 3, 4, 5 and 6 (d) 2, 4 and 5

61. The conductivity of an intrinsic semicoductor is (symbols have the usual meanings) where e is electron charge

(a) Generally less than that of a doped semiconductor.

(b) Given by $\sigma_i = en_i(\mu_n - \mu_p)$

(c) Given by $\sigma_i = en_i(\mu_n + \mu_p)$

(d) Given by $\sigma_i = n_i(\mu_n + \mu_p)$

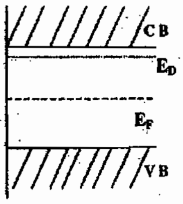
62. In an intrinsic semiconductor, as the temperature increases the conductivity σ, electron concentration n, and mobility μ vary as follows:

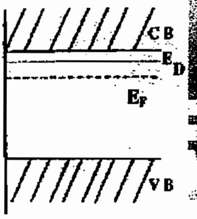
(a) σ decreases, n increases and μ decreases (b) σ decreases, n is unaffected and μ decreases
(c) σ increases, n increases and μ decreases (d) σ increases, n increases and μ is unaffected

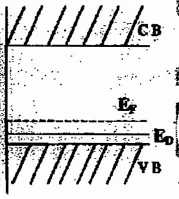
63. A Ge semiconductor is doped with acceptor impurity concentration of 10^{15} atoms /cm^3. For the given hole mobility of $1800\ \text{cm}^2/\text{V-s}$, the resistivity of this material is:

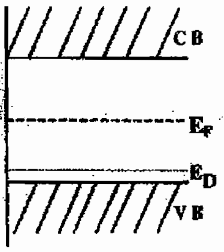
(a) 0.288 Ω cm (b) 0.694 Ω cm (c) 3.472 Ω cm (d) 6.944 Ω cm

64. Identify the CORRECT energy band diagram for silicon doped with Arsenic. Here CB, VB, E_D and E_F conduction band, valence band, impurity level and Fermi level, respectively.

(a)


(b)






65. The net charge of an n-type semiconductor is
(a) positive (b) zero (c) negative (d) dependent on the dopant density

66. For a intrinsic semiconductor, m_e^* and m_h^* are respectively the effective masses of electrons and holes near the corresponding band edges. At a finite temperature, the position of hte Fermi level

(a) Depends on m_e^* but not on m_h^* (b) Depends on m_h^* but not on m_e^*

(c) Depends on m_e^* and m_h^* (d) depends neither on m_e^* nor on m_h^*

67. When a pentavalent impurity is added to Si, it becomes
(a) an insulator (b) a conductor
(c) an intrinsic semiconductor (d) an *n*-type semiconductor

68. An n-type semiconductor is
(a) positively charged (b) negatively charged
(c) electrically neutral (d) positively or negatively charged

69. When a pure semiconductor is heated, its resistance
(a) increases (b) decreases linearly
(c) decreases exponentially (d) remains the same

70. A sample of Si has electron and hole mobilities of 0.13 and 0.05 m^2/V-s respectively at 300K. It is doped with P and Al with doping densities of $1.5 \times 10^{21}/m^3$ and $2.5 \times 10^{21}/m^3$ respectively. The conductivity of the doped Si sample at 300K is

(a) $8\Omega^{-1}m^{-1}$ (b) $32\Omega^{-1}m^{-1}$ (c) $20.8\Omega^{-1}m^{-1}$ (d) $83.2\Omega^{-1}m^{-1}$

71. For a pure germanium semiconductor cooled in liquid nitrogen, the average density of conduction electrons is about $n = 10^{12}$ per cm^3. At this temperature the electron and hole mobilities are equal and have the common value $\mu = 5.0\times 10\ cm^2V^{-1}s^{-1}$. If a potential of 100V is applied across a 1 cm cube of this cooled germanium sample, the current observed can be estimated as

(a) 80 μA (b) 160 mA (c) 16 mA (d) 16 A

72. The donor concentration in a sample of n-type silicon is increased by a factor of 100. The shift in the position of the Fermi level at 300K, assuming the sample to be non degenerate is ________ meV.

$(k_BT = 25meV$ at 300K$)$

73. An electron makes a transition from the valence band to the conduction band in an indirect band gap semiconductor. Which of the following is NOT true ?
(a) The energy of the electron increases (b) A phonon is involved in the process
(c) A photon is absorbed in the process (d) There is no momentum change in the electron

74. A beam of X-ray of intensity I_0 is incident normally on a metal sheet of thickness 2 mm. The intensity of the transmtted beam is $0.025I_0$. The linear absorption coefficient of the metal sheet (in m^{-1}) is ________ (upto one decimal places) **[GATE 2015]**

75. The band gap of an intrinsic [illegible] $= 0.72$ eV and m_h [illegible] At 300K, the Fermi level with respect to the edge of th[illegible] [illegible] upto three decimal places) $k_B = 1.38\times10^{-23} JK^{-1}$ **[GATE 2015]**

76. The concentration of electrons, n and holes, p for an intrinsic semiconductor at a temperature T can be expressed as $n = p = AT^{3/2}\exp\left(-\frac{E_g}{2k_BT}\right)$, where E_g is the band gap and A is a constant. If the mobility of both types of carries is proportional to $T^{-3/2}$, then the log of the conductivity is a linear function of T^{-1} with slope

(a) $\frac{E_g}{(2k_B)}$ (b) $\frac{E_g}{k_B}$ (c) $\frac{-E_g}{(2k_B)}$ (d) $\frac{-E_g}{k_B}$ **[NET June 2015]**

77. Which one of the following statements is NOT TRUE?
(a) Entropy decreases markedly on cooling a superconductor below the critical temperature, T_C.
(b) The electronic contribution to the heat capacity in the superconducting state has an exponential form with an argument proportional to T^{-1}, suggestive of an energy gap
(c) A type I superconductor is a perfect diamagnet.
(d) Critical temperature of superconductors does not vary with the isotopic mass.

78. The majority carriers in n-type semiconductor have an average drift velocity V in a direction perpendicular to a uniform magnetic field B. The electric field E induced due to Hall effects acts in the direction:
(a) V×B (b) B×V (c) along V (d) opposite to V

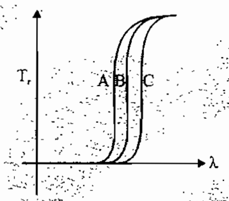
79. The longest wavelength that can be absorbed by silicon, which has the band-gap of 1.12 eV, is $1.1\mu m$. If the longest wavelength that can be absorbed by another material is $0.87\ \mu m$, then the bandgap of this material is:
(a) 1.416 eV (b) 0.886 eV (c) 0.854 eV (d) 0.706 eV

80. A particular green LED emits light of wavelength 5490Å. The energy bandgap of the semiconductor material used there is (Planck's constant = 6.626×10^{-34} J-s):
(a) 2.26 eV (b) 1.98 eV (c) 1.17 eV (d) 0.74 eV

81. An infra-red LED is usually fabricated from:
(a) Ge (b) Si (c) Ga As (d) Ga As P

82. Consider the following statements for a photo conducting material:
1. Its dark conductivity is small.
2. With the absorption of radiation, equal numbers of electrons and of holes are produced.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 nor 2.

83. Photovoltaic devices may be made from various semiconductors as optical materials for which the transmission coefficient (T_r) is shown in the figure given below as a function of photon wavelength.



Which one of the following is the correct sequence of A, B and C pertaining to Ge, Ga, As and Si, respectively?
(a) A–B–C (b) B–A–C (c) C–A–B (d) C–B–A

84. Photons of energy 1.53×10[illegible] incident on a photodi[illegible] responsivity of 0.65 A/W. If the optical power level is 10 μ[illegible] o current [illegible]
(a) 64 nA (b) [illegible] 2.1 μA [illegible] μA

85. An LED made usig GaAs e[illegible]
(a) Visile region (b) [illegible]
(c) Infra red region (d) Microwave frequency region.

86. The light emitting diode (LED) emits light of a particular colour because
(a) It is fabricated from a fluorescent material.
(b) Transition between energy levels of the carriers takes place while crossing the p-n junction.
(c) Heat generated in the diode is converted into light.
(d) The band gap of the semi-conductor material used in fabrication of the diode is equal to the energy h_v of the light photon.

87. Given that the band gap of cadmium sulphide is 2.5 eV, the maximum photon wavelength, for electron-hole pair generation will be
(a) 5400 μm (b) 540 μm (c) 5400Å (d) 540 Å

88. Hall effect is observed in a specimen when it (metal or a semiconductor) is carrying current and is placed in a magnetic field. The resultant electric field inside the specimen will be in
(a) A direction normal to both current and magnetic field.
(b) The direction of current
(c) A direction antiparallel to magnetic field
(d) An arbitrary direction depending upon the conductivity of the specimen.

89. The wavelength of light emitted by a GaAs laser s 8670×10^{-10} m. Given Planck's constant = 6.626×10^{-34} Js, velocity of light = 2.998×10^{8} m/s and 1 eV = 1.602×10^{-19} J, the energy gap in GaAs is

(a) 0.18 eV (b) 0.7 eV (c) 1.43 eV (d) 2.39 eV

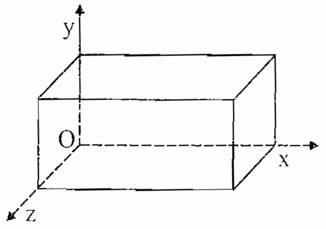
90. A doped Germanium crystal of length 2 cm, breadth 1 cm and width 1 cm, carries a current of 1 mA along its length parallel to +x axis. A magnetic field of 0.5 T is applied along +z axis. Hall voltage of 6 mV is measured with negative polarity at y = 0 plane. The sign and concentration of the majority charge carrier are, respectively [Given: $e = 1.6 \times 10^{-19}$ C]

(a) Positive and 5.2×10^{19} m^{-3}

(b) Negative and 5.2×10^{19} m^{-3}

(c) Positive and 10.4×10^{19} m^{-3}

(d) Negative and 10.4×10^{19} m^{-3}



91. An electron of wavevector $\vec{k}_e$, velocity υ_e and effective mass m_e is removed from a filled energy band. The resulting hole has wavevevtor $\vec{k}_h$, velocity υ_h, and effective mass m_h. Which one of the following statements is correct?

(a) $\vec{k}_h = -\vec{k}_e; \upsilon_h = \upsilon_e; m_h = -m_e$ (b) $\vec{k}_h = \vec{k}_e; \upsilon_h = \upsilon_e; m_h = m_e$

(c) $\vec{k}_h = \vec{k}_e; \upsilon_h = -\upsilon_e; m_h = -m_e$ (d) $\vec{k}_h = -\vec{k}_e; \upsilon_h = \upsilon_e; m_h = m_e$

92. LED is a

(a) p-n diode (b) Thermistor (c) Gate (d) Transistor

93. As per Hall effect, if any specimen carrying a current I is placed in a transverse magnetic field B, then an electric field E is induced in the specimen in the direction.

(a) Parallel to I (b) Perpendicular to B and parallel to I
(c) Parallel to I and B (d) Perpendicular to both I and B.

94. Which one of the following is the correct relationship between the band gap of a material used in a photo detector and the energy of the incident photon?

(a) $E_g \geq hc/\lambda$ (b) $h\upsilon > E_g$ (c) $h\upsilon \geq E_g$ (d) $h\upsilon \leq E_g$

95. Two pure specimen of a semiconductor material are taken. One is doped with 10^{18} cm^{-3} number of donors and the other is doped with 10^{16} cm^{-3} number of acceptors. The minority carrier density in the first specimen is 10^{7} cm^{-3}. What is the minority carrier density in the other specimen?

(a) 10^{16} cm^{-3} (b) 10^{27} cm^{-3} (c) 10^{18} cm^{-3} (d) 10^{9} cm^{-3}

96. In an extrinsic semiconductor the Hall coefficient R_H.

(a) Increases with increase of temperature (b) Decreaes with increase of temperature
(c) Is independent of the change of temperature (d) Changes with the change of magnetic field.

97. Which of the following statements related to the Hall effect?
1. A potential difference is developed across a current-carrying metal strip when the strip is placed in a transverse magnetic field.
2. The Hall effect is very weak in metals but is large in semiconductors
3. The Hall effect is very weak in semiconductors but is large in metals.
4. It is applied in the measurement of the magnetic field intensity.
Select the correct answer using the code given below:
(a) 1, 2 and 3 (b) 2 and 4 (c) 1, 3 and 4 (d) 1, 2 and 4

98. Germanium and silicon photo-sensors have their maximum spectral response in the

(a) Infrared region (b) Ultraviolet region (c) Visible region (d) X-ray region

99. Measurement of Hall effect coefficient in a semiconductor provides information on the

(a) Sign and mass of charge carriers (b) Mass and concentration of charge carriers
(c) Sign of charge carriers alone (d) Sign and concentration of charge carriers.

100. Light of wavelength 660 nm and power of 1 mW is incident on a semiconductor photodiode with an absorbing layer of thickness of $(\ln 4)\ \mu m$.

A. If the absorption coefficient at this wavelength is 10^4 cm^{-1} and if 1% power is lost on reflection at the surface, the power absorbed will be

(a) 750 μW (b) 675 μW (c) 250 μW (d) 225 μW

B. The generated photo-current for a quantum efficiency of unity will be

(a) 360 μA (b) 400 μA (c) 133 μA (d) 120 μA

101. The number density of electrons in the conduction band of a semiconductor at a given temperature is 2×10^{19} m^{-3}. Upon lightly doping this semiconductor with donor impurities, the number density of conduction electrons at the same temperature becomes 4×10^{20} m^{-3}. The ratio of majority to minority charge carrier concentration is _______________. **[GATE 2016]**

102. A *pn* junction was formed with a heavily doped (10^{18} cm^{-3}) *p*-region and lightly doped (10^{14} cm^{-3}) *n*-region. Which of the following statement(s) is(are) correct?

(A) the width of the depletion layer will be more in *n*-side of the junction
(B) the width of the depletion layer will be more in the *p*-side of the function
(C) the width of the depletion layer will be same on both sides of the junction.
(D) if the *pn* junction is reverse biased, then the width of the depletion region increases.

103. Consider a 20 μm diameter p-n junction fabricated in silicon. the donor density is 10^{16} per cm^3. The charge developed on the n-side is $1.6\times10^{-13}\,C$. Then the width (in μm) of the depletion region on the n-side of the p-n junction is _______________

CSIR-UGC-NET/JRF|GATE-PHYSICS

SOLID STATE PHYSICS

Assignment-5

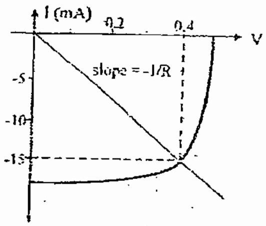
1. Monochromatic light of wavelength 660 nm and intensity $100\ \text{mW}/\text{cm}^2$ falls on a solar cell of area $30\ \text{cm}^2$. The conversion efficiency of the solar cell is 10%. If each coverted photon results in an electron-hole pair, what is the maximum circuit current supplied by the solar cell? (Take $h = 6.6\times10^{-34}\,\text{J}-\text{s},\ c = 3\times10^{8}\,\text{m/s}$ and $e = 1.6\times10^{-19}\,\text{C}$).

   (a) 160 mA (b) 320 mA (c) 1600 mA (d) 3200 mA

2. In the Hall effect the sign of the Hall voltage depends upon the carrier concentration (n,p) and mobility $(\mu_n\ \mu_p)$ of electrons and holes, respectively. The Hall voltage in intrinsic and n-type semiconductors is negative because

   (a) $n\mu_n > p\mu_p$ (b) $n > p, \mu_n < \mu_p$ (c) $n > p, \mu_n = \mu_p$ (d) $n = p, \mu_n > \mu_p$

3. The I-V characteristics of a solar cell of dimension 1 cm × 1 cm illuminated with light of intensity $600\ \text{Wm}^{-2}$ are shown in the figure. If the cell is used to drive a load R (see the load line in the figure) the efficiency of the solar cell is:



   (a) 1% (b) 5[illegible] (c) 20% (d) [illegible]0%

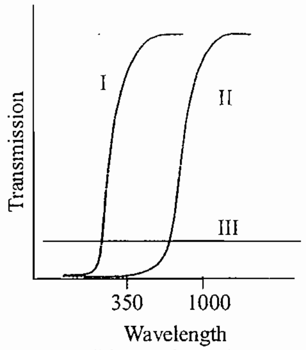
4. In an experiment to measure [illegible] of a particular sample [illegible]nerated Hall voltage was found to be too small to measure. [illegible]ement, you would

   (a) Increase the magnetic fie[illegible] (b) Increase the temperature of the sample
   (c) Use a longer and thinner sample (d) Use a shorter and thicker sample.

5. An LED operates at 1.5 V and 5 mA in forward bias. Assuming an 80% external efficiency of the LED, how many photons are emitted per second?

   (a) 5.0×10^{16} (b) 1.5×10^{16} (c) 0.8×10^{16} (d) 2.5×10^{16}

6. The experimentally measured transmission spectra of metal, insulator and semiconductor thin films are shown in the figure. It can be inferred that I, II and III correspond respectively, to

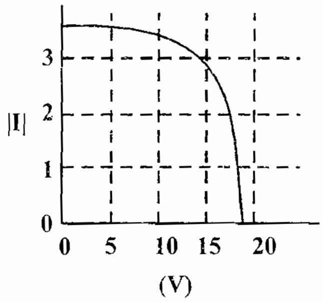


   (a) insulator, semiconductor and metal (b) semiconductor, metal and insulator
   (c) metal, semiconductor and insulator (d) insulator, metal and semiconductor

7. A magnetic field sensor based on the Hall effect is to be fabricated by implanting. As into a Si film of thickness 1 μm. The specifications require a magnetic field sensitivity of 500 mV/Tesla at an excitation current of 1 mA. The implanation dose is to be adjusted such that the average carrier density, after activation, is

(a) 1.25×10^{26} m^{-3} (b) 1.25×10^{22} m^{-3} (c) 4.1×10^{21} m^{-3} (d) 4.1×10^{20} m^{-3}

8. The output characteristics of a solar panel at a certain level of irradiance is shown in the figure below.



If the solar cell is to power a load of 5 Ω, the power drawn by the load is:

(a) 97W (b) 73 W (c) 50 W (d) 45 W

9. If the gap between ground state and excited state energy levels of electrons in an atom is 1eV. Estimate the wavelength of the light the atom will absorb? The value of the Planck's constant is 6.6×10^{-34} Js.

(a) 1 mm (b) 1.24 μm (c) 1.24 angstrom (d) 1cm

10. The difference between the electron and hole of a semiconductor laser is 1.5 eV and the band gap of the semiconductor is 1.43 eV. The upper and lower frequency limits of the laser will be respectively

(a) 3.3×10^{14} and 9.9×10^{13} Hz (b) 3.7×10^{14} and 3.5×10^{14} Hz

(c) 6.28×10^{14} and 3.1×10^{14} Hz (d) 1.16×10^{15} and 8.97×10^{14} Hz

11. The power density of sunlight incident on a solar cell is 100 mW/cm^2. Its short circuit current density is 30 mA/cm^2 and the open circuit voltage is 0.7V. If the fill factor of the solar cell decreases from 0.8 to 0.5 then the percentage efficiency will decrease from

(a) 42.0 to 26.2 [illegible] (c) 21.0 to [illegible] (d) 16.8 to 10.5

12. In a Hall effect experiment [illegible] intrinsic semicon[illegible] is negative. This is because (symbols carry usual meaning) **[GATE 2015]**

(A) $n \approx p$ (B) $n > p$ (C) $\mu_e > \mu_h$ (D) $m_e^* > m_h^*$

(a) A (b) B (c) C (d) D

13. Ionic crystals are very good insulators at room temperature conducting at higher temperatures. This is due to:

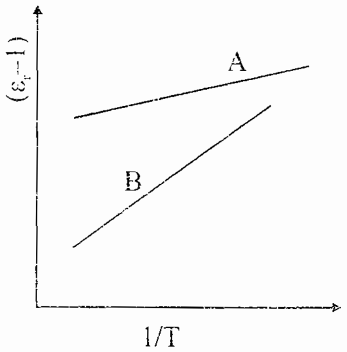
(a) Thermal expansion of the ionic solid

(b) Generation of more free electrons

(c) Transition of large number of electrons from valence band

(d) Movement of larger number of ions in vacancies

14. Two dielectric materials A and B exhibit both ionic and orientational polarizabilities. The variation of the susceptibilities $c(=e_r-1)$ with temperature T is shown in the figure, where e_r is the relative dielectric constant. It can be inferred from the figure that:



(a) A is more polar and it has a smaller value of ionic polarizability than that of B
(b) A is more polar and it has a higher value of ionic polarizability than that of B
(c) B is more polar and it has a higher value of ionic polarizability than that of A
(d) B is more polar and it has a smaller value of ionic polarizability than that of A

15. If the static dielectric constant of NaCl crystal is 5.6 and its optical refractive index is 1.5, the ratio of its electric polarizability to its total polarizability is:
(a) 0.5 (b) 0.7 (c) 0.8 (d) 0.9

16. The dielectric constant of a material at optical frequencies is mainly due to
(a) Ionic polarizability. (b) Electronic polarizability
(c) Dipolar polarizability (d) Ionic and dipolar polarizability.

17. Magnetic long range order is typically exhibited by:
(a) noble metals (b) alkali metals
(c) inert gas solids (d) transition metals

18. Which one of the following is TRUE for a semiconductor p–n junction with no external bias?
(a) The total charge in the junction is not conserved
(b) The p side of the junction is positively charged
(c) The p side of the junction is negatively charged
(d) No charge develops anywhere in the junction

19. Which one of the following is the first order phase transition?
(a) Vaporization of a liquid at its boiling point
(b) Ferromagnetic to paramagnetic
(c) Normal liquid He to superfluid He
(d) Superconducting to normal state

20. Identify which one is a first order phase transition?
(a) A liquid to gas transition at its critical temperature.
(b) A liquid to gas transition close to its triple point.
(c) A paramagnetic to ferromagnetic transition in the absence of a magnetic field.
(d) A metal to superconductor transition in the absence of a magnetic field.

21. A second order phase transition is one in which:
(a) the plot of entropy as a function of temeprature shows a discontinuity
(b) the plot of specific heat as a function of temperature shows a discontinuity
(c) the plot of volume as a function of pressure shows a discontinuity
(d) the plot of comprehensibility as a function of temperature is continuous

22. A ferromagnetic material has a Curie temperature 100 K. Then:

(a) its succeptibility is double when it is cooled from 300 K to 200 K
(b) all the atomic magnets in it get oriented in the same direction above 100 K
(c) the plot of inverse susceptibility versus temperature is linear with a slope T_C
(d) the plot if its succeptibility versus temperature is linear with an intercept T_C

23. The energy of a ferromagnet as a function of magnetization M is given by:

$$F(M) = F_0 + 2(T - T_C) M_2 + M_4, F_0 > 0$$

The number of minima in the function F(M) for $T > T_C$ is:

(a) 0 (b) 1 (c) 3 (d) 4

24. When liquid oxygen is poured down close to a strong bar magnet, the oxygen stream is:

(a) repliecd towards the lower field because it is diamagnetic
(b) attracted towards the higher field because it is diamagnetic
(c) repelled towards the lower field becaue it is paramagnetic
(d) attracted towards the higher field because it is paramagnetic

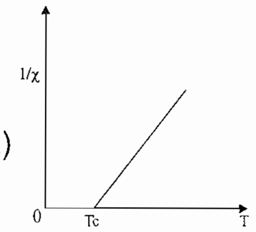
25. A ferromagnetic mixture of iron and copper having 75% atoms of Fe exhibits a saturation magnetization of 1.3×10^6 A m^{-1}. Assume that the total number of atoms per unit volume is 8×10^{28} m^{-3}. The magnetic moment of an iron atom, in terms of the Bohr Magneton, is:

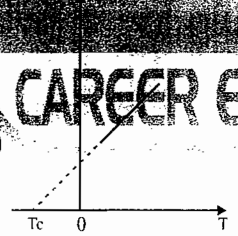
(a) 1.7 (b) 2.3 (c) 2.9 (d) 3.8

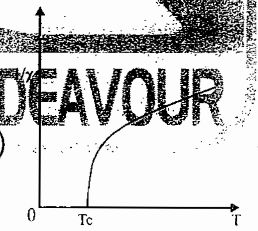
26. Which one of the following statements is TRUE?

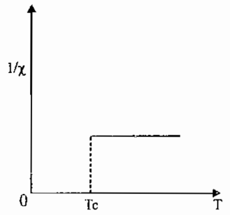
(a) Magnetic tapes are made of Iron.
(b) Permanent magnets are made from ferrites.
(c) Ultrasonic transducers are made from quartz crystals.
(d) Optoelectronic devices are made from soft ferrites

27. The plot of inverse magnetic susceptibility $\frac{1}{\chi}$ versus temperature T of an antiferromagnetic sample corresponds to:

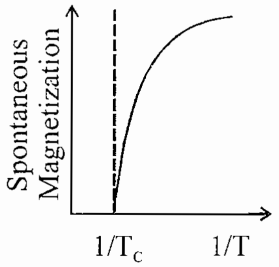
(a)


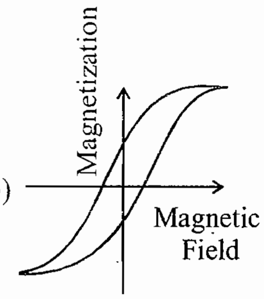
(b)


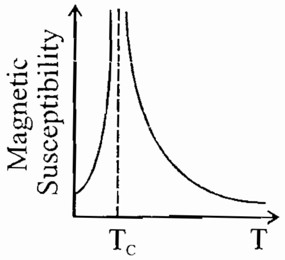
(c)


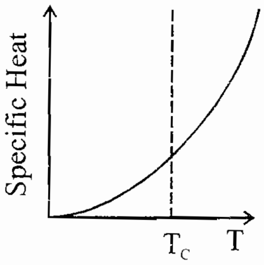
(d)


28. In an experiment involving a ferromagnetic medium, the following observations were made. Which one of the plots does NOT correctly represented the property of the medium? (T_C is the Curie temperature)

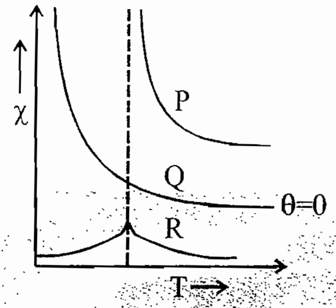
(a)


(b)


(c) 

(d) 

29. The dependence of the magnetic susceptibility (χ) of a material with temperature (T) can be represented by $\chi \propto \frac{1}{T-\theta}$, where θ is the Curie-Weiss temperature. The plot of magnetic susceptibility versus temperature is sketched in the figure, as curves P, Q and R with curve Q having $\theta = 0$. Which one of the following statements is correct?



(a) Curve R represents a paramagnet and Q a ferromagnet.
(b) Curve Q represents a ferromagnet and P an anterromagnet.
(c) Curve R represents an antiferromagnet and Q a paramagnet.
(d) Curve R represents an antiferromagnet and Q a ferromanget.

30. The temperature (T) depend[illegible]magnetic susceptibility [illegible]agnetic substance with a Curie temperature (T_c) is given by

(a) $\frac{C}{T-T_c}$, for $T < T_c$ [illegible] $\frac{C}{[illegible]}$, for $T > T_c$

(c) $\frac{C}{T+T_c}$, for $T > T_c$ (d) [illegible] $T+T_c$ for all temperature.

31. Assume that the free energy of a magnetic system has an expansion in the order parameter M of the form $F(M,T) = a(T-T_C)M^2 + bM^4 + cM^6$, with a, b and c > 0. As the temperature is lowered below T_C, the system undergoes a phase transition. The behaviour of the order parameter just below the transition, where $(T-T_C)$ is very small, is best described by

(a) $M \propto (T_C - T)^{-1/2}$ (b) $M \propto (T_C - T)^{1/2}$ (c) $M \propto (T_C - T)$ (d) $M \propto (T_C - T)^3$

32. The magnetization M of a system in equilibrium at temperature T is given by the equation $M = \tanh\left(M/T\right)$, where all quantities are in dimensionless units. This system will show

(a) a first order phase transition at $T = 1$ (b) a second order phase transition at T=1

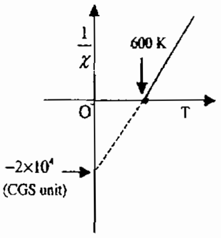
(c) no phase transition at any T (d) a second order phase transition at $T = 1/2$.

33. The (Pauli) paramagnetic susceptibility of a metal at a low temperature T is

(a) proportional to $\frac{1}{T}$ (b) independent of T

(c) proportional T^2 (d) proportional to T

34. Inverse susceptibility $(1/\chi)$ as a function of temperature, T for a material undergoing paramagnetic to ferromagnetic transition is given in the figure, where O is the origin. the values of the Curie constant, C and the Weiss molecular field constant, λ, in CGS units, are



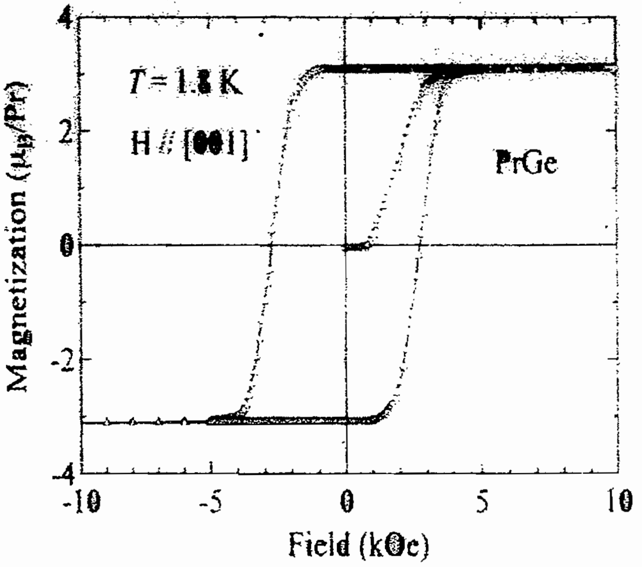
(a) $C=5\times10^{-5}, \lambda=3\times10^{-2}$ (b) $C=3\times10^{-2}, \lambda=5\times10^{-5}$

(c) $C=3\times10^{-2}, \lambda=2\times10^{-4}$ (d) $C=2\times10^{4}, \lambda=3\times10^{-2}$

35. The magnetization M of a ferromagnet, as a function of the temperature T and the magnetic field H, is described by the equaltion $M=\tanh\left(\frac{T_c}{T}M+\frac{H}{T}\right)$. In these units, the zero-field magnetic susceptibility in terms of $M(0)=M(H=0)$ is given by

(a) $\frac{1-M^2(0)}{T-T_c(1-M^2(0))}$ (b) $\frac{1-M^2(0)}{T-T_c}$ (c) $\frac{1-M^2(0)}{T+T_c}$ (d) $\frac{1-M^2(0)}{T}$

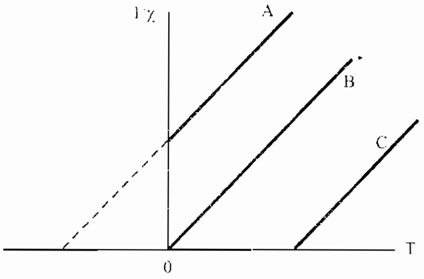
36. The Curie temperature of a single crystal of PrGe is known to be 41 K. The magnetization data of this sample is measured at 1.8K for the [illegible] applied parallel to [illegible] direction is shown in the figure on the left. At a temperature of 38 K [illegible] hysteresis loop in the [illegible]



(a) Have the same width (b) decrease in width

(c) increase in width (d) shrink to a line.

37. The magnetic susceptibility χ of three samples A, B and C, is measured as a function of their absolute temperature T, leading to teh graphs shown below.



From these graphs, the magnetic nature of the samples can be inferred to be

(a) A:anti-ferromagnet B: paramagnet C: ferromagnet
(b) A:diamagnet B: paramagnet C: anti-ferromagnet
(c) A:paramagnet B: anti-ferromagnet C: ferromagnet
(d) A:anti-ferromagnet B: diamagnet C: paramagnet.

38. A contribution to the heat capacity is large in magnetic materials at the temperature at which magnetic moments

(a) are ordered
(b) are disordered
(c) are antiparallel to each other
(d) become zero

39. The physical phenomenon that cannot be used for memory storage applications is

(a) large variation in magnetoresistance as a function of applied magnetic field
(b) variation in magnetization of a ferromagnet as a function of applied magnetic field
(c) variation in polarization of a ferroelectric as a function of applied electric field
(d) variation in resistance of a metal as a function of applied electric field

40. For Nickel, the number density is 8×10^{23} atoms/cm^3 and electronic configuration is $1s^2\,2s^2 2p^6 3s^2 3p^6 3d^8 4s^2$. The value of the saturation magnetization of Nickel in its ferromagnetic states is ________ $\times 10^9$ A/m.

(Given the value of Bohr ma[illegible]$^{-21}$ Am^2)

41. A solid material is found to have a temperature independent magnetic susceptibility, $\chi = C$. Which of the following statements is correct? **[GATE 2016]**

(a) If C is positive, the material is a diamagnet
(b) If C is positive, the material is a ferromagnet
(c) If C is negative, the material could be a type I superconductor
(d) If C is positive, the material could be a type I superconductor

42. In the Meissner state the magnetic susceptibility of a super conductor in S.I. units is:

(a) $-\frac{1}{4\pi}$ (b) $\frac{1}{4\pi}$ (c) -1 (d) $-\frac{1}{\mu_a}$

43. Which one of the following statements about superconductors is NOT true?

(a) A type I superconductor is completely diamagnetic
(b) All type II superconductor exhibits Meissner effect upto second critical magnetic field (H_{C_2}).
(c) A type II superconductor exhibits zero reistance upto the second critical magnetic field
(d) Both type I and type II superconductors exhibit sharp fall in resistance at the superconducting transition temperature.

44. A solid superconductor is placed in an external magnetic field and then cooled below its criticalk temperature. The superconductor.

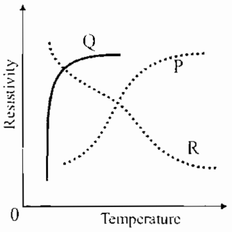
(a) retains its magnetic flux becuase the surface current supports it.

(b) expels out its magnetic flux because it behaves like a paramagnetic material.

(c) expels out its magnetic flux because it behaves like an anti-ferromagnetic material

(d)expels out its magnetic flux because the surface current induces a field in the direction opposite to the applied magnetic field.

45. Variation of electrical resistivity r with temperature T of three solids is sketched (on different scales) in the figure, as curves P, Q and R.



Which of the following statements describes the variations most appropriately?

(a) P is for a superconductor and R for a semiconductor

(b) Q is for a superconductor and P for a conductor

(c) Q is for a superconductor and R for a conductor

(d) R is for a superconductor, and P for a conductor

46. The critical magnetic field for a solid in superconducting state

(a) Does not depend upon temperature (b) Increases if the temperature

(c) Increases if the temperature decreases (d) Does not depend on the transition temperature.

47. Which one of the following statements is NOT TRUE?

(a) Entropy decreases markedly on cooling a superconductor below the critical temperature, T_C.

(b) The electronic contribution to the heat capacity in the superconducting state has an exponential form with an argument proportional to T^{-1}, suggestive of an energy gap

(c) A type I superconductor [illegible] diamagnet.

(d) Critical temperature of s[illegible] not vary with the is[illegible]ss.

48. A superconducting ring is c[illegible] critical temperature (T_C). The total magnetic flux that passes through the ring is:

(a) zero (b) $n\frac{h}{2e}$ (c) $\frac{nh}{4\pi e}$ (d) $\frac{ne^2}{hc}$

49. Experimental study shows that there exists an energy gap in a superconductor. Which one of the following statements is incorrect?

(a) The gap separates the lowest excited state in a superconductor from the ground state.

(b) The temperature dependence of electronic specific heat indicates the existence of an energy gap.

(c) The existence of an energy gap is accounted for in the BCS theory.

(d)The energy gap in supercondocturs is similar to the energy gap which occurs in insulators.

50. An external magnetic field of magnitude H is applied to a Type-I superconductor at a temperature below the transition ponit. Then which one of the following statements is NOT true for H less than the ciritical field H_C?

(a) the sample is diamagnetic

(b) it magnetization varies linearly with H

(c) the lines of magnetic induction are pushed out from the sample

(d) the sample exhibits mixed states of magnetization near H_C

51. The thermal conductivity of a given material reduces when it undergoes a transition from its normal state to the superconducting state. The reason is:

(a) The cooper pairs cannot transfer energy to the lattice.
(b) Upon the formation of cooper pairs, the lattice becomes less efficient in heat transfer.
(c) The electrons in the normal state lose their ability to transfer heat because of their coupling to the cooper pairs.
(d) The heat capacity increases on transition to the superconducting state leading to a reduction in thermal conductivity.

52. A flux quantum (fluxoid) is approximately equal to 2×10^{-7} gauss-cm^2. A type II superconductor is placed in a small magnetic field, which is then slowly increased till the field starts penetrating the superconductor. The strength of the field at this point is $\frac{2}{\pi}\times10^5$ gauss

A. The penetration depth of this superconductor is

(a) 100Å	(b) 10Å	(c) 1000Å	(d) 314 Å

B. The applied field is further increased till superconductivity is completely destroyed. The strength of the field is now $\frac{8}{\pi}\times10^5$ gauss. The correlation length of the superconductor is:

(a) 20Å	(b) 200Å	(c) 628 Å	(d) 2000Å

53. Let ξ and λ denote the coherence length and the penetration depth of a superconductor. For a type-I, superconductor

(a) $\xi<\lambda$	(b) $\xi>\lambda$	(c) $\xi=\lambda$	(d) $\lambda=0$

54. The magnetic susceptibility of a type I superconductor at a temperature T below the transition temperature T_c

(a) increases linearly from a negative value to zero as temperature increases from 0 to T_c
(b) decreases linearly from a positive value to zero as temperature increases from 0 to T_c
(c) is negative and constant [illegible] (d) is positive [illegible] all T

55. The isotope effect in superc[illegible] al temperature
(a) increases as the isotopic mass increases (b) decreases as the isotopic mass increases
(c) increases linearly with the isotopic mass (d) decreases linearly with the isotopic mass.

56. Considering the BCS theory of superconductors, which one of the following statements is NOT CORRECT? (*h* is the Planck's constant and *e* is the electronic charge)

(a) Presence of energy gap at temperatures below the critical temperature
(b) Different critical temperatures or isotopes
(c) Quantization of magnetic flux in superconducting ring in the unit of $\left(\frac{h}{e}\right)$
(d) Presence of Meissner effect

57. **Group-I** contains elementary excitations in solids. **Group-II** gives the associated fields with these excitations. MATCH the excitations with their associated field and select your answer as per codes given below.

Group-I	**Group-II**
(P) phonon	(i) photon + lattice vibration
(Q) plasmon	(ii) electron + elastic deformation
(R) polaron	(iii) collective electron oscillations
(S) polariton	(iv) elastic wave

Codes:

(a) (P–iv), (Q-iii), (R-i), (S-ii) (b) (P–iv), (Q-iii), (R-ii), (S-i)

(c) (P–i), (Q-iii), (R-ii), (S-iv) (d) (P–iii), (Q-iv), (R-ii), (S-i)

58. The order of magnitude of the energy gap of a typical superconductor is:

(a) 1 MeV (b) 1 KeV (c) 1 eV (d) 1 meV

59. The voltage required to produce 483.6 MHz through a Josephson junction is

(a) 1nV (b) 1 μV (c) 1mV (d) 1V

60. The transition temperature of a BCS superconductor with an average atomic mass of 199.5 is 4.185K. What is the transition temperature for an average atomic mass of 203.4?

(a) 4.185 (b) 4.145 (c) 4.226 (d) 4.105

61. A solid sample has the property that, when cooled below a certain temperature, it expels any small applied magnetic field from within the material. Which of the following best describes this sample in the cooled state?

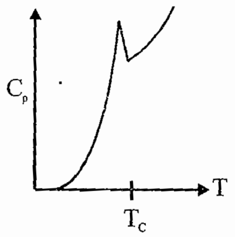
(a) Paramagnet (b) Diamagnet (c) Ferromagnet (d) Anti-ferromagnet

62. In a low temperature experiment, the resistance of a sensor is used as a thermometer. In order to have better sensitivity in the range 100 mK to 1.0 K, which material would make the best sensor?

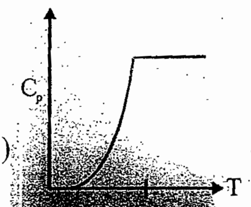
(a) insulator (b) p-n junction (c) pure semiconductor (d) metal

63. The plot of specific heat versus temperature across the superconducting transition temperature (T_C) is most appropriately represented by

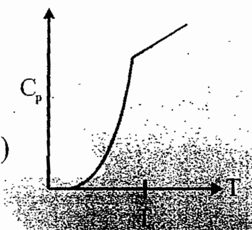
(a)



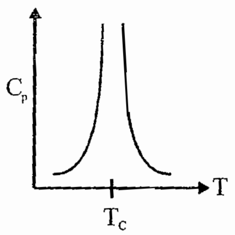
(b)



(c)



(d)



64. A superconducting ring carr[illegible] presence of a magnetic field $\vec{B}$ normal to the plane of the ring. Identify the ***incorrect*** statement.

(a) The flux passing through the superconductor is quantized in units of hc/e

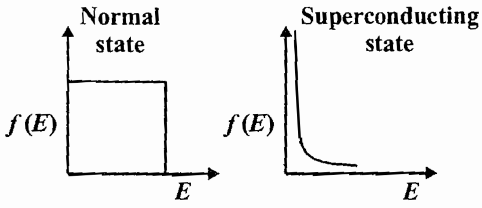
(b) The current and the magnetic field in the superconductor are time independent

(c) The current density $\vec{J}$ and $\vec{B}$ are related by the equation $\vec{\nabla}\times\vec{J}+\Lambda^2\vec{B}=0$, where Λ is a constant

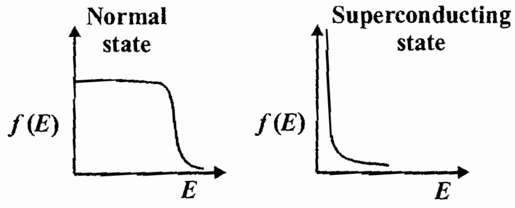
(d) The superconductor shows an energy gap which is proportional to the transition temperature of the superconductor

65. Which one of the following represents the electron occupancy for a superconductor in its normal and superconducting states? **[GATE 2015]**

(A)



(B)



66. The critical magnetic fields of a super-conductor at temperatures 4 K and 8 K are 11 mA/m and 5.5 mA/m respectively. The transition temperature is approximately **[NET June 2015]**

(a) 8.4 K (b) 10.6 K (c) 12.9 K (d) 15.0 K

67. A DC voltage V is applied across a Josephson junction between two superconductors with a phase difference ϕ_0. If I_0 and k are constants that depend on the properties of the junction, the current flowing through it has the form

(a) $I_0 \sin\left(\frac{2eVt}{\hbar}+\phi_0\right)$ (b) $kV \sin\left(\frac{2eVt}{\hbar}+\phi_0\right)$ (c) $kV \sin\phi_0$ (d) $I_0 \sin\phi_0 + kV$

1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13

$d_{111} = \frac{n}{d_{nhh}}$

CSIR-UGC-NET/JRF | GATE - PHYSICS

Assignment - 6 : Solid State Physics

1. Calculate c/a ratio and packing fraction for hcp crystal. 0.74

2. Sodium transfoms from bcc to hcp crystal structure at about 23 kelvin without change of density in the cubic phase $a = 4.23$ Å and the hcp phase c/a has ideal value of 1.63. Find the lattice constant.

3. (a) Calcualte the diffraction angle (2θ) for the first three peaks in aluminium powder diffraction pattern. Note that the crystal structure of the Aluminium is fcc and lattice constant is 0.404 nm. Wavelength of wave $\lambda = 1Å$.

   (b) If the lattice constant of Al was larger than 0.404 nm. Describe how pattern would change?

4. In a powder X-ray diffractoin pattern using X-rays of wave length 1.54Å, three consecutive first order peaks are observed at '2θ' values of 60°, 90° and 112°. If system is cubic and peaks at 90°. Corresponds to (220) planes.

   (a) Calculate the value of lattice constant (a)

   (b) Obtain the miller indices and identify the system.

5. A body-centred cubic (bcc) cell has a lattice constant a

   (a) How many atoms are there in a cubic unit cell?

   (b) What is the spacing between the (1 1 1) planes?

   (c) What will be the Bragg's diffraction angle for scattering from the (100) planes of the crystal?

6. A transmission electron microscope is use to produce a diffraction ring for the polycrystalline sample of copper the ring (1 1 1) is 12 nm from centre. How far would (200) ring be from the centre.

7. Calculate the lattice constant for a rock salt crystal (density = 2180 kg/m³), assuming that it has fcc lattice. Mol. wt. of NaCl = 58.5.

8. The distance between consecutive (1 1 1) planes in a cubic crystal is 2Å. Determine the lattice parameter and the atomic diamagnetic.

9. In a tetragonal crystal, the lattice parameters a = b = 2.42Å, and c = 1.74Å. Deduce the interplanar spacing between consecutive (101) planes.

10. Determine the Miller indices of a plane that makes an intercept of 3Å, 4Å, and 5Å on the coordinate axes of an orthorhombic crystal with a: b: c = 1:2:5.

11. Find the Miller indices of a plane that makes intercepts on a, b and c axes equal to 4Å, 6Å and 4Å in a tetragonal crystal with the cla ratio as 2.

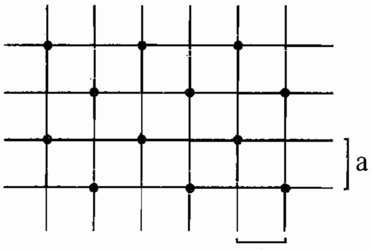
12. Determine the value of "i" in the following Miller-Bravais indices: (10.3), (11.3), (01.4), $(1\bar{1}4)$, (10.4), $(2\bar{2}.4)$, $(2\bar{2}.3)$ (11.0), (32.5) and $(3\bar{4}.2)$

13. Calculate the number of atoms per unit area in (100), (110) and (111) plane of a bcc crystal with the lattice parameter of 2.5Å.

h k l

1a 2b 5c

14) Figure shows a hypothetical two-dimensional crystal consisting of atoms arranged on a square grid.



Figure

Draw primitive unit cell in the crystal and calculate the area of primitive cell.

15. Define lattice constant of a material. Given the silicon lattice constant $a = 0.543$ nm, calculate the Si-Si bond length. Also calculate the density of silicon, in g cm^{-3}. [Atomic mass of silicon = 28.09, Avogadro's number = 6.022×10^{23} mol^{-1}].

1,2

CSIR-UGC-NET/JRF | GATE - PHYSICS

Assignment - 7 : Solid State Physics

1. As we know for diffraction studies of crystal, the wavelength of incident wave should be around 1 Å. X-rays, neutrons and electrons used to explore the crystal structure, then compare the energies of X-rays, neutrons and electrons for 1 Å wavelength also find out which one better for the studies of
(i) Crystal lattice structure (ii) Surface structure (iii) Magnetic structure

2. (a) Show that as diffraction angle increases accuracy in calculation of lattice parameter increases

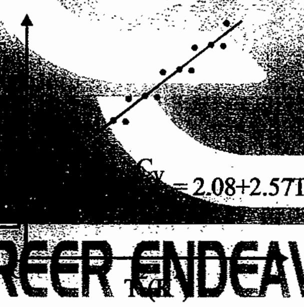
(b) In an X-ray diffraction experiment using $\lambda_{cu} = 1.542\text{Å}$, the (311) planeof a cubic lattice gives a dffraction peak at $\theta = 45^\circ$ with an uncertaininty of $\pm 0.2^\circ$. Calculate the value of lattice parameter and estimated error.

3. Consider the monoatomic lattice in which second nearest neighbour interaction is also considered. The dispersion relation for phonon is given by

$$\omega^2 = \left(\frac{4C}{m}\right)\sin^2\left(\frac{Ka}{2}\right)\left[1+4\cos^2\left(\frac{Ka}{2}\right)\right]$$

Calculate the velocity of sound in the solid.

4. The experimental heat capacity of potassium (K) is plotted in the graph below:



from this graph calculate

(a) Fermi temperature (T_F) (b) Fermi energy (E_F)

(c) Debye temperature (θ_D) (d) Debye frequency (ν_D)

5. (a) Consider two branches of a phonon spectrum of a one dimensional lattice $\omega_1(\bar{k}) = A|\sin(k)|$ and $\omega_2(\bar{k}) = B(2+\cos(k))$

(a) in the Debye phonon dispersion approximations give the phonon dispersion reation.
(b) Band width of optical branch, Band width of acoustical branch and Band gap

8. Take the Fermi energy of silver to be 5.52 eV.

(a) Find the corresponding velocity of conduction electron.

(b) If the resistivity of silver at room temperature is 1.62×10^{-8} Ωm estimate the average time between collisions.

(c) Determine the mean free path. Assume the number of conduction electrons as 5.86×10^{28} m^{-3}.

9. Aluminium is trivalent with atomic weight 27 and density 2.7 g/cm^3, while the mean collision time between electrons is 4×10^{-14}s. Calculate the current flowing through an aluminium wire 20m long and 2mm^2 cross-sectional area when a potential of 3V is applied to its ends.

10. The Fermi energy in gold is 5.54 eV (a) calculate the average energy of the free electrons in gold at 0°K. (b) Find the corresponding speed of free electrons (c) When temperature is necessary for the average kinetic energy of gas molecules to posses this value?

11. Find the probability of occupancy of a state of energy (a) 0.05eV above the Fermi energy (b) 0.05eV below the Fermi energy (c) equal to the Fermi energy. Assume a temperature of 300K.

12. What is the probability at 400K that a state at the bottom of the conduction band is occupied in silicon. Given that $E_g = 1.1$eV.

13. The Debye temperature θ for iron is known to be 360K. Calculate ν_m, the maximum frequency.

14. A fcc crystal of cube side a. Calculate the fermi energy for (111) plane and compare this fermi energy to the fermi energy of (100) plane. Here we are assuming that fcc crystal giving two conduction electrons per atom.

15. Sodium metal with bcc structure has radius of sodium atom to be 1.85 Å. Calculate
(a) Fermi energy, fermi velocity and diameter of fermi sphere.
(b) Electrical resistavity from drude model. (Given: $\tau = 3 \times 10^{-14}$ second)

16. Lattice vectors of rectangular primitive lattice is given as $\vec{a} = 2\hat{i}$, $\vec{b} = \hat{j}$ (a and b are in Å).
(i) Calculate the area of all the Brillouin zones.
(ii) Radius of fermi circle.
(iii) Fermi energy.
(iv) Average velocity of electrons at which they are moving in crystal.

18. Calculate the fermi energy of electron gas in copper. The density of copper at 300 K is 8.96 g/cm and atomic weight = 63.5. Take one conduction electron per atom.

19. A body centred cubic metal having electrons as charge carries density of electrons 2.64×10^{28} /m^3. Using free electron theory calculate:
(i) Fermi energy of metal in eV
(ii) Lattice constant of solid in Å
(iii) Fermi momentum.

20. Copper has monovalent fcc structure. Calculate the diameter of fermi sphere and shortest distance in first Brillouin zone.

20. $E(\bar{k}) = E_0 - 24\left[\cos(k_x a) + \cos(k_y a)\right]$ where $k^2 = k_x^2 + k_y^2$
Calculate:
(i) Band width of lowest band.
(ii) Plot first Brillouin zone.
(iii) E–k diagram along [10] and [11] direction.

21. A hypothetical semi conductor has a conduction band that can be described by $E_{cb} = E_1 - E_2 \cos(ka)$

and a valence band that can be described by $E_{vb} = E_3 - E_4 \sin^2\left(\frac{ka}{2}\right)$

where $E_3 < (E_1 - E_2)$ and $\frac{-\pi}{a} \le k \le \frac{\pi}{a}$

(i) Sketch the variation of the energy of two bands as function of k for $-\frac{\pi}{a} \le k \le \frac{\pi}{a}$

(ii) Band width of the conduction band and that of the valence band?

(iii) Band gap of the material?

(iv) What is the effective mass of the electron at the bottom of the conduction band?

22. For a solid energy is given by $E(k) = E_0 - c\cos\left(\frac{k_x a}{2}\right)\cos\left(\frac{k_y a}{2}\right)$

(a) Expand $E(\bar{k})$. (b) Band width of the band (c) Effective mass at $(0, \pi/a)$ and at $(0,0)$.

23. The energy band relation in a linear chain with interatomic distance a is given by $E_k = E_0 - T\cos(ka)$. When width of this band is increased by 20%. Calculate the percentage change in the effective mass of electron at the bottom of the band.

24. Assume the potential energy of two particles in the field of each other is given by

$$U(R) = \frac{-A}{R} + \frac{B}{R^9}$$

Where A and B are constants.

(a) Calculate equilibrium distance.

(b) Show that stable configuration, the energy of attracton is nine times the energy of repulsion.

(c) Show that potential energy of the system under stable configuration is

$$\frac{8A}{9R}$$

CSIR-UGC-NET/JRF | GATE - PHYSICS

Assignment - 8 : Solid State Physics

1. The band structure of a simple cubic lattice is given by $E = E_0 - A(\cos ka + \cos ka + \cos ka)$ where a is the lattice constant and A is a positive constant.
   (a) Find the effective mass of the electron at values of k corresponding to the top and bottom the band.
   (b) For a particular simple cubic solid with a lattice constant a = 2.5Å, the effective mass of the electron at the bottom of the band is equal to the free electron mass. Find the value of the constant A and the width of the band in eV.
2. A certain metal has a body centered cubic space lattice with cube side a. The electrons are assume to be free.
   (a) If the metal is monovalent, what is the radius of fermi sphere?
   (b) For the electrons density of part (a) the fermi sphere does not reach the Brillouin zone boundary. What is the minimum value of the free electron density so that the fermi sphere just touches the zone boundary?
3. An organic conductor has recently been found by optical studies to have $\omega_p = 1.80 \times 10^{15}\ \text{sec}^{-1}$ for plasma frequency and $\tau = 2.83 \times 10^{-15}$ sec for the electron relaxation time at room temperature.
   (a) Calculate electrical conductivity
   (b) From crystal and chemical structure; the conduction electron concentration is $4.7 \times 10^{21}\ /\text{cm}^3$. Calculate effective mass m*.
4. Calculate the hall coefficient of sodium (Na) on based on free electron theory. Sodium has bcc structure and side of cube is a = 4.28Å.
5. A sample of silicon of thickness 100 μm is doped with 10^{23} phosphorus atoms/m^3. If the sample is kept in a magnetic field of 0.2 wb/m^2 and a current of 1mA is passed through the sample, calculate the hall voltage produced.
6. Hall coefficient of a body-centred cubic metal having electrons as charge carriers is $R_H = -2.36 \times 10^{10}\ \text{m}^3/\text{C}$. Using free electron theory. Calculate the lattice constant of metal.
7. A silicon bar of length l = 10.0 mm and cross-section $a^2 = 0.5 \times 0.5\ \text{mm}^2$ carries current I = 5mA along its length in x-direction. It is placed between pole pieces of an electron magnet of field B_z = 1.6 tesla along z-direction. A pairs of voltage probles, plaed across the cross-section of the sample and perpendicular to the current and field, measures a voltage, $V_H = -5$mV.
8. The donor concentration of an n-type semiconductor (E_g = 2.303 eV) is $N_D = 2 \times 10^{16}\ /\text{cm}^3$. Find the temperature (in terms of Boltzman constant) at which the semiconductor starts behaving like an intrinsive semiconductor. The density of states in the conduction and the valence bands are $4 \times 10^9\ /\text{cm}^3$ and $1 \times 10^{19}/\text{cm}^3$ respectively.
9. (a) Show that minimum conductivity of a semi conductor sample occurs. When $n_0 = n_i\sqrt{\frac{\mu_p}{\mu_n}}$
   (b) What is the expression for the minimum conductivity $\sigma_{\min}$?
   (c) Calculate $\sigma_{\min}$ for Si at 300K and compare with the intrinsic conductivity.

10. A Si solar cell has a short circulated current of 100 mA and an open circuit voltage of 0.8 volt under full solar illumination. The fill factor is 0.7. What is the maximum power delivered to a load by this cell?

11. A Si solar cell 2cm × 2cm with Ith = 32nA has an optical generation rate of 10^{18} EHA cm^{3}–sec within $L_p = L_n = 2\mu m$ of the junction. If the depletion width is 1 μm, calculate the short-circuit current and open circuit voltage for this cell?

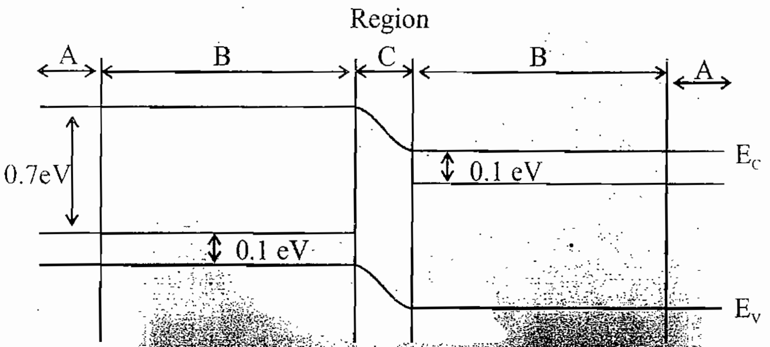
12. If one LED made from a semiconductor with a band gap of 2.5 eV. What wavelength of light will it emit? Can you use it to efficiently detect photons of wavelength 1900 nm and 100 nm?

13. A solar cell has a short circuit current of 50 mA and an open circuit voltage of 0.7V under full illumination. What is the maximum power delivered by this cell if the fill factor is 0.8?

14. A photo conductive cell, 1×1×1 nm/Cds crystal is illuminated with light of wavelength 450 nm and intensity 1 mW/cm^2. Assuming quantum efficiency of unity. Calculate the number of electron hole pairs (EHP/second) generated per second. Given the energy band gap of Cds, E_g = 2.6 eeV.

15. What wavelength of light can be observed by a silicon photodetector band gap silicon = 1.1 eV.

16. Consider a band diagram of a simple light-emitting diode. Assume essentially all recombination is direct and result in light emission. The forward biased current consists of holes injected from a conctact on the left and electrons injected from a contact on the right.



(a) In which region would you expect the optical recombination rate to be greatest?

(b) What is the approximate energy of emitted photons is eV?

(c) For steady state current [illegible] all photons escape, what is the optical output power consists with your answer to part (b)

(d) If the voltage drop across depletion region is [illegible] volt, what is the separation of quasi fermi level in region C. How can this be less than the total forward bias of 1.4 volt?

17. A 0.46μm thick sample of GaAs is illuminated with monochromatic light of $h\nu = 2$eV. The absorption coefficient α is 5×10^4 cm^{-1}. The power incident on the sample is 10 mW. (Band gap E_g = 1.43 eV)

(a) Find the total energy absorbed by the sample per second (J/sec).

(b) Find the number of photons per second given off from recombination events assuming perfect quantum efficiency.

18. A Si pin photodiode has an active light receiving area of dimensions (0.4mm × 0.4 mm). When radiation of wavelength 700 nm and intensity 0.1 mW/cm^2 is incident, it generated a photon current of 20 nA. Calculate the

(a) Responsivity in units of A/W, and the

(b) Qunatum efficiency of hte photodiode at 700 nm (within an accuracy of 1%).

19. (a) Calculate the static dielectric constant (ε_r) of solid argon using classius-Mossotti equation and the following data?

$\alpha_e = 1.7\times10^{-40}\ F.m^2$, atomic mass of Avogadro number

Ar = 39.95 g/mole,
N = 6.02×10^{23} per mole, density of solid Ar
= 1.8 gm/cm^3.

(b) Will result be affected if (ε_r) is calculated from $P = \varepsilon_0(\varepsilon_r - 1)E = N\alpha_e E$. Give reason for your answer.

20. Iron has bcc structure with lattice constant a = 2.86Å. The saturation magnetisation of iron is 1.74×10^6 amp/m. Calcualte magnetic moment (μ) of Fe atom.

21. Calculate saturation magnetisation for Nickel and cobalt having densities equal to 8.67 and 9.04 gm/cm^3, respectively. Number of Bohr magnetons per atom are 1.72 for Co and 0.54 for Ni. Atomic masses are $M_{CO} = 58.93$ and $M_{Ni} = 58.69$.

22. The binding energy of a copper pair at absolute zero according to BCS theory is a about 3.5 KT$_c$. Where 'K' is Boltzmann constant and T$_c$ is critical temperature. The critical temperature of type I super conductor is 10K.
(a) What is the energy gap in electron volt.
(b) Calculate the wavelength of photons whose energy is the just sufficient to break the copper pair in this super conductivity
(c) In which region of electromagnetic spectrum do this photon lie?

23. In lead (Pb) the binding energy of Cooper pair is 10 meV. Calculate
(i) Critical temperature of Pb.
(ii) Maximum wavelength of photons require to break the cooper pair.

24. For BCS super conductor $T_C = 5K$, $E_F = 5$ eV, and *n* density of conduction electrons = 10^{29}/m^3. Evaluate
(a) Approximate magnitude of energy gap in (eV)
(b) Density of super electrons close to T = 0K.

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GATE Previous Years' Questions

1. The mean momentum of a nucleon in a nucleus with mass number A varies as **[GATE 2000]**
(a) A (b) A^2 (c) $A^{-2/3}$ (d) $A^{-1/3}$

2. The masses of hydrogen atom, neutron and ${}_{92}U^{238}$ are given by 1.0078, 1.0087 and 238.0508 a.m.u., respectively. The binding energy of ${}_{92}U^{238}$ is therefore approximately equal to **[GATE 2000 & 03]**
(Taking 1 a.m.u. = 931.64 MeV)
(a) 120 MeV (b) 1500 MeV (c) 1600 MeV (d) 1800 MeV

3. The volume of a nucleus in an atom is proportional to the **[GATE 2004]**
(a) mass number (b) proton number
(c) neutron number (d) electron number

4. As one moves along the line of stability from ^{56}Fe to ^{235}U nucleus, the nuclear binding energy per particle decreases from about 8.8 MeV to 7.6 MeV. This trend is mainly due to the:
(a) short range nature of the nuclear forces. **[GATE 2004]**
(b) long range nature of the coulomb forces.
(c) tensor nature of the nuclear forces.
(d) spin dependence of the nuclear forces.

may for short

5. The order of magnitude of the binding energy per nucleon in a nucleus is: **[GATE 2006]**
(a) 10^{-5} MeV (b) 10^{-3} MeV (c) 0.1 MeV (d) 10 MeV

6. The experimentally measured spin g-factors of proton and neutron indicate that **[GATE 2006]**
(a) both proton and neutron are elementary point particles.
(b) both proton and neutron are not elementary point particles.
(c) while proton is an elementary point particle, neutron is not
(d) while neutron is an elementary point particle, proton is not.

has Internal structure

7. An ${}_8O^{16}$ nucleus is spherical and has a charge radius R and a volume $V = \frac{4}{3}\pi R^3$. According to empirical observations of the charge radii, the volume of the ${}_{54}Xe^{128}$ nucleus assumed to be spherical is
(a) 8 V (b) 2 V (c) 6.75 V (d) 1.89 V **[GATE 2008]**

8. A particle is confined within a spherical region of radius one femtometer (10^{-15} m). Its momentum can be expressed to be about **[GATE 2010]**
(a) $20\frac{keV}{c}$ (b) $200\frac{keV}{c}$ (c) $200\frac{MeV}{c}$ (d) $2\frac{GeV}{c}$

9. The mean kinetic energy of a nucleon in a nucleus of atomic weight A varies as A^n, where n is_______ (upto two decimal places) **[GATE 2015]**

ANSWER KEY

1. (d) **2. (d)** **3. (a)** **4. (b)** **5. (d)** **6. (b)** **7. (a)**
8. (c) **9. (–0.66 to –0.67)**

CSIR-UGC-NET Previous Years' Questions

1. The radius of a ${}^{64}_{29}\text{Cu}$ nucleus is measured to be 4.8×10^{-13} cm. **[NET June 2011]**

(A) The radius of a ${}^{27}_{12}\text{Mg}$ nucleus can be estimated to be

(a) 2.86×10^{-13} cm (b) 5.2×10^{-13} cm (c) 3.6×10^{-13} cm (d) 8.6×10^{-13} cm

(B) The root-mean-square (rms) energy of a nucleon in a nucleus of atomic number A in its ground state varies as:

(a) $A^{4/3}$ (b) $A^{1/3}$ (c) $A^{-1/3}$ (d) $A^{-2/3}$

2. The intrinsic electrric dipole moment of a nucleus ${}^{A}_{Z}X$ **[NET Dec. 2013]**

(a) increases with Z, but independent of A
(b) decreases with Z, but independent of A
(c) is always zero
(d) increases with Z and A

ANSWER KEY

1_A(c) **1_B(c)** **3. (c)**

TIFR Previous Years' Questions

1. A proton is accelerated to a high energy E and shot at a nucleus of Oxygen $\left({}^{16}_{8}\text{O}\right)$. In order to penetrate the Coulomb barrier and reach the surface of the Oxygen nucleus, E must be at least **[TIFR 2012]**

(a) 3.6 MeV (b) 1.8 MeV (c) 45 keV (d) 180 eV

ANSWER KEY

1. (a)

JEST Previous Years' Questions

1. The stable nucleus that has $\frac{1}{3}$ the radius of ${}^{189}\text{Os}$ nucleus is, **[JEST 2016]**

(a) ${}^{7}\text{Li}$ (b) ${}^{16}\text{O}$ (c) ${}^{4}\text{He}$ (d) ${}^{14}\text{N}$

ANSWER KEY

1. (a)

Other Examinations Previous Years' Questions

1. If M_e, M_p and M_H are the rest masses of electron, proton and hydrogen atom in the ground state (with energy -13.6 eV), respectively, which of the following is exactly true? (c is the speed of light in free space)

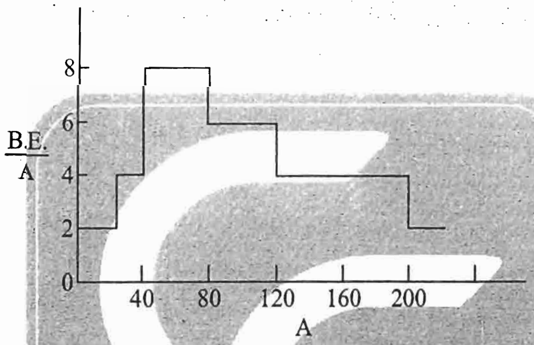
(a) $M_H = M_p + M_e$ (b) $M_H = M_p + M_e - \frac{13.6\text{ eV}}{c^2}$ (c) $M_H = M_p + M_e + \frac{13.6\text{ eV}}{c^2}$

(d) $M_H = M_p + M_e + K$, where $K \neq \pm\frac{13.6\text{ eV}}{c^2}$ or zero.

2. A neutron of mass $m_n = 10^{-27}$ kg is moving inside a nucleus. Assume the nucleus to be a cubical box of size 10^{-14} m with impenetrable walls. Take $\hbar \approx 10^{-34}$ Js and $1\text{ MeV} \approx 10^{-13}\text{ J}$. An estimate of the energy in MeV of the neutron is:

(a) 80 MeV (b) $\frac{1}{8}$ MeV (c) 8 MeV (d) $\frac{1}{80}$ MeV

3. The following histogram represents the binding energy per particle (B.E./A) in MeV as a function of the mass number A of a nucleus.



A nucleus with mass number $A = 180$ fissions into two nuclei of equal masses. In the process

(a) 180 MeV of energy is released. (b) 180 MeV of energy is absorbed.
(c) 360 MeV of energy is released. (d) 360 MeV of energy is absorbed.

4. A nucleus has a size of 10^{-15} m. Consider an electron bound within a nucleus. The estimated energy of this electron is of the order of

(a) 1 MeV (b) 10^2 MeV (c) 10^4 MeV (d) 10^6 MeV

5. Two spherical nuclei have mass numbers 216 and 64 with their radii R_1 and R_2 respectively. The ratio $\frac{R_1}{R_2}$ is:

(a) 1.0 (b) 1.5 (c) 2.0 (d) 2.5

6. According to the uncertainty principle, the kinetic energy of an electron confined to a spherical region of volume 10^{-33} m^3 is of the order of

(a) 10^{-10} J (b) 10^{-12} J (c) 10^{-14} J (d) 10^{-16} J

7. The radius of the nucleus of the Ra atom, which carries an electric charge +88e, is 7.0×10^{-15} m. What should roughly be the speed of a proton, if it has to reach as close as 1.0×10^{-14} m from the centre of the nucleus? [The radius of the cloud of orbital electrons of the Ra atom is approximately 5.0×10^{-11} m.]

(a) 6.7×10^9 m/s (b) 3.1×10^8 m/s (c) 1.4×10^5 m/s (d) 4.9×10^7 m/s

ANSWER KEY

1. (b) **2. (c)** **3. (c)** **4. (b)** **5. (b)** **6. (d)** **7. (d)**

GATE Previous Years' Questions

1. The angular momentum and parity of ${}^{17}_{8}O$ nucleus, according to the nuclear shell model (including spin-orbit coupling), is: **[GATE 1998]**

(a) 0^+ (b) $\frac{1}{2}^-$ (c) $\frac{3}{2}^+$ (d) $\frac{5}{2}^+$

2. The asymmetry terms in the Weizsacker semi-empirical mass formula is because of **[GATE 2002]**

(a) non-spherical shape of the nucleus.
(b) non-zero spin of nucleus.
(c) unequal number of protons and neutrons inside the nucleus.
(d) odd number of protons inside the nucleus.

3. The nuclear spins of ${}_6C^{14}$ and ${}_{12}Mg^{25}$ nuclei are respectively, **[GATE 2002]**

(a) zero and half integer (b) half integer and zero
(c) an integer and half integer (d) both half integer

4. The spin and parity of ${}_4Be^9$ nucleus, as predicted by the shell model, are respectively. **[GATE 2002]**

(a) 3/2 and odd (b) 1/2 and odd (c) 3/2 and even (d) 1/2 and even

Common Data for Q.5 and Q.6:

The nucleus ${}^{41}Ca$ can be described by the single particle shell model.

5. The single particle states occupied by the last proton and the last neutron, respectively, are given by **[GATE 2004]**

(a) $d_{5/2}$ and $f_{7/2}$ (b) $d_{3/2}$ and $f_{5/2}$ (c) $d_{5/2}$ and $f_{5/2}$ (d) $d_{3/2}$ and $f_{7/2}$

6. The ground state angular momentum and parity of ${}^{41}Ca$ are: **[GATE 2004]**

(a) $\left(\frac{7}{2}\right)^-$ (b) $\frac{3}{2}^+$ (c) $\frac{5}{2}^+$ (d) $\frac{5}{2}^-$

7. Which of the following expressions for total binding energy B of a nucleus is correct $(a_1, a_2, a_3, a_4 > 0)$? **[GATE 2005]**

(a) $B = a_1A - a_2A^{2/3} - a_3\dfrac{Z(Z-1)}{A^{1/3}} - a_4\dfrac{(A-2Z)^2}{A} + \delta$

(b) $B = a_1A + a_2A^{2/3} - a_3\dfrac{Z(Z-1)}{A^{1/3}} - a_4\dfrac{(A-2Z)^2}{A} + \delta$

(c) $B = a_1A + a_2A^{1/3} - a_3\dfrac{Z(Z-1)}{A^{1/3}} - a_4\dfrac{(A-2Z)^2}{A} + \delta$

(d) $B = a_1A - a_2A^{1/3} - a_3\dfrac{Z(Z-1)}{A^{1/3}} - a_4\dfrac{(A-2Z)^2}{A} + \delta$

8. According to shell model, the ground state of ${}^{15}_{8}O$ nucleus is **[GATE 2005]**

(a) $\frac{3}{2}^+$ (b) $\frac{1}{2}^+$ (c) $\frac{3}{2}^-$ (d) $\frac{1}{2}^-$

9. According to the shell model the ground state spin of the ${}^{17}O$ nucleus is **[GATE 2007]**

(a) $\frac{3}{2}^+$ (b) $\frac{5}{2}^+$ (c) $\frac{3}{2}^-$ (d) $\frac{5}{2}^-$

10. The following gives a list of pairs containing **[GATE 2008]**

(i) a nucleus (ii) one of its properties

Find the pair which is in appropriate.

(a) (i) ${}_{10}Ne^{20}$ nucleus, (ii) stable nucleus;

(b) (i) a spheroidal nucleus, (ii) an electric quadrupole moment;

(c) (i) ${}_{8}O^{16}$ nucleus, (ii) nucleus spin $J = \frac{1}{2}$;

(d) (i) U^{238} nucleus, (ii) binding energy = 1785 MeV (approx);

11. The four possible configurations of neutrons in the ground state of ${}_{4}Be^{9}$ nucleus according to the shell model, and the associated nuclear spin are listed below. Choose the correct one. **[GATE 2008]**

(a) $\left({}^{1}s_{1/2}\right)^2\left({}^{1}p_{3/2}\right)^3;\ J = \frac{3}{2}$ (b) $\left({}^{1}s_{1/2}\right)^2\left({}^{1}p_{1/2}\right)^2\left({}^{1}p_{3/2}\right)^1;\ J = \frac{3}{2}$

(c) $\left({}^{1}s_{1/2}\right)^2\left({}^{1}p_{3/2}\right)^4;\ J = \frac{1}{2}$ (d) $\left({}^{1}s_{1/2}\right)^2\left({}^{1}p_{3/2}\right)^2\left({}^{1}p_{1/2}\right)^1;\ J = \frac{1}{2}$

12. The mass difference between the pair of mirror nuclei ${}_{6}C^{11}$ and ${}_{5}B^{11}$ is given to be $\Delta \mathrm{MeV/c^2}$. According to the semi empirical mass formula, the mass difference between the pair of mirror nuclei ${}^{17}_{9}F$ and ${}^{17}_{8}O$ will approximately be (rest mass of proton $m_p = 938.27$ MeV/c^2 and rest mass of neutron $m_n = 939.57$ MeV/c^2). **[GATE 2008]**

(a) $1.39\,\Delta\mathrm{MeV/c^2}$ (b) $(1.39\Delta + 0.5)\mathrm{MeV/c^2}$

(c) $1.86\,\Delta\mathrm{MeV/c^2}$ (d) $(1.6\,\Delta + 0.78)\mathrm{MeV/c^2}$

13. Consider the following expression for the mass of a nucleus with Z protons and A nucleons.

$$M(A,Z) = \frac{1}{c^2}\left[f(A) + yZ + zZ^2\right]$$

Here, $f(A)$ is a function of A. **[GATE 2009]**

$$y = -4a_A$$

$$z = a_c A^{-1/3} + 4a_A A^{-1}$$

a_A and a_c are constants of suitable dimensions. For a fixed A, the expression of Z for the most stable nucleus is

(a) $Z=\dfrac{A/2}{\left[1+\left(\dfrac{a_c}{a_A}\right)A^{2/3}\right]}$ (b) $Z=\dfrac{A/2}{\left[1-\left(\dfrac{a_c}{4a_A}\right)A^{2/3}\right]}$

(c) $Z=\dfrac{A/2}{\left[1+\left(\dfrac{a_c}{4a_A}\right)A^{2/3}\right]}$ (d) $Z=\dfrac{A}{\left(1+A^{2/3}\right)}$

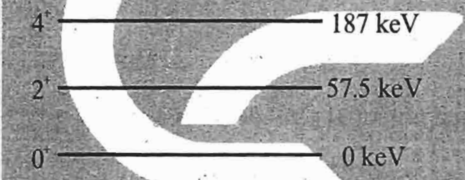
14. Consider a nucleus with N neutrons and Z protons. If m_p, m_n and $B.E.$ represents the mass of the proton, the mass of the neutron and the binding energy of the nucleus respectively and c is the velocity of light in free space, the mass of the nucleus is given by **[GATE 2009]**

(a) Nm_n+Zm_p (b) Nm_p+Zm_n (c) $Nm_n+Zm_p-\dfrac{B.E.}{c^2}$ (d) $Nm_p+Zm_n+\dfrac{B.E.}{c^2}$

15. In the nuclear shell model the spin parity of ${}^{15}_{7}N$ is given by **[GATE 2010]**

(a) $\dfrac{1}{2}^-$ (b) $\dfrac{1}{2}^+$ (c) $\dfrac{3}{2}^-$ (d) $\dfrac{3}{2}^+$

16. The first three energy level of ${}^{228}Th_{90}$ are shown below **[GATE 2010]**



The expected spin-parity and energy of the next level are given by

(a) (6^+; 400 keV) (b) (6^+; 300 keV) (c) (2^+; 400 keV) (d) (4^+; 300 keV)

17. The semi-empirical mass formula for the binding energy of nucleus contains a surface correction term. This term depends on the mass number A of the nucleus as **[GATE 2011]**

(a) $A^{-1/3}$ (b) $A^{1/3}$ (c) $A^{2/3}$ (d) A

18. According to the single particle nuclear shell model, the spin parity of the ground state of ${}^{17}_{8}O$ is

(a) $\dfrac{1}{2}^-$ (b) $\dfrac{3}{2}^-$ (c) $\dfrac{3}{2}^+$ (d) $\dfrac{5}{2}^+$ **[GATE 2011]**

19. Total binding energies of O^{15}, O^{16} and O^{17} are 111.96 MeV, 127.62 MeV and 131.76 MeV, respectively. The energy gap between ${}^1p_{1/2}$ and ${}^1d_{5/2}$ neutron shells for the nuclei whose mass number is close to 16, is: **[GATE 2012]**

(a) 4.1 MeV (b) 11.5 MeV (c) 15.7 MeV (d) 19.8 MeV

20. The electromagnetic form factor $F(q^2)$ of a nucleus is given by, $F(q^2)=\exp\left|-\dfrac{q^2}{2Q^2}\right|$ **[GATE 2013]**

where Q is a constant. Given that $F(q^2)=\dfrac{4\pi}{q}\int_0^\infty rdr\ \rho(r)\sin qr$

$$\int d^3r\,\rho(r) = 1$$

where $\rho(r)$ is the charge density, the root mean square radius of the nucleus is given by

(a) $1/Q$ (b) $\sqrt{2}/Q$ (c) $\sqrt{3}/Q$ (d) $\sqrt{6}/Q$

21. A proton is confined to a cubic box, whose sides have length $10^{-12}\ m$. What is the minimum kinetic energy of the proton? The mass of proton is $1.67 \times 10^{-27}\ kg$ and Planck's constant is $6.63 \times 10^{-34}\ Js$. **[GATE 2013]**

(a) $1.1 \times 10^{-17}\ J$ (b) $3.3 \times 10^{-17}\ J$ (c) $9.9 \times 10^{-17}\ J$ (d) $6.6 \times 10^{-17}\ J$

Statement for Linked Answer Q. 22 and 23:

In the Schmidt model of nuclear magnetic moments, we have,

$$\vec{\mu} = \frac{e\hbar}{2Mc}(g_l\vec{l} + g_s\vec{S})$$

where the symbols have their usual meaning

22. For the case $J = l + 1/2$, where J is the total angular momentum, the expectation value of $\vec{S}\cdot\vec{J}$ in the nuclear ground state is equal to, **[GATE 2013]**

(a) $(J-1)/2$ (b) $(J+1)/2$ (c) $J/2$ (d) $-J/2$

23. For the O^{17} nucleus ($A = 17, Z = 8$), the effective magnetic moment is given by,

$$\vec{\mu}_{eff} = \frac{e\hbar}{2Mc} g\vec{J}$$

where g is equal to, ($g_s = 5.59$ for proton and -3.83 for neutron) **[GATE 2013]**

(a) 1.12 (b) −0.77 (c) −1.28 (d) 1.28

24. In the nuclear shell model, the potential is modeled as $V(r) = \frac{1}{2}m\omega^2r^2 - \lambda\vec{L}\cdot\vec{S},\ \lambda > 0$. The correct spin-parity and isospin assignments for the ground state of ^{13}C is **[GATE 2015]**

(a) $\frac{1^-}{2};\frac{-1}{2}$ (b) $\frac{1^+}{2};\frac{-1}{2}$ (c) $\frac{3^+}{2};\frac{1}{2}$ (d) $\frac{3^-}{2};\frac{-1}{2}$

25. According to the nuclear shell model, the respective ground state spin-parity values of $^{15}_{8}O$ and $^{17}_{8}O$ nuclei are **[GATE 2016]**

(a) $\frac{1}{2}^+, \frac{1}{2}^-$ (b) $\frac{1}{2}^-, \frac{5}{2}^+$ (c) $\frac{3}{2}^-, \frac{5}{2}^+$ (d) $\frac{3}{2}^-, \frac{1}{2}^-$

ANSWER KEY

1. (d) **2. (c)** **3. (a)** **4. (a)** **5. (d)** **6. (a)** **7. (a)**
8. (d) **9. (b)** **10. (c)** **11. (a)** **12. (b)** **13. (c)** **14. (c)**
15. (a) **16. (a)** **17. (c)** **18. (d)** **19. (b)** **20. (c)** **21. (c)**
22. (b) **23. (b)** **24. (a)** **25. (b)**

CSIR-UGC-NET Previous Years' Questions

1. The difference in the Coulomb energy between the mirror nuclei ${}^{49}_{24}\mathrm{Cr}$ and ${}^{49}_{25}\mathrm{Mn}$ is 6.0 MeV. Assuming that the nuclei have a spherically symmetric charge distribution and that e^2 is approximately 1.0 MeV-fm, the radius of the ${}^{49}_{25}\mathrm{Mn}$ nucleus is **[NET Dec. 2011]**

   (a) $4.9\times10^{-13}\,\mathrm{m}$ (b) $4.9\times10^{-15}\,\mathrm{m}$ (c) $5.1\times10^{-13}\,\mathrm{m}$ (d) $5.1\times10^{-15}\,\mathrm{m}$

2. According to the shell model the spin and parity of the two nuclei ${}^{125}_{51}\mathrm{Sb}$ and ${}^{89}_{38}\mathrm{Sr}$ are, respectively. **[NET Dec. 2011]**

   (a) $\left(\frac{5}{2}\right)^+$ and $\left(\frac{5}{2}\right)^+$ (b) $\left(\frac{5}{2}\right)^+$ and $\left(\frac{7}{2}\right)^+$ (c) $\left(\frac{7}{2}\right)^+$ and $\left(\frac{5}{2}\right)^+$ (d) $\left(\frac{7}{2}\right)^+$ and $\left(\frac{7}{2}\right)^+$

3. The single particle enertgy difference between the p-orbitals (i.e. $p_{3/2}$ and $p_{1/2}$) of the nucleus ${}^{114}_{50}\mathrm{Sn}$ is 3 MeV. The energy difference between the states in its $1f$ orbital is **[NET Dec. 2012]**

   (a) –7 MeV (b) 7 MeV (c) 5 MeV (d) – 5 MeV

H.W

4. The binding energy of a light nucleus (Z, A) in MeV is given by the approximate formula

$$B(A,Z)\approx 16A-20A^{2/3}-\frac{3}{4}Z^2A^{-1/3}+30\frac{(N-Z)^2}{A}$$ **[NET June 2013]**

   where $N = A - Z$ is the neutron number. The value of Z of the most stable isobar for a given A is

   (a) $\frac{A}{2}\left(1-\frac{A^{2/3}}{160}\right)^{-1}$ (b) $\frac{A}{2}$ (c) $\frac{A}{2}\left(1-\frac{A^{2/3}}{120}\right)^{-1}$ (d) $\frac{A}{2}\left(1+\frac{A^{4/3}}{64}\right)$

5. According to the shell model, the total angular momentum (in units of $\hbar$) and the parity of the ground state of the ${}^{7}_{3}\mathrm{Li}$ nucleus is **[NET Dec. 2013]**

   (a) $\frac{3}{2}$ with negative parity (b) $\frac{3}{2}$ with positive parity

   (c) $\frac{1}{2}$ with positive parity (d) $\frac{7}{2}$ with negative parity

6. A permanently deformed even-even nucleus with $J^P = 2^+$ has rotational energy 93 keV. The energy of the next excited state is **[NET June 2014]**

   (a) 372 keV (b) 310 keV (c) 273 keV (d) 186 keV

H.W

7. If the binding energy B of a nucleus (mass number A and charge Z) is given by **[NET Dec. 2014]**

$$B=a_VA-a_SA^{2/3}-a_{sym}\frac{(2Z-A)^2}{A}-\frac{a_cZ^2}{A^{1/3}}$$ where $a_V = 16$ MeV, $a_S = 16$ MeV, $a_{sym} = 24$ MeV and $a_c = 0.75$ MeV, then the Z for the most stable isobar for a nucleus with $A = 216$ is

   (a) 68 (b) 72 (c) 84 (d) 92

8. Let us approximate the nuclear potential in the shell model by a 3-dimensional isotropic harmonic oscillator. Since the lowest two energy levels have angular momenta $l = 0$ and $l = 1$ respectively, which of the following two nuclei have magic numbers of protons and neutrons ? **[NET June 2015]**

(a) ${}^{4}_{2}$He and ${}^{16}_{8}$O (b) ${}^{2}_{1}$D and ${}^{8}_{4}$Be (c) ${}^{4}_{2}$He and ${}^{8}_{4}$Be (d) ${}^{4}_{2}$He and ${}^{12}_{6}$Be

H.W 9. Of the nuclei of mass number $A = 125$, the binding energy calculated from the liquid drop model (given that the coefficients for the Coulomb and the asymmetry energy are $a_C = 0.7$MeV and $a_{sym} = 22.5$ MeV respectively) is a maximum for **[NET Dec. 2015]**

(a) ${}^{125}_{54}$Xe (b) ${}^{125}_{53}$I (c) ${}^{125}_{52}$Te (d) ${}^{125}_{51}$Sb

10. The electric quadrupole moment of an odd proton nucleus is $\frac{(2j-1)}{2(j+1)}\langle r^2\rangle$, where j is the total angular momentum. Given that $R_0 = 1.2$ fm, what is the value, in barn, of the quadrupole moment of the ^{27}Al nucleus in the shell model? **[NET Dec. 2015]**

(a) 0.043 (b) 0.023 (c) 0.915 (d) 0

H.W 11. Let E_S denote the contribution of the surface energy per nucleon in the liquid drop model. The ratio $E_S\left({}^{27}_{13}\text{Al}\right) : E_S\left({}^{64}_{30}\text{Zn}\right)$ is **[NET June 2016]**

(a) 2:3 (b) 4:3 (c) 5:3 (d) 3:2

12. According to the shell model, the nuclear magnetic moment of the ${}^{27}_{13}$Al nucleus is (Given that for a proton $g_l = 1, g_s = 5.586$, and for a neutron $g_l = 0, g_s = -3.826$). **[NET June 2016]**

(a) $-1.913\,\mu_N$ (b) $14.414\,\mu_N$ (c) $4.793\,\mu_N$ (d) 0

ANSWER KEY

1. (b)	**2. (d)**	**3. (b)**	**4. (a)**	**5. (a)**	**6. (b)**	**7. (c)**
8. (a)	**9. (c)**	**10. (a)**	**11. (b)**	**12. (c)**		

TIFR Previous Years' Questions

1. In the semi-empirical mass formula, the volume (V), surface (S), coulomb (C), and pairing (P) contributions to the binding energy of a nucleus $^{A}_{Z}X$ vary with mass number A as **[TIFR 2015]**

(a) $V \propto A$, $S \propto A^{2/3}$, $C \propto A^{-1/3}$, $P \propto A^{-3/4}$

(b) $V \propto A$, $S \propto A^{1/3}$, $C \propto A^{-1/3}$, $P \propto A^{-3/4}$

(c) $V \propto A$, $S \propto A^{-2/3}$, $C \propto A^{1/3}$, $P \propto A^{-3/4}$

(d) $V \propto A^{2}$, $S \propto A^{2/3}$, $C \propto A^{-1/3}$, $P \propto A^{-3/4}$

2. The Weizäcker semi-empirical mass formula for an odd nucleus with Z protons and A nucleons may be written as $M(Z, A) = \alpha_1 A + \alpha_2 A^{2/3} + \alpha_3 A + \alpha_4 Z^2$ where the α_i are constants independent of Z, A. For a given A, if Z_A is the number of photons of the most stable isobar, the total energy released when an unstable nuclide undergoes a single β^- decay to (Z_A, A) is **[TIFR 2016]**

(a) α_3 (b) α_4 (c) $\alpha_4 - \alpha_3$ (d) $\alpha_1 + \alpha_2$

ANSWER KEY

1. (a) **2. (b)**

Other Examinations Previous Years' Questions

1. A nucleus may be modelled as a drop of liquid consisting of the nucleons (protons and neutrons). In this model, the dominant contribution to the nuclear binding energy is from the volume, which is proportional to A, the total number of nucleons. Then the two important subdominant contributions from the surface tension and the coulomb repulsion of the protons are, proportional to

(a) $A^{2/3}$ and $Z/A^{1/3}$ respectively
(b) $A^{2/3}$ and $Z^2/A^{1/3}$ respectively
(c) $A^{1/3}$ and $Z^2/A^{2/3}$ respectively
(d) $A^{1/2}$ and $Z^2/A^{1/3}$ respectively

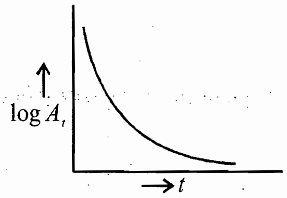
ANSWER KEY

1. (b)

GATE Previous Years' Questions

1. A nucleus having mass number 240 decays by a emission to the grond state of its daughter nucleus. The Q value of the process is 5.26 MeV. The energy (in MeV) of the a particle is: **[GATE 2005]**
(a) 5.26 (b) 5.17 (c) 5.13 (d) 5.09

2. The plot of log A_t is versus time t, where, A_t is activity, as shown in the figure, corresponds to decay



[GATE 2005]

(a) from only one kind of radioactive nuclei having same half-life.
(b) from only neutron activated nuclei
(c) from a mixture of radioactive nuclei having different half-lives.
(d) which is unphysical.

3. The evidence for the non-conservation of parity in β-decay has been obtained from the observation that the β-intensity **[GATE 2005]**

(a) anti-parallel to the nuclear spin directions is same as that along the nuclear spin direction

(b) anti-parallel to the nuclear spin direction is not the same as that along the nuclear spin direction

(c) shows a continuous distribution as a function of momentum

(d) is independent of the nuclear spin direction

4. Which one of the following disintegration series of the heavy elements will give ^{209}Bi as a stable nucleus?
(a) Thorium series (b) Neptunium series **[GATE 2006]**
(c) Uranium series (d) Actinium series

5. Consider Fermi theory of β-decay **[GATE 2006]**
(I) The number of final states of electrons corresponding to momenta between p and $(p + dp)$ is
(a) independent of p (b) proportion to pdp
(c) proportional to p^2dp (d) proportional to p^3dp
(II) The number of emitted electrons with momentum (p) and energy (E) in the allowed approximation is proportional to (E_0 is the total energy given up by the nucleus)
(a) $(E_0 - E)$ (b) $p(E_0 - E)$ (c) $p^2(E_0 - E)^2$ (d) $p(E_0 - E)^2$

6. Half-life of a radioisotope is 4×10^8 years. If these are 10^3 radioactive nuclei in a sample today, the number of such nuclei in the sample 4×10^9 years ago were. **[GATE 2007]**

(a) 128×10^3 (b) 256×10^3 (c) 512×10^3 (d) 1024×10^3

7. Fission fragments are generally radioactive as: **[GATE 2007]**
(a) they have excess in neutrons
(b) they have excess of protons
(c) they are products of radioactive nuclides
(d) their total kinetic energy is of the order of 200 MeV

Statement for Linked Answer Q.8 and Q.9

Consider the β-decay of a free neutron at rest in the laboratory.

8. Which of the following configurations of the decay products correspond to the largest energy of the anti-neutrino $\bar{\nu}$? (rest mass of electron, $m_e = 0.51\ MeV/c^2$, rest mass of proton $m_p = 938.27\ MeV/c^2$ and rest mass of neutron $m_n = 939.57\ MeV/c^2$) **[GATE 2008]**

(a) In the laboratory, proton is produced at rest
(b) In the laboratory, momenta of proton, electron and the anti-neutrino all have the same magnitude.
(c) In the laboratory, proton and electron fly-off with (nearly) equal and opposite momenta.
(d) In the laboratory, electron is produced at rest.

9. Using the result of above problem, answer the following. Which of the following represents approximately the maximum allowed energy of the anti-neutrino $\bar{\nu}$? **[GATE 2008]**

(a) 1.3 MeV (b) 0.8 MeV (c) 0.5 MeV (d) 2.0 MeV

10. The disintegration energy is defined to be the difference in the rest energy between the initial and final states. Consider the following process: **[GATE 2009]**

$$ {}^{240}_{94}Pu \rightarrow {}^{236}_{92}U + {}^{4}_{2}He $$

The emitted α particle has a kinetic energy 5.17 MeV. The value of the disintegration energy is

(a) 5.26 MeV (b) 5.17 MeV (c) 5.08 MeV (d) 2.59 MeV

11. In the β-decay of neutron, $n \rightarrow p + e^- + \bar{\nu}_e$, the anti-neutrino $\bar{\nu}_e$ escapes detection. Its existence is inferred from the measurement of **[GATE 2011]**

(a) energy distribution of electrons (b) angular distribution of electrons
(c) helicity distribution of electrons (d) forward-backward asymmetry of electrons

12. In the β-decay process, the transition $2^+ \rightarrow 3^+$, is **[GATE 2013]**

(a) allowed both by Fermi and Gamow-Teller selection rule
(b) allowed by Fermi and but not by Gamow-Teller selection rule
(c) not allowed by Fermi but allowed by Gamow-Teller selection rule
(d) not allowed both by Fermi and Gamow-Teller selection rule

13. A nucleus X undergoes a first forbidden β-decay to a nucleus Y. If the angular momentum (I) and parity (P), denoted by I^P as $\frac{7}{2}^-$ for X, which of the following is a possible I^P value for Y?

(a) $\frac{1}{2}^+$ (b) $\frac{1}{2}^-$ (c) $\frac{3}{2}^+$ (d) $\frac{3}{2}^-$ **[GATE 2014]**

ANSWER KEY

1. (b) **2. (c)** **3. (c)** **4. (b)** **5. (c)** **6. (d)** **7. (a)**
8. (c) **9. (a)** **10. (a)** **11. (a)** **12. (c)** **13. (c)**

CSIR-UGC-NET Previous Years' Questions

1. The ground state of ${}^{207}_{82}\text{Pb}$ nucleus has spin-parity $J^P = \frac{1^-}{2}$, while the first excited state has $J^P = \frac{5^-}{2}$. The electromagnetic radiation emitted when the nucleus makes a transition from the first excited state to the ground state are **[NET June 2012]**
(a) E2 and E3 (b) M2 and E3 (c) E2 and M3 (d) M2 and M3

2. A radioactive element X decays to Y, which in turn decays to a stable element Z. The decay constant from X to Y is λ_1, and that from Y to Z is λ_2. If, to begin with, there are only N_0 atoms of X, at **short** times ($t \ll 1/\lambda_1$ as well as $1/\lambda_2$) the number of atoms of Z will be **[NET June 2016]**
(a) $\frac{1}{2}\lambda_1\lambda_2 N_0 t^2$ (b) $\frac{\lambda_1\lambda_2}{2(\lambda_1+\lambda_2)}N_0 t$ (c) $(\lambda_1+\lambda_2)^2 N_0 t^2$ (d) $(\lambda_1+\lambda_2)N_0 t$

ANSWER KEY

1. (c) **2. (a)**

TIFR Previous Years' Questions

1. A detector is used to count the numbr of γ-rays emitted by a radioactive source. If the number of counts recorded in exactly 20 seconds is 10000, the error in the counting rate per second is **[TIFR 2010]**

(a) ± 5.0 (b) ± 22.4 (c) ± 44.7 (d) ± 220.0

2. A lead container contains 1 gm of a $^{60}_{27}Co$ radioactive source. It is known that a $^{60}_{27}Co$ nucleus emits a β-particle of energy 316 KeV followed by two γ-emissions of energy 1173 and 1333 KeV respectively. Which of the following experimental methods would be the best way to determine the lifetime of the $^{60}_{27}Co$ source?

(a) Measure the change in temperature of the source **[TIFR 2010]**

(b) Measure the weight of the source now and again after one year

(c) Measure the recoil momentum of the nucleus during β-emission

(d) Measure the number of γ-photons emitted by this source

3. An excited atomic electron undergoes a spontaneous transition, $3d_{3/2} \to 2p_{1/2}$ **[TIFR 2011]**
The interaction responsible for this transition must be of the type

(a) electric dipole (E1) OR magnetic quadrupole (M2)

(b) electric dipole (E1) OR magnetic dipole (M1)

(c) electric quadrupole (E2) OR magnetic quadrupole (M2)

(d) electric quadrupole (E2) OR magnetic dipole (M1)

4. A cloud Chamber of width 0.01 m is filled with pure nitrogen gas (N_2) at normal temperature and pressure. A beam of α–particles, when incident normally no the chamber, make tracks which are visible under strong illumination. Whenever an α–particle $\left({}^{4}_{2}\mathrm{He}\right)$ has a nuclear collision with a ${}^{14}_{7}\mathrm{N}$ nucleus, the track shows a distinct bend. The radius of a nucleus is given by $r = r_0 A^{1/3}$ where $r_0 = 1.217 \times 10^{-15}$ m and A is the atomic mass number. If the α particles move at non-relativistic speeds, and the total number of incident α particles is 10^7, the number of such distinct bends is approximately. **[TIFR 2011]**

(a) 100 (b) 200 (c) 300 (d) 400

(e) 500 (f) 600

5. Let E_N be the energy released when one mole of pure ^{235}U undergoes controlled fission, and E_C be the energy released when one mole of pure carbon undergoes complete combustion. The ratio E_N/E_C will have the order of magnitude **[TIFR 2013]**

(a) 10^4 (b) 10^8 (c) 10^9 (d) 10^6

6. A standard radioactive source is known to decay by emission of γ-rays. The source is provided to a student in a thick sealed capsule of unbreakable plastic and she is asked to find out the half-life. Which of the following would be the most useful advice to the student? **[TIFR 2014]**

(a) The half-life cannot be measured because the initial concentration of the source is not given.

(b) Mount the source in front of a gamma ray detector and count the number of photons detected in one hour.

(c) Measure the mass of the source at different times with an accurate balance having a least count of 1 mg. Plot these values on a curve and fit it with an exponential decay law.

(d) Mount the source in front of a gamma ray detector and count the number of photons detected in a specific time interval. Repeat this experiment at different times and note how the count changes.

7. An alpha particle of energy E is shot towards a gold nucleus $\left({}^{197}_{79}Au\right)$. At distances much larger than the nuclear size R_N, the dominant force is the Coulomb repulsion, but at distances comparable to the nuclear size the dominant force is the strong nuclear attraction. These combine to form a potential barrier of height V_C. If $E < V_C$, the probability that the alpha particle will fuse with the gold nucleus can be written (in terms of a dimensionless positive constant k) as **[TIFR 2014]**

(a) zero (b) $\dfrac{kE}{\sqrt{k^2E^2+V_C^2}}$ (c) $k\left(1-\dfrac{E}{V_C}\right)$ (d) $\exp\left(-\dfrac{kV_C}{E}\right)$

8. Which of the following radioactive decay chains is it possible to observe ? **[TIFR 2015]**

(a) ${}^{206}_{82}Pb \to {}^{202}_{80}Hg \to {}^{202}_{79}Au$ (b) ${}^{210}_{83}Bi \to {}^{210}_{84}Po \to {}^{206}_{82}Pb$

(c) ${}^{214}_{88}Ra \to {}^{210}_{86}Rn \to {}^{207}_{82}Pb$ (d) ${}^{206}_{82}Pb \to {}^{202}_{80}Hg \to {}^{202}_{79}Au$

9. In an experiment, ${}^{197}_{79}Au$ nuclei were bombarded with neutrons leading to formation of ${}^{198}_{79}Au$, which is unstable. The half-life of ${}^{198}_{79}Au$ was measured to be 2.25 days and it was found later that this measured half-life was an underestimate by 10%. The corresponding percentage error in the estimated population of ${}^{198}_{79}Au$ after 9 days is **[TIFR 2015]**

(a) 10% (b) 25% (c) 2.5% (d) 15%

ANSWER KEY

1. (b) **2. (d)** **3. (a)** **4. ( )** **5. (d)** **6. (d)** **7. (d)**

8. (b) **9. (b)**

JEST Previous Years' Questions

1. ^{238}U decays with a half life of 4.51×10^9 years, the decay series eventually ending at ^{206}Pb, which is stable. A rock smaple analysis shows that the ratio of the numbers of atoms of ^{206}Pb to ^{238}U is 0.0058. Assuming that all the ^{206}Pb has been produced by the decay of ^{238}U and that all other half lives in the chain are negligible, the age of rock sample is **[JEST 2013]**

   (a) 38×10^6 years (b) 48×10^6 years (c) 38×10^7 years (d) 48×10^7 years

2. In the mixture of isotopes normally found on the earth at the present time, $^{238}_{92}U$ has on aboundance of 99.3% and $^{235}_{92}U$ has an aboundance of 0.7%. The measured lifetimes of these isotopes are 6.52×10^9 years and 1.02×10^9 years respectively. Assuming that they were equally aboundant when the earth was formed, the estimated age of the earth, in years is **[JEST 2014]**

   (a) 6.0×10^9 (b) 1.0×10^9 (c) 6.0×10^8 (d) 1.0×10^8

3. The half-life of a radioactive nuclear source is 9 days. The fraction of nuclei which are left undercayed after 3 days is: **[JEST 2016]**

   (a) $\frac{7}{8}$ (b) $\frac{1}{3}$ (c) $\frac{5}{6}$ (d) $\frac{1}{2^{1/3}}$

ANSWER KEY

1. (a) **2. (a)** **3. (d)**

Other Examinations Previous Years' Questions

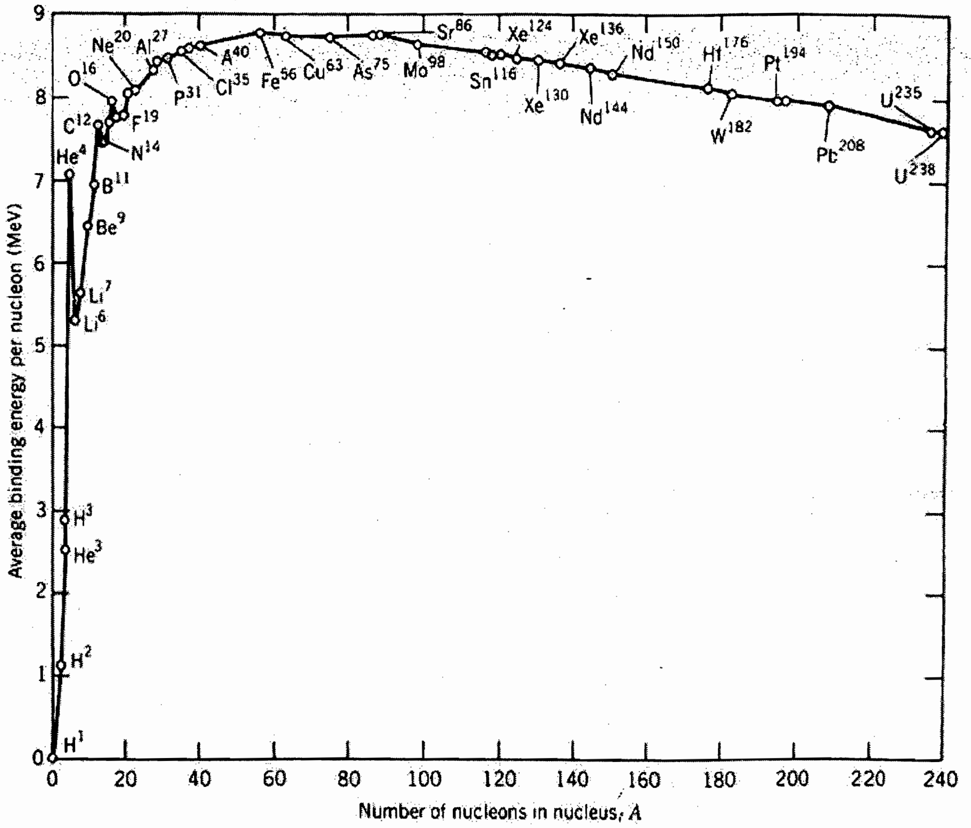
1. The activity of a radioactive sample is decreased to 75% of the initial value after 30 days. The half-life (in days) of the sample is approximately [You may use $\ln 3 \approx 1.1$, $\ln 4 \approx 1.4$]

(a) 38 (b) 45 (c) 59 (d) 69

2. $^{60}_{27}\text{Co}$ is a radioactive nucleus of half-life $2\ell n\, 2\times10^{8}$ s. The activity of 10 g of $^{60}_{27}\text{Co}$ in disintegration per second is:

(a) $\frac{1}{5}\times10^{10}$ (b) 5×10^{10} (c) $\frac{1}{5}\times10^{14}$ (d) 5×10^{14}

3. The variation of binding energy per nucleus with respect to the mass of nuclei is shown in the figure



Consider the following reactions:

(1) $^{238}_{92}U \rightarrow {}^{206}_{82}Pb + 10p + 22n$ (2) $^{238}_{92}U \rightarrow {}^{206}_{82}Pb + 8\,^{4}_{2}He + 6e^{-}$

Which one of the following statements is true for the given decay modes of $^{238}_{92}U$?

(a) both 1 and 2 are allowed (b) both 1 and 2 are forbidden

(c) 1 is forbidden and 2 is allowed (d) 1 is allowed and 2 is forbidden

4. A particular radioisotope has a half-life of 5 days. In 15 days the probability of decay in percentage will be

____

5. The radioactivity of a sample of Co^{55} decreases by 4% every hour. (The decay product is not radioactive). The half-life of Co^{55} is approximately.

(a) 1 hour (b) 1 day (c) 1 month (d) 1 year.

6. In order to determine the age of ancient wooden tools, radiocarbon dating method is used. This is done by measuring the fraction of radioactive isotope ^{14}C of carbon compared to the normal (non-radioactive) isotope ^{12}C in a sample. An old sample is found to contain 1/10 times the fraction of ^{14}C as compared to a fresh piece of wood. Given that the half-life of ^{14}C is 5570 years, the approximate age of the old sample is:
(a) 557 years (b) 12800 years (c) 18500 years (d) 55700 yearas.

7. A radioactive nucleus X decays to Y with a mean lifetime $\tau_2 = (= \tau_1/2)$. If N_0 nuclei of X (but no nuclei of Y) are present at t=0, how many nuclei of Y will be there when the number of X nuclei becomes $N_0/2$?
(a) $N_0/2e$ (b) $N_0/4$ (c) $N_0/2$ (d) N_0/e

8. Radon has a half-life of 3.8 days. If we start with 10.24 gm of radon, the amount of it which will be left after 38 days is
(a) 10^{-4}gm (b) 10^{-2} gm (c) 10^{-6} gm (d) 10^{-3} gm

9. If the age of the universe is 10^{10} years, the humans have existed for 10^6 years. If we take the age of the universe to be a day, how many seconds have the humans existed?
(a) 2.40 sec (b) 86.40 sec (c) 8.64 sec (d) 24 sec

ANSWER KEY

1. (d) **2. (d)** **3. (c)** **4. (87 to 88)** **5. (b)** **6. (c)** **7. (b)**
8. (d) **9. (c)**

GATE Previous Years' Questions

1. Typical energies released in a nuclear fission and a nuclear fusion reactions are respectively:
(a) 50 MeV and 1000 MeV (b) 200 MeV and 1000 MeV
(c) 1000 MeV and 50 MeV (d) 200 MeV and 10 MeV **[GATE 2002]**

2. An atomic bomb consisting of ${}^{235}U$ explodes and releases an energy of 10^{14} J. It is known that each ${}^{235}U$ which undergoes fission releases 3 neutrons and about 200 MeV of energy. Further, only 20% of the ${}^{235}U$ atoms in the bomb undergoes fission: **[GATE 2003]**

(i) the total number of neutrons released is about

(a) 4.7×10^{24} (b) 9.7×10^{24} (c) 1.9×10^{25} (d) 3.7×10^{25}

(ii) The mass of ${}^{235}U$ in the bomb is about

(a) 1.5 kg (b) 3.0 kg (c) 6.1 kg (d) 12 kg.

3. A thermal neutron having speed v impinges on a U^{235} nucleus. The reaction cross-section is proportional to **[GATE 2004]**

(a) v^{-1} (b) v (c) $v^{1/2}$ (d) $v^{-1/2}$

4. The threshold temperature above which the thermonuclear reaction: **[GATE 2005]**

$${}_2^3He+{}_2^3He\rightarrow{}_2^4He+2{}_1^1H+12.86\,\text{MeV}$$

can occur is (use $e^2/4\pi\varepsilon_0=1.44\times10^{-15}$ MeV-m)

(a) 1.28×10^{10} K (b) 1.28×10^{9} K (c) 1.28×10^{8} K (d) 1.28×10^{7} K

5. By capturing an electron ${}_{25}Mn_{29}^{54}$ transforms into ${}_{24}Cr_{30}^{54}$ releasing **[GATE 2006]**

(a) a neutrino (b) an anti neutrino (c) an α-particle (d) a positron

6. Fission fragments are generally radioactive as **[GATE 2007]**
(a) they have excess of neutrons (b) they have excess of protons
(c) they are products of radioactive nuclide (d) their total kinetic energy is of the order of 200 MeV.

7. A neutron scatters elastically from a heavy nucleus, the initial and final states of the neutron have the **[GATE 2007]**

(a) same energy (b) same energy and linear momentum

(c) same energy and angular momentum (d) same linear and angular momenta

Linked Answer Q.8 and Q.9

A 16 μA beam of alpha particles, having cross-sectional area 10^{-4} m^2, is incident on a Rhodium target of thickness 1 μm. This produces neutrons through the reaction,

$$\alpha+{}^{100}\text{Rh}\longrightarrow{}^{101}\text{Pd}+3\text{n}$$

8. The number of α particles hitting the target per second is **[GATE 2007]**

(a) 0.5×10^{14} (b) 1.0×10^{14} (c) 2.0×10^{20} (d) 4.0×10^{20}

9. The neutrons are observed at the rate of 1.806×10^{8} s^{-1}. If the density of rhodium is approximated as 10^4 kg m^{-3} the cross-section for the reaction (in barns) is **[GATE 2007]**
(a) 0.1 (b) 0.2 (c) 0.4 (d) 0.8

10. The energy released in the fission of 1 kg Uranium (approximately [in Joule]): **[GATE 2008]**
(a) 10^{14} (b) 10^{17} (c) 10^{16} (d) 10^{10}

11. The atomic masses of $^{152}_{63}Eu$, $^{152}_{62}Sm$, $^{1}_{1}H$ and neutron are 151.921749, 151.919756, 1.007825 and 1.008665 in atomic mass units (a.m.u.), respectively. Using the above information, the Q-value of the reaction $^{152}_{63}Eu + n \rightarrow {}^{152}_{62}Sm + p$ is_______ $\times 10^{-3}$ a.m.u. (upto three decimal places) **[GATE 2015]**

12. Consider the reaction $^{54}_{25}Mn + e^{-} \rightarrow {}^{54}_{24}Cr + X$. The particle X is **[GATE 2016]**

(a) γ (b) ν_e (c) n (d) π^0

ANSWER KEY

1. (d) 2. (b) 3. (a) 4. (a) 5. (a) 6. (a) 7. (a)

8. (a) 9. (a) 10. (a) 11. (2.833) 12. (b)

TIFR Previous Years' Questions

1. The binding energy per nucleon for ^{235}U is 7.6 MeV. The ^{235}U nucleus undergoes fission to produce two fragments, both having binding energy per nucleon 8.5 MeV. The energy released, in Joules, from the complete fission of 1 kg of ^{235}U is therefore, **[TIFR 2010]**

(a) 8000 (b) 10^{35} (c) 450 (d) 20000

(e) 8.7×10^{13} (f) 5.0×10^{8}

2. A fast-moving ^{14}N nucleus collides with an α particle at rest in the laboratory frame, giving rise to the reaction

$$^{14}N + \alpha \to {}^{17}O + p$$ **[TIFR 2011]**

Given the masses 14.00307 a.m.u. and 16.99913 a.m.u. for ^{14}N and ^{17}O nuclei respectively, and 4.00260 a.m.u. and 1.00783 a.m.u. for α and p respectively, the minimum kinetic energy in the laboratory frame of the ^{14}N nucleus must be

(a) 4.20 MeV (b) 1.20 MeV (c) 5.41 MeV (d) 1.55 MeV

3. In a nuclear reactor, Plutonium ($^{239}_{94}Pu$) is used as fuel, releasing energy by its fission into isotopes of Barium ($^{146}_{54}Ba$) and Strontium ($^{91}_{38}Sr$) through the reaction **[TIFR 2012]**

$$^{239}_{94}Pu + {}^{1}_{0}n \longrightarrow {}^{146}_{56}Ba + {}^{91}_{38}Sr + 3 \times {}^{1}_{0}n$$

The binding energy (*B.E.*) per nucleon of each of these nuclides is given in the table below:

Nuclide	$^{239}_{94}Pu$	$^{146}_{54}Ba$	$^{91}_{38}Sr$
B.E. per nucleon (MeV)	7.6	8.2	8.6

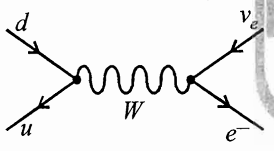
Using this information, one can estimate the number of such fission reactions per second in a 100 MW reactor as

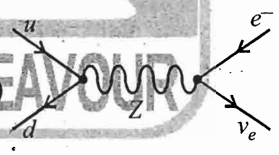
(a) 3.9×10^{18} (b) 7.8×10^{18} (c) 5.2×10^{19} (d) 5.2×10^{18}

(e) 8.9×10^{17}

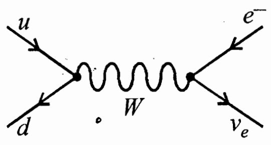
4. The process of electron capture $p + e^{-} \to n + \nu_e$ **[TIFR 2013]**

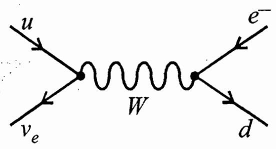
takes place at the quark level through the Feynman diagram

(a)


(b)


time

(c)


(d)


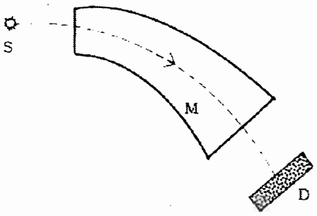
5. A gold foil, having $N(0)$ number of ^{197}Au nuclides per cm^2, is irradiated by a beam of thermal neutrons with a flux of F neutrons-cm^{-2}-s^{-1}. As a result, the nuclide ^{198}Au, with a half-life τ of several years, is produced by the reaction **[TIFR 2013]**

$$^{197}\text{Au} + n \to {}^{198}\text{Au} + \gamma$$

which has a cross section of σ cm^2. Assuming that the gold foil has 100% abundancy of ^{197}Au nuclides, the maximum number of ^{198}Au nuclides that can accumulate at any time in the foil is proportional to

(a) $\sigma\tau F\, N(0)$ (b) $\dfrac{\tau}{\sigma F} N(0)$ (c) $\dfrac{1}{\sigma\tau F} N(0)$ (d) $\dfrac{\sigma F}{\tau} N(0)$

6. In a beta decay experiment, an electromagnet M and a detector D are used to measure the energy of electrons (β^-), as shown in the figure. **[TIFR 2014]**



The detector D is capable of detecting either electons (β^-) or positrons (β^+). Now the β^- source is replaced with a β^+ source, and we would like to measure the energy of the positrons (β^+) using the same setup. Which of the following is correct?

(a) This can be done quite easily, if the polarity of current in the coils of the electromagnet is reversed.

(b) This is can done trivially, without changing anything, since the detector D can detect either β^- or β^+.

(c) There is no way to do this with the given set up, since β^+ will have to be converted into β^-, which is obviously not possible.

(d) This cannot be done since the magnet does not have a symmetric shape.

7. It is well-known that the energy of the Sun arises from the fusion of hydrogen nuclei (protons) inside the core of the Sun. This takes place through several mechanisms, each resulting in emission of energy.

Which of the following reactions is NOT possible during the proton fusion inside the Sun? **[TIFR 2014]**

(a) ${}^1_1H + {}^1_1H \longrightarrow {}^2_2He$ (b) ${}^2_1H + {}^1_1H \longrightarrow {}^3_2He$

(c) ${}^1_1H + {}^1_1H \longrightarrow {}^2_1H + e^+ + v_e$ (d) ${}^1_1H + {}^1_1H + {}^1_1H + {}^1_1H \longrightarrow {}^4_2He + 2e^+$

8. Consider a process in which atoms of Actinium-226 $({}^{226}_{89}Ac)$ get converted to atoms of Radium-226 $({}^{226}_{88}Ra)$ and the yield of energy is 0.64 MeV per atom. This occurs through **[TIFR 2016]**

(a) Both $p \to n + e^+ + v_e$ and $p + e^- \to n + v_e$

(b) Both $p \to n + e^+ + v_e$ and $n \to p + e^- + \bar{v}_e$

(c) Only $p \to n + e^+ + v_e$.

(d) Only $p + e^- \to n + v_e$

ANSWER KEY

1. (e) **2. (d)** **3. (a)** **4. (c)** **5. (a)** **6. (a)** **7. (d)**
8. (d)

JEST Previous Years' Questions

1. If hydrogen atom is bombarded by energetic electrons, it will emit **[JEST 2014]**

(a) K_α X-rays (b) β-rays (c) Neutrons (d) None of the above

ANSWER KEY

1. (d)

GATE Previous Years' Questions

1. Identify the reaction which has the same transition probability as $\pi^+ + p \rightarrow \pi^+ + p$ **[GATE 2000]**

   (a) $\pi^+ + n \rightarrow \pi^+ + n$ (b) $\pi^- + p \rightarrow \pi^0 + n$ (c) $\pi + n \rightarrow \pi + n$ (d) $\pi^0 + p \rightarrow \pi^+ + n$

2. Which of the following functions describes the nature of interaction potential $V(r)$ between two quarks inside a nucleon? (r is the distance between the quarks and a and b are positive constants of suitable dimensions). **[GATE 2001]**

   (a) $V(r) = \frac{a}{r} + br$ (b) $V(r) = -\frac{a}{r} + br$ (c) $V(r) = \frac{a}{r} - br$ (d) $V(r) = -\frac{a}{r} - br$

3. Which of the following reactions violates lepton number conservation? **[GATE 2001]**

   (a) $e^+ + e^- \rightarrow v_e + \overline{v}_e$ (b) $e^- + p \rightarrow v_e + n$ (c) $e^+ + n \rightarrow p + v_e$ (d) $\mu^- \rightarrow e^- + v_e + \overline{v}_e$

4. The cross-sections of the reactions **[GATE 2001]**

   $p + \pi^- \rightarrow \Sigma^- + K^+$ and $p^- + \pi^+ \rightarrow \Sigma^- + K^-$

   at a given energy are the same due to

   (a) baryon number conservation (b) time reversal invariance

   (c) charge conjugation (d) parity conservation

5. The Baryon number of proton, the lepton number of proton, the baryon number of electron and the lepton number of the electron are respectively **[GATE 2002]**

   (a) zero, zero, one and zero (b) one, one, zero and one

   (c) one, zero, zero and one (d) zero, one, one and zero

6. The nucleus of the atom ${}^9Be_4$ consists of **[GATE 2003]**

   (a) 13 up quarks and 13 down quarks (b) 13 up quarks and 14 down quarks

   (c) 14 up quarks and 13 down quarks (d) 14 up quarks and 14 down quarks

7. Which one of the following nuclear reactions is possible? **[GATE 2003]**

   (a) ${}^{14}N_7 \longrightarrow {}^{13}C_6 + \beta^+ + v_e$ (b) ${}^{13}N_7 \longrightarrow {}^{13}C_6 + \beta^+ + v_e$

   (c) ${}^{13}N_7 \longrightarrow {}^{13}C_6 + \beta^+$ (d) ${}^{13}N_7 \longrightarrow {}^{13}C_7 + \beta^+ + v_e$

8. Suppose that a neutron at rest in free space decays into a proton and an electron. This process would violate **[GATE 2003]**

   (a) conservation of charge (b) conservation of energy

   (c) conservation of linear momentum (d) conservation of angular momentum

9. A stationary particle in free space is oberved to spontaneously decay into two photons. This imples that

   (a) the particle carries electric charge **[GATE 2003]**

   (b) the spin of the particle must be greater than or equal to 2

   (c) the particle is a boson

   (d) the mass of the particle must be greater than or equal to the mass of the hydrogen atom

10. Choose the particle with zero Baryon number from the list given below **[GATE 2004]**

    (a) pion (b) neutron (c) proton (d) Δ^+

11. Which one of the following reactions is allowed? **[GATE 2004]**

(a) $p \rightarrow n + e^+$ (b) $p \rightarrow e^+ + \nu_e$ (c) $p \rightarrow \pi^+ + \gamma$ (d) $\bar{p} + n \rightarrow \pi^- + \pi^0$

12. Which of the following decay is forbidden? **[GATE 2005]**

(a) $\mu^- \longrightarrow e^- + \nu_\mu + \bar{\nu}_e$ (b) $\pi^+ \longrightarrow \mu^+ + \nu_\mu$

(c) $\pi^+ \longrightarrow e^+ + \nu_e$ (d) $\mu^- \longrightarrow e^+ + e^- + e^-$

13. The interaction potential between two quarks, separated by a distance r inside a nucleon, can be described by (a, b and β are positive cosntants) **[GATE 2006]**

(a) $ae^{-\beta r}$ (b) $\frac{a}{r} + br$ (c) $-\frac{a}{r} + br$ (d) $\frac{a}{r}$

14. Which one of the following nuclear processes is forbidden? **[GATE 2006]**

(a) $\bar{\nu} + p \rightarrow n + e^+$ (b) $\pi^- \rightarrow e^- + \nu_e + \pi^0$

(c) $\pi^- + p^- \rightarrow n + K^+ + K^-$ (d) $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

15. Weak nuclear forces act on **[GATE 2006]**
(a) both hadrons and leptons (b) hadrons only
(c) all particles (d) all charged particles

16. The strangeness quantum number is conserved in **[GATE 2007]**
(a) strong, weak and electromagnetic interactions
(b) weak and electromagnetic interactions
(c) strong and weak interactions
(d) strong and electromagnetic interactions.

17. A relativistic particle travels a length of 3×10^{-3} m in air before decaying. The decay process of the particle is dominated by: **[GATE 2007]**

(a) strong interactions (b) electromagnetic interactions.
(c) weak interactions (d) gravitational interactions

18. The strange baryon Σ^+ has the quark structure **[GATE 2007]**
(a) uds (b) uud (c) uus (d) $u\bar{s}$

19. According to the quark model, the K^+ meson is composed which of the following quarks
(a) uud (b) $u\bar{c}$ (c) $u\bar{s}$ (d) $s\bar{u}$ **[GATE 2008]**

20. Choose the correct statement from the following: **[GATE 2008]**

(a) The reaction $K^+K^- \longrightarrow p\,p^-$ can proceed irrespective of the kinetic energies of K^+ and K^-.

(b) The reaction $K^+K^- \longrightarrow pp^-$ is forbidden by the Baryon number conservation.

(c) The reaction $K^+K^- \longrightarrow 2\gamma$ is forbidden by strangeness conservation.

(d) The decay $K^0 \longrightarrow \pi^+\pi^-$ proceeds via weak interaction.

21. A neutral pi meson (π^0) has a rest-mass of approximately 140 MeV/c^2 and a lifetime of τ sec. A π^0 produced in the laboratory is found to decay after 1.25 τ sec into two photons. Which of the following sets represents a possible set of energies of the two photons as seen in the laboratory?
(a) 70 MeV and 70 MeV (b) 35 MeV and 100 MeV **[GATE 2008]**
(c) 75 MeV and 100 MeV (d) 25 MeV and 150 MeV

22. In the quark model which one of the following represents a proton? **[GATE 2009]**

(a) udd (b) uud (c) $u\bar{b}$ (d) $c\bar{c}$

23. Let $|n\rangle$ and $|p\rangle$ denote the isospin state with $I=\frac{1}{2}$, $I_3=\frac{1}{2}$ and $I=\frac{1}{2}$, $I_3=-\frac{1}{2}$ of a nuclear respectively.

Which one of the following two nuclear state has $I=0, I_3=0$? **[GATE 2009]**

(a) $\frac{1}{\sqrt{2}}(|nn\rangle-|pp\rangle)$ (b) $\frac{1}{\sqrt{2}}(|nn\rangle+|pp\rangle)$

(c) $\frac{1}{\sqrt{2}}(|np\rangle-|pn\rangle)$ (d) $\frac{1}{\sqrt{2}}(|np\rangle+|pn\rangle)$

24. The basic process underlying the neutron β-decay is **[GATE 2010]**

(a) $d\rightarrow u+e^-+\bar{v}_e$ (b) $d\rightarrow u+e^-$ (c) $s\rightarrow u+e^-+\bar{v}_e$ (d) $u\rightarrow d+e^-+\bar{v}_e$

25. Match the reactions on the left with the associated interactions on the right **[GATE 2010]**

(1) $\pi^+\rightarrow\mu^++v_\mu$ (i) strong

(2) $\pi^0\rightarrow\gamma+\gamma$ (ii) electromagnetic

(3) $\pi^0+n\rightarrow\pi^-+p$ (iii) weak

(a) 1-iii, 2-ii, 3-i (b) 1-i, 2-ii, 3-iii (c) 1-ii, 2-i, 3-iii (d) 1-iii, 2-i, 3-ii

26. The quark content of Σ^+, K^-, π^- and p is indicated. **[GATE 2010]**

$|\Sigma^+\rangle=|\text{uus}\rangle$; $|K^-\rangle=|\text{s}\bar{\text{u}}\rangle$; $|\pi^-\rangle=|\text{d}\bar{\text{u}}\rangle$; $|p\rangle=|\text{uud}\rangle$

In the process $\pi^-+p\rightarrow K^-+\Sigma^+$;

Considering strong interactions only, which of the following is true?

(a) the process is allowed because, $\Delta S=0$

(b) the process is allowed because, $\Delta I_3=0$

(c) the process is not allowed because, $\Delta S\neq 0$ and $\Delta I_3\neq 0$

(d) the process is not allowed because the baryon number is violated.

27. A π^0 meson at rest decays into two photons, which move along the x-axis. They are both detected simultaneously after a time, t = 10 s. In an inertial frame moving with a velocity v = 0.6 c in the direction of one of the photons, the time interval between the two detections is **[GATE 2010]**

(a) 15 s (b) 0 s (c) 10 s (d) 20 s

28. The isospin and the strangeness of Ω^- baryon are **[GATE 2011]**

(a) $1,-3$ (b) $0,-3$ (c) $1,3$ (d) $0,3$

29. Which one of the following sets corresponds to fundamental particles? **[GATE 2012]**

(a) proton, electron and neutron (b) proton, electron and photon

(c) electron, photon and neutrino (d) quark, electron and meson

30. Choose the CORRECT statement from the following **[GATE 2012]**

(a) Neutron interacts through electromagnetic interaction

(b) Electron does not interact through weak interaction

(c) Neutrino interacts through weak and electromagnetic interaction

(d) Quark interacts through strong interaction but not through weak interaction

31. The decay process $n \to p^+ + e^- + \bar{\nu}_e$ violates **[GATE 2013]**

(a) baryon number (b) lepton number (c) isospin (d) strangeness

32. The isospin (I) and baryon number (B) of the upquark is **[GATE 2013]**

(a) $I = 1, B = 1$ (b) $I = 1, B = 1/3$ (c) $I = 1/2, B = 1$ (d) $I = 1/2, B = 1/3$

33. Consider the decay of a pion into a muon and an anti-neutrino $\pi^- \to \mu^- + \bar{\nu}_\mu$ in the pion rest frame. $m_\pi = 139.6\,\text{MeV/c}^2$, $m_\mu = 105.7\,\text{MeV/c}^2, m_\nu \approx 0$. The energy (in MeV) of the emitted neutrino, to the nearest integer is _______. **[GATE 2013]**

34. Consider the process $\mu^+ + \mu^- \to \pi^+ + \pi^-$. The minimum kinetic energy of the muons (μ) in the centre of mass frame required to produce the pion (π) pairs at rest is ________ MeV. **[GATE 2014]**

(Given: $m_\mu = 105\,\text{MeV/c}^2, m_\pi = 140\,\text{MeV/c}^2$)

35. Which one of the following three-quark states (qqq), denoted by X, CANNOT be a possible baryon? The corresponding electric charge is indicated in the superscript **[GATE 2014]**

(a) X^{++} (b) X^{+} (c) X^{-} (d) X^{--}

36. Which one of the following high energy processes is allowed by conservation laws? **[GATE 2014]**

(a) $p + \bar{p} \to \Lambda^0 + \Lambda^0$ (b) $\pi^- + p \to \pi^0 + n$

(c) $n \to p + e^- + \nu_e$ (d) $\mu^+ \to e^- + \gamma$

37. If the half-life of an elementary particle moving with speed 0.9c in the laboratory frame is 5×10^{-8}s, then the proper half-life is _____$\times10^{-8}$ s. ($c = 3\times10^8$ m/s) **[GATE 2014]**

38. Which one of the following is a fermion? **[GATE 2014]**

(a) α particle (b) ${}_4Be^7$ nucleus (c) hydrogen atom (d) deuteron

39. The decay $\mu^+ \to e^+ + \gamma$ is forbidden, because it violates **[GATE 2015]**

(a) momentum and lepton number conservations
(b) baryon and lepton number conservations
(c) angular momentum conservation
(d) lepton number conservation

40. In the $SU(3)$ quark model, the triplet of mesons (π^+, π^0, π^-) has **[GATE 2016]**

(a) Isospin = 0, Strangeness = 0 (b) Isospin = 1, Strangeness = 0

(c) Isospin = ½, Strangeness = +1 (d) Isospin = ½, Strangeness = –1

ANSWER KEY

1. (b) 2. (b) 3. (c) 4. (c) 5. (c) 6. (b) 7. (b)
8. (d) 9. (c) 10. (a) 11. (d) 12. (d) 13. (c) 14. (b)
15. (c) 16. (d) 17. (c) 18. (c) 19. (c) 20. (d) 21. ( )
22. (b) 23. (c) 24. (a) 25. (a) 26. (c) 27. ( ) 28. (b)
29. (c) 30. (a) 31. (c) 32. (d) 33. (30) 34. (35) 35. (d)
36. (b) 37. (2.2) 38. (b) 39. (d) 40. (b)

CSIR-UGC-NET Previous Years' Questions

1. A beam of pions $\left(\pi^{+}\right)$ is incident on a proton target, giving rise to the process $\pi^{+}p \rightarrow n+\pi^{+}+\pi^{+}$

[NET June 2011]

(A) Assuming that the decay proceeds through strong interactions, the total isospin I and its third component I_3 for the decay products, are

(a) $I=\frac{3}{2}, I_3=\frac{3}{2}$ (b) $I=\frac{5}{2}, I_3=\frac{5}{2}$ (c) $I=\frac{5}{2}, I_3=\frac{3}{2}$ (d) $I=\frac{1}{2}, I_3=-\frac{1}{2}$

(B) Using isospin symmetry, the cross-section for the above process can be related to that of the process

(a) $\pi^{-}n \rightarrow p\pi^{-}\pi^{-}$ (b) $\pi^{-}\overline{p} \rightarrow \overline{n}\pi^{-}\pi^{-}$ (c) $\pi^{+}n \rightarrow p\pi^{+}\pi^{-}$ (d) $\pi^{+}\overline{p} \rightarrow n\pi^{+}\pi^{-}$

2. Consider the decay process $\tau^{-} \rightarrow \pi^{-}+V_{\tau}$ in the rest frame of the τ^{-}. The masses of τ^{-}, π^{-} and ν_{τ} are M_{τ}, M_{π} and zero repectively.

[NET June 2011]

(A) The energy of π^{-} is:

(a) $\frac{\left(M_{\tau}^{2}-M_{\pi}^{2}\right)c^{2}}{2M_{\tau}}$ (b) $\frac{\left(M_{\tau}^{2}+M_{\pi}^{2}\right)c^{2}}{2M_{\tau}}$ (c) $\left(M_{\tau}-M_{\pi}\right)c^{2}$ (d) $\sqrt{M_{\tau}M_{\pi}}\,c^{2}$

(B) The velocity is π^{-} is:

(a) $\frac{\left(M_{\tau}^{2}-M_{\pi}^{2}\right)c}{M_{\tau}^{2}+M_{\pi}^{2}}$ (b) $\frac{\left(M_{\tau}^{2}-M_{\pi}^{2}\right)c}{M_{\tau}^{2}-M_{\pi}^{2}}$ (c) $\frac{M_{\pi}c}{M_{\tau}}$ (d) $\frac{M_{\tau}c}{M_{\pi}}$

3. The dominant interactions underlying the following processes **[NET June 2012]**

A. $K^{-}+p \rightarrow \Sigma^{-}+\pi^{+}$ B. $\mu^{-}+\mu^{+} \rightarrow K^{-}+K^{+}$ C. $\Sigma^{+} \rightarrow p+\pi^{0}$ are

(a) A: strong, B: electromagnetic and C: weak (b) A: strong, B: weak and C: weak

(c) A: weak, B: electromagnetic and C: strong (d) A: weak, B: electromagnetic and C: weak

4. If a Higgs boson of mass m_H with a speed $\beta=\frac{v}{c}$ decays into a pair photons, then the invariant mass of the photon pair is

[NET June 2012]

[**Note:** The invariant mass of a system of two particles, with four-momenta p_1 and p_2 is $\left(p_1+p_2\right)^2$]

(a) βm_H (b) m_H (c) $m_H/\sqrt{1-\beta^2}$ (d) $\beta m_H/\sqrt{1-\beta^2}$

5. Consider the following particles: the proton p, the neutron n, the neutral pion π^0 and the delta resonance Δ^+. When ordered in terms of decreasing lifetime, the correct arrangement is as follows: **[NET Dec. 2012]**

(a) π^0, n, p, Δ^+ (b) p, n, Δ^+, π^0 (c) p, n, π^0, Δ^+ (d) Δ^+, n, π^0, p

6. The muon has mass 105 MeV/c^2 and mean lifetime 2.2 μs in its rest frame. The mean distance traversed by muon of energy 315 MeV/c^2 before decaying is approximately **[NET Dec. 2012]**

(a) 3×10^5 km (b) 2.2 cm (c) 6.6 μm (d) 1.98 km

7. A spin-1/2 particle A undergoes the decay $A \rightarrow B+C+D$ where it is known that B and C are also spin-1/2 particles. The complete set of allowed values of the spin of the particle D is **[NET June 2013]**

(a) $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \ldots\ldots$ (b) 0, 1 (c) $\frac{1}{2}$ only (d) $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ldots$

8. Muons are produced through the annihilation of particle a and its antiparticle, namely the process $a + \bar{a} \to \mu^+ + \mu^-$. A muon has a rest mass of 105 MeV/c^2 and its proper life time is $2\,\mu$s. If the center of mass energy of the collision is 2.1 GeV in the laboratory frame that coincides with the center-of-mass frame, then the fraction of muons that will decay before they reach a detector placed 6km away from the interaction point is **[NET June 2013]**

(a) e^{-1} (b) $1 - e^{-1}$ (c) $1 - e^{-2}$ (d) e^{-10}

9. Consider the following ratios of the partial decay widths $R_1 = \frac{\Gamma(\rho^+ \to \pi^+ + \pi^0)}{\Gamma(\rho^- \to \pi^- + \pi^0)}$ and $R_2 = \frac{\Gamma(\Delta^{++} \to \pi^+ + p)}{\Gamma(\Delta^- \to \pi^- + n)}$. If the effects of electromagnetic and weak interactions are neglected, then R_1 and R_2 are, respectively, **[NET Dec. 2013]**

(a) 1 and $\sqrt{2}$ (b) 1 and 2 (c) 2 and 1 (d) 1 and 1

10. The recently-discovered Higgs boson at the LHC experiment has a decay mode into a photon and a Z boson. If the rest masses of the Higgs and Z boson are 125 GeV/c^2 and 90 GeV/c^2 respectively, and the decaying Higgs particle is at rest, the energy of the photon will approximately be **[NET June 2014]**

(a) $35\sqrt{3}$ GeV (b) 35 GeV (c) 30 GeV (d) 15 GeV

11. In a classical model, a scalar (spin-0) meson consists of a quark and an antiquark bound by a potential $V(r) = ar + \frac{b}{r}$, where $a = 200$ MeV fm^{-1} and $b = 100$ MeV fm. If the masses of the quark and antiquark are negligible, the mass of the meson can be estimated as approximately **[NET June 2014]**

(a) 141 MeV/c^2 (b) 283 MeV/c^2 (c) 353 MeV/c^2 (d) 425 MeV/c^2

12. Consider the four processes **[NET Dec. 2014]**

(i) $p^+ \to n + e^+ + \nu_e$ (ii) $\Lambda^0 \to p^+ + e^+ + \nu_e$

(iii) $\pi^+ \to e^+ + \nu_e$ (iv) $\pi^0 \to \gamma + \gamma$

Which of the above is/are forbidden for free particles?

(a) only (ii) (b) (ii) and (iv) (c) (i) and (iv) (d) (i) and (ii)

13. In deep inelastic scattering electrons are scattered off protons to determine if a proton has any internal structure. The energy of the electron for this must be at least **[NET Dec. 2014]**

(a) 1.25×10^9 eV (b) 1.25×10^{12} eV (c) 1.25×10^6 eV (d) 1.25×10^8 eV

14. The reaction ${}^2_1\mathrm{D} + {}^2_1\mathrm{D} \to {}^4_2\mathrm{He} + \pi^0$ cannot proceed via strong interactions because it violates the conservations of **[NET June 2015]**

(a) angular momentum (b) electric charge (c) baryon number (d) isospin

15. The charm quark is assigned a charm quantum number $C = 1$. How should the Gellmann-Nishijima formula for electric charge be modified for four flavours of quarks? **[NET June 2015]**

(a) $I_3 + \frac{1}{2}(B - S - C)$ (b) $I_3 + \frac{1}{2}(B - S + C)$

(c) $I_3 + \frac{1}{2}(B + S - C)$ (d) $I_3 + \frac{1}{2}(B + S + C)$

16. Consider the following processes involving free particles **[NET Dec. 2015]**

(i) $\bar{n} \to \bar{p} + e^{+} + \bar{\nu}_e$ (ii) $\bar{p} + n \to \pi^{-}$

(iii) $p + n \to \pi^{+} + \pi^{0} + \pi^{0}$ (iv) $p + \bar{\nu}_e \to n + e^{-}$

Which of the following statements is true?

(a) Process (i) obeys all conservation laws

(b) Process (ii) conserves baryon number, but violates energy-momentum conservation

(c) Process (iii) is not allowed by strong interactions, but is allowed by weak interactions

(d) Process (iv) conserves baryon number, but violates lepton number conservation

17. The decay constants f_p of the heavy pseudoscalar mesons, in the heavy quark limit, are related to their masses m_p by the relation $f_p = \frac{a}{\sqrt{m_p}}$, where a is an empirical parameter to be determined. The values $m_p = 6400 \pm 160$ MeV and $f_p = 180 \pm 15$ MeV correspond to uncorrelated measurement of a meson. The error on the estimate of a is **[NET June 2016]**

(a) 175 $(\text{MeV})^{3/2}$ (b) 900 $(\text{MeV})^{3/2}$ (c) 1200 $(\text{MeV})^{3/2}$ (d) 2400 $(\text{MeV})^{3/2}$

18. In the large hadron collider (LHC), two equal energy proton beams traverse in opposite directions along a circular path of length 27 km. If the total center of mass energy of a proton-proton pair is 14 TeV, which of the following is the best approximation for the proper time taken by a proton to traverse the entire path?

(a) 12 ns (b) 1.2 μs (c) 1.2 ns (d) 0.12 μs **[NET June 2016]**

ANSWER KEY

1_A(c) **1_B(b)** **2_A(b)** **2_B(a)** **3. (a)** **4. (b)** **5. (c)**

6. (d) **7. (d)** **8. (b)** **9. (d)** **10. (c)** **11. (b)** **12. (d)**

13. (a) **14. (d)** **15. (d)** **16. (b)** **17. (c)** **18. (a)**

TIFR Previous Years' Questions

1. A spin-$\frac{1}{2}$ particle A decays to two other particles B and C. If B and C are of spin-$\frac{1}{2}$ and spin-1 respectively, then a complete list of the possible values of the orbital angular momentum of the final state (i.e. B + C) is **[TIFR 2013]**

(a) 0, 1 (b) $\frac{1}{2}, \frac{3}{2}$ (c) 0, 1, 2 (d) 0, ±1

2. The interaction strength of the recently-discovered Higgs boson (mass approximately 125 GeV/c^2) with any other elementary particle is proportional to the mass of that particle. Which of the following decay processes will have the greatest probability? **[TIFR 2014]**

(a) Higgs boson decaying to a top quark + a top anti-quark
(b) Higgs boson decaying to a bottom quark + a bottom anti-quark
(c) Higgs boson decaying to an electron and a positron
(d) Higgs boson decaying to a neutrino-antineutrino pair.

3. Cosmic ray muons generated at the top of the Earth's atmosphere decay according to the radioactive decay law **[TIFR 2014]**

$$N(t) = N(0)\exp\left(-\frac{0.693t}{T_{1/2}}\right)$$

where $N(t)$ is the number of muons at time t, and $T_{1/2} = 1.52\ \mu s$ is the proper half-life of the muon. Immediately after generation, most of these muons shoot down towards the Eearth's surface. Some of these muons decay on the way, but their interaction with the atmosphere is negligible.
An observer on the top of a mountain of height 2.0 km above mean sea level detects muons with the speed 0.98c over a period of time and counts 1000 muons. The number of muons of the same speed detected by an observer at mean sea level in the same period of time would be

(a) 232 (b) 539 (c) 839 (d) 983

4. A spin-2 nucleus absorbs a spin-½ electron and is then observed to decay to a stable nucleus in two stages, recoiling against an emitted invisible particle in the first stage and against an emitted spin-1 photons in the second stage. If the stable nucleus is spinless, then the set of all possible spin values of the invisible particle is **[TIFR 2014]**

(a) {1/2, 5/2} (b) {3/2, 7/2} (c) {1/2, 3/2, 5/2} (d) {1/2, 3/2, 5/2, 7/2}

5. Consider the following reaction involving elementary particles: **[TIFR 2015]**

(A) $\pi^- + p \rightarrow K^- + \Sigma^+$

(B) $K^- + p \rightarrow K^- + \rho^+$

Which of the following statements is true for strong interactions ?
(a) (A) and (B) are both forbidden (b) (B) is allowed but (A) is forbidden
(c) (A) is allowed but (B) is forbidden (d) (A) and (B) are both allowed

6. In a fixed target experiment, a proton of total energy 200 GeV is bombarded on a proton at rest and produces a nucleus ${}^Z_AN$ and its anti-nucleus ${}^Z_A\bar{N}$

$$p + p \rightarrow {}^Z_AN + {}^Z_A\bar{N} + p + p$$

The heaviest nucleus ${}^Z_AN$ that can be created has atomic mass number $A =$

(a) 15 (b) 9 (c) 5 (d) 4

7. Consider the hyperon decay (1) $\Lambda \to n + \pi^0$ followed by (2) $\pi^0 \to \gamma\gamma$. If the isospin component, baryon number and strangeness quantum numbers are denoted by I_z, B and S respectively, then which of the following statements is completely correct? **[TIFR 2016]**

(a) In (1) I_z is not conserved, B is conserved, S is not conserved;
In (2) I_z is conserved, B is conserved, S is conserved.

(b) In (1) I_z is conserved, B is not conserved, S is not conserved;
In (2) I_z is conserved, B is conserved, S is conserved.

(c) In (1) I_z is not conserved, B is conserved, S is not conserved;
In (2) I_z is not conserved, B is conserved, S is conserved.

(d) In (1) I_z is not conserved, B is conserved, S is conserved;
In (2) I_z is conserved, B is conserved, S is conserved.

ANSWER KEY

1. (c) **2. (b)** **3. (b)** **4. (d)** **5. (a)** **6. (b)** **7. (a)**

JEST Previous Years' Questions

1. A K-meson (with a rest mass of 494 MeV) at rest decays into a muon (with a rest mass of 106 MeV) and a neutrino. The energy of the neutrino, which can be taken as massless, approximately

   (a) 120 MeV (b) 236 MeV (c) 300 MeV (d) 388 MeV **[JEST 2013]**

2. The reaction $e^+ + e^- \rightarrow \gamma$ is forbidden because, **[JEST 2015]**

   (a) lepton number is not conserved (b) linear mementum is not conserved

   (c) angular momentum is not conserved (d) charge is not conserved

ANSWER KEY

1. (b) **2. (c)**

Other Examinations Previous Years' Questions

1. The ratio F_C/F_G of the electostatic Coulomb force F_C to the gravitational force F_G between the proton and the electron in the first Bohr orbit (or radius rB) of a hydrogen atom is closest to the following value.

   (a) 2×10^{39} (b) 2×10^{49} (c) $2\times10^{42} r_B$ (d) $-2\times10^{35}/r_B$

2. In the large Hadron Collider (LHC) in CERN, protons will be accelerated to an energy of 7 TeV(tetraelectron volt), i.e., $7\times10^{12}\ eV$. (As a matter of fact, two proton beams, each with 7 TeV will collide so that the energy of collision in the centre of mass frame is 14 TeV). The speed v of a proton of 7 TeV energy is in the range

   (a) $0.999999000c < v < 0.999999999c$ (b) $0.999900c < v < 0.99999c$
   (c) $0.9900c < v < 0.9999c$ (d) $0.90c < v < 0.99c$

3. An ^{57}Fe nucleus decays from an excited state to the ground state by emitting a gamma ray. The energy of the emitted photon is 14.4 eV when the nucleus is held fixed. If the nucleus is free to recoil, then the energy of the photon emitted will be

   (a) lower by approximately 2 keV (b) lower by approximately 2 meV
   (c) higher by approximately 2 eV (d) higher by approximately 20 eV
   (e) lower by approximately 20 meV

4. Muons of kinetic energy E are produced in collision with a target in a laboratory. The mass of a muon is 106 MeV/c² and its half-life is 1.4×10^{-6} s in its rest frame. What should be the minimum value of E if more than half the muons created at the target are to reach a detector 840 m away?

   (a) 106 MeV (b) 212 MeV (c) 130 MeV (d) 189 MeV (e) 162 MeV

5. A free neutron is unstable and decays into a proton, an electron and an anti-neutrino : $n \to p + e^- + \bar{v}_e$. The rest masses of these particles may be taken to be $m_n = 939.6$ MeV, $m_p = 938.3$ MeV, $m_e = 0.51$ MeV and $m_{\bar{v}} = 0$, so that the change in the total rest mass in the decay is 0.79 MeV. If, in a particular decay, the neutron as well as the proton (created due to the decay) are at rest, then the energy of the anti-neutrino is

   (a) 1.05 MeV (b) 0.39 MeV (c) 0.78 MeV (d) 0.55 MeV (e) 0.40 MeV

ANSWER KEY

1. (a) **2. (b)** **3. (b)** **4. (c)** **5. (d)**

GATE Previous Years' Questions

1. An admissible potential between the proton and the neutron in a deutron is **[GATE 2000]**
(a) Coulomb (b) Harmonic oscillator
(c) Finite square potential (d) Infinite square well.

2. Nuclear forces are **[GATE 2002]**
(a) spin dependent and have no non-central part
(b) spin dependent and have a non-central part
(c) spin independent and have no non-central part
(d) spin independent and have a non-central part

3. With reference to nuclear forces which of the following statements is NOT true? **[GATE 2005]**
The nuclear forces are
(a) short range (b) charge independent (c) velocity dependent (d) spin independent

4. To explain the observed magnetic moment of deutron ($0.0857\,\mu_N$), its ground state wavefunction is taken to be an admixture of S and D states, the expectation values of the z-component of the magnetic momenta in pure S and pure D-states are $0.8797\,\mu_N$ and $0.3101\,\mu_N$ respectively. The contribution of the D-state to the mixed ground state is approximately **[GATE 2006]**
(a) 40% (b) 4% (c) 0.4% (d) 0.04%

5. In the deuterium + tritium ($d+t$) fusion more energy is released as compared to deuterium + deuterium ($d+d$) fusion because **[GATE 2007]**
(a) tritium is radioactive
(b) more nucleus participate in fusion
(c) the coulomb parrier is lower for the $d+t$ system then $(d+d)$ system
(d) the reaction product ^{4}He is more tightly bound

6. A heavy nucleus is found to contain more neutrons than protons. This fact is related to which one of the following statement **[GATE 2008]**
(a) the nuclear force between neutrons is stronger than that between protons.
(b) the nuclear force between protons is of a shorter range than those between neutrons, so that a smaller number of protons are held together by the nuclear force.
(c) protons are unstable, to their number in a nucleus diminishes.
(d) it costs more energy to add a proton to a (heavy) nucleus, than a neutron because of the Coulomb repulsion between protons.

7. Pick the wrong statement **[GATE 2009]**
(a) the nuclear force is independent of electric charge
(b) the Yukawa potential is proportinoal to $r^{-1}\exp\left(\frac{m}{h}r\right)$. Where, r is the seperation between two nucleons.
(c) The range of nuclear force is of the order of $10^{-15}-10^{-14}$ m
(d) the nucleons interact among each other by the exchange of mesons.

8. The ground state wavefunction of deuteron is in a superposition of s and d states. Which of the following is NOT true as a consequence? **[GATE 2010]**
(a) It has a non-zero quadruple moment
(b) The neutron-proton potential is non-central
(c) The orbital wavefunction is not spherically symmetric
(d) The Hamiltonian does not conserve the total angular momentum

9. To detect trace amounts of a gaseous species in a mixture of gases, the preferred probing tools is

(a) ionization spectroscopy with X-rays (b) NMR spectroscopy **[GATE 2010]**
(c) ESR spectroscopy (d) laser spectroscopy

10. A neutron passing through a detector is detected because of **[GATE 2011]**
(a) the ionization it produces
(b) the scintillation light it produces
(c) the electron-hole pair it produces
(d) the secondary particles produced in a nuclear reaction in the detector medium.

11. In case of a Geiger-Muller (GM) counter, which one of the following statements is **CORRECT**?

(a) Multiplication factor of the detector is of the order of 10^{10}. **[GATE 2012]**
(b) Type of the particles detected can be identified.
(c) Energy of the particles detected can be distinguished.
(d) Operating voltage of the detector is few tens of Volts.

12. Deuteron has only one bound state with spin parity 1^+, isospin 0 and electric quadrupole moment 0.286 efm^2. These data suggest that the nuclear forces are having **[GATE 2012]**
(a) only spin and isospin dependence (b) no spin dependence and no tensor components
(c) spin dependence but no tensor components (d) spin dependence along with tensor components

13. Consider the scattering of neutrons by protons at very low energy due to a nuclear potential of range r_0. Given that, **[GATE 2013]**

$$\cot\left(kr_0+\delta\right)\approx-\frac{\gamma}{k}$$

where δ is the phase shift, k the wave number and $(-\gamma)$ the logarithmic derivative of the deuteron ground state wave function, the phase shift is

(a) $\delta\approx-\frac{k}{\gamma}-kr_0$ (b) $\delta\approx-\frac{\gamma}{k}-kr_0$ (c) $\delta\approx\frac{\pi}{2}-kr_0$ (d) $\delta\approx-\frac{\pi}{2}-kr_0$

14. The value of the magnetic field required to maintain non-relativistic protons of energy 1 MeV in a circular orbit of radius 100 mm is _______ Tesla. **[GATE 2014]**

(Given: $m_p=1.67\times10^{-27}$ kg, $e=1.6\times10^{-19}$ C

15. Which of the following statements is **NOT** correct? **[GATE 2016]**
(a) A deuteron can be disintegrated by irradiating it with gamma rays of energy 4 MeV.
(b) A deuteron has no excited states
(c) A deuteron has no electric quadrupole moment
(d) The $^1S_0$ state of deuteron cannot be formed

16. An alpha particle is accelerated in a cyclotron. It leaves the cyclotron with a kinetic energy of 16 MeV. The potential difference between the D electrodes is 50 kilovolts. The number of revolutions the alpha particle makes in its spiral path before it leaves the cyclotron is _______ **[GATE 2016]**

ANSWER KEY

1. (d) **2. (b)** **3. (d)** **4. (b)** **5. (a)** **6. (d)** **7. (c)**
8. (d) **9. (c)** **10. (d)** **11. (a)** **12. (d)** **13. (a)** **14. (1.44)**
15. (c) **16. (80)**

CSIR-UGC-NET Previous Years' Questions

1. An atom of mass M can be excited to a state of mass $M + \Delta$ by photon capture. The frequency of a photon which can cause this transition is: **[NET Dec. 2011]**

(a) $\Delta c^2 / 2h$ (b) $\Delta c^2 / h$ (c) $\Delta^2 c^2 / 2Mh$ (d) $\Delta(\Delta + 2M)c^2 / 2Mh$

ANSWER KEY

1. (d)

CAREER ENDEAVOUR

TIFR Previous Years' Questions

1. An atom is capable of existing in two states: a ground state of mass M and an excited state of mass M + Δ. If the transtion from the ground state to the excited state proceeds by the absorption of a photon, the photon frequency in the laboratory frame (where the atom is initially at rest) is **[TIFR 2010]**

(a) $\frac{\Delta c^2}{h}$ (b) $\frac{\Delta c^2}{h}\left(1+\frac{\Delta}{2M}\right)$ (c) $\frac{Mc^2}{h}$ (d) $\frac{\Delta c^2}{h}\left(1-\frac{\Delta}{2M}\right)$

(e) $\frac{Mc^2}{h}\left(1+\frac{\Delta}{2M}\right)$ (f) $\frac{Mc^2}{h}\left(1-\frac{\Delta}{2M}\right)$

2. Given that the ionization energies of Hydrogen (^1H) and Lithium (^3Li) are 13.6 eV and 5.39 eV, respectively, the effective nuclear charge experienced by the valence electron of a ^3Li atom may be estimated in terms of the proton charge e as **[TIFR 2011]**

(a) 0.63 e (b) 1.26 e (c) 1.59 e (d) 3.00 e

ANSWER KEY

1. (b) **2. (b)**

JEST Previous Years' Questions

1. The binding energy of the k-shell electron in a Uranium atom (Z = 92, A = 238) will be modified due to (i) screening caused by other electrons and (ii) the finite extent of the nucleus as follows: **[JEST 2013]**

(a) increases due to (i), remains unchanged due to (ii).
(b) decreases due to (i), decreases due to (ii).
(c) increases due to (i), increases due to (ii).
(d) decreases due to (i), remains unchanged due to (ii).

2. A hydrogen atom in its ground state is collided with an electron of kinetic energy 13.377 eV. The maximum factor by which the radius of the atom would increase is **[JEST 2014]**

(a) 7 (b) 8 (c) 49 (d) 64

3. If the Rydberg constant of an atom of finite nuclear mass is αR_∞, where R_∞ is the Rydberg constant corresponding to an infinite nuclear mass, the ratio of the electronic to nuclear mass of the atom is:

(a) $\frac{(1-\alpha)}{\alpha}$ (b) $\frac{(\alpha-1)}{\alpha}$ (c) $(1-\alpha)$ (d) $\frac{1}{\alpha}$ **[JEST 2016]**

ANSWER KEY

1. (b) **2. (d)** **3. (a)**

Other Examinations Previous Years' Questions

1. A proton moves in a circular orbit in a plane which is perpendicular to a uniform magnetic field of strength B. When the kinetic energy of the proton changes from $0.25\ m_pc^2$ to m_pc^2 (where m_p is the rest mass of the proton) the freqency of rotation changes from ω_1 to ω_2. The ratio ω_1/ω_2 is

(a) 1 (b) 8 (c) 8/5 (d) $1/\sqrt{8}$ (e) $\sqrt{8}$

ANSWER KEY

1. (c)